About Wind Turbine Power Estimation and Measurement

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Abstract: *Power of wind turbine offers various values if there are calculated with different formulas. The reason consists from different models considered and the influence of various factors. The article tries to compare and conclude about the relevance of these factors and influences on the true values. Also there are given characteristic curves measured on a wind power plant designed from Politehnica University of Timisoara.*

Keywords: *Wind turbines, parameters and characteristic curves, analytical and numerical values for power, measurements data.*

1. Introduction

In the subtle play between the energy available in the atmospheric ocean and the harvesting possibilities of it by industrial applications and human using are a lot of enhancing and perturbing factors and parameters.
This situation makes that from the design power to the developed power of a realized wind power plant to occur differences. The sources of these discrepancies are in the wind characteristics and the wind aggregate peculiarities. The wind is an air flow with rapid change in value and direction in turbulent regime and with gusts. Wind's energy is harnessed by wind power plants in which the fluid mechanic energy is transformed in stereo-mechanic energy and then in electric energy through a chain of machines and conversion equipments.
In the following chapters there are given formulas and values for aerodynamic, stereo-mechanic and electric powers associated with different physical models and measurement. The efficiency of wind turbines is smaller than that of the electrical machines and convertors and for this reason it is important to investigate them.

2. Ciugud wind aggregate plant power

The wind power plant Ciugud (Alba County) was designed for a stereo-mechanical power of 5 kW. The wind turbine Ciugud is a horizontal axial rapid wind turbine (HAWT) type. The measured parameters of the wind aggregate operation through a SCADA system are given in [1]. The kinetic power of the wind, "P_1", and the maximum theoretical power of the wind turbines for different aerodynamic models, "P_2" and "P_3", and various diameters of the runners in function of air density, "ρ", swept area, "S", and mean wind velocity, "v", from [2], [3], [8] are given with the formulas:

$$P_1 = \frac{1}{2} \cdot \rho \cdot S \cdot v^3 \tag{1}$$

$$P_2 = \frac{16}{27} \cdot \rho \cdot S \cdot v^3 \tag{2}$$

$$P_3 = \frac{8}{27} \cdot \rho \cdot S \cdot v^3 \tag{3}$$

The results for air density ρ = 1.2 kg / m^3, wind mean velocity v_0 = 8.5 m/s, diameters D and number of blades z are given in Table 1.

TABLE 1

z	D	S	P_1	P_2	P_3	Observations
-	[m]	[m^2]	W	W	W	runner
5	5	19.63495409	7234.989708	8574.802617	4287,401308	Ciugud - A
3	7	38.4845	14180.57614	16806.60876	8403.304378	Ciugud - B

The maximum aerodynamic power, "P_4", from statistical data in function of the similitude coefficient, "K_P", from [2]:

$$P_4 = K_P \cdot D^2 \cdot v^3 \tag{4}$$

Numerical results are calculated in Table 2:

TABLE 2

D	K_P	P_4	Observations
[m]	-	[W]	-
5	0.15	2302.96875	Slow machine
	0.2	3070.625	Rapid machine
7	0.15	4513.81875	Slow machine
	0.2	6018.425	Rapid machine

The maximum power, "P_5", extracted from the wind turbine in function of the runner radius, "R", and power coefficient, "C_P", with the formula from [3] is:

$$P_5 = C_P \cdot \frac{\rho}{2} \cdot \pi \cdot R^2 \cdot v^3 \tag{5}$$

Power coefficients are given in [1] and [3] and the results are offered in Table 3.

TABLE 3

R	C_P	P_5	Observations about C_P
[m]	-	[W]	The source
2.5	0.45	3255.74536	[4] Fig. 5.14
	0.37699	2727.518769	[1] graphic, page 427
	0.3	2170.496912	[3] page115
	0.47	3400.445162	[3] page115
3.5	0.45	6381.260921	[4] Fig. 5.14
	0.37699	5345.936788	[3] page115
	0.3	4254.173947	[3] page 115
3.5	0.47	6664.872518	[3] page115
2.5	16/27 x0.47	2015.078611	[3] page 45
3.5	16/27x0.47	3949.554077	[3] page 45

Analytic aerodynamic power of the wind turbine runner neglecting the finite extension of the blades is:

$$P_6 = \frac{z\cdot\omega\cdot\rho}{2}\cdot\int_{r_o}^{R_o}\left\{C_L\left[i(r)\right]\cdot v - C_D\left[i(r)\right]\cdot\omega\cdot r\right\}\cdot\sqrt{\omega^2\cdot r^2+v^2}\cdot b(r)\cdot r\,dr \qquad (6)$$

where:

$\omega = \frac{\pi\cdot n}{30}$ - angular velocity of the runner

b (r) - blade chord,

R_0 , r_0 , - maximum and minimum radius of the blades,

C_L , C_D , - lift and drag coefficients of the airfoils.

The rated speed of rotation for Ciugud wind turbine is:

$$n_0 = 120 \quad rev/\min$$

the blade chord in function of radius for Ciugud - A:

$$b(r) = 0.48333 - 0.1111\cdot r \qquad (7)$$

and for Ciugud – B:

$$b(r) = 0.95 - 0.17\cdot r \qquad (8)$$

The lift and drag coefficients of an airfoil depends from airfoil's geometry and a lot of other parameters. Using for the blade airfoils the same family namely NACA 4415 (d/l = 15 %) in tip (outside) zone, NACA 4418 (d/l = 18 %) in middle zone and NACA 4421 (d/l = 21 %) in hub zone in an interval for angles of attack between i = 0^0 ….. 15^0, the coefficients are independent of the maximum thickness "d/l" and equal with:

$$C_L\left[i(r)\right] = C_L(i) = 0.0685\cdot i + 0.302 \qquad (9)$$

$$C_D\left[i(r)\right] = C_D(i) = 0.0002857143\cdot i^2 + 0.0028282571\cdot i + 0.01414286 \qquad (10)$$

These relations were obtained from the mean experimental characteristics of the airfoils [7]. Approximate analytic values of the power are obtained from relation (6) considering constant values along the blade and taking into account the values of the centre of the blade with the radius r_x. The data from [6] for Ciugud - B are in Table 4:

TABLE 4

		r_0	r_x	R_0
r	**[m]**	1.5	2.5	3,5
b	**[m]**	0.695	0.525	0.355
i	**[^{0}]**	8.93	7.54	6.65

Neglecting the resistant term from the integral rel. (6) because:

$$C_D\left[i(r)\right]\cdot\omega_0\cdot r \ll C_L\left[i(r)\right]\cdot v_0 \qquad (11)$$

With these approximations rel. (6) becomes:

$$P_6 = \frac{z \cdot \rho \cdot \omega_0}{2} \cdot C_L(i_x) \cdot v_0 \cdot b_x \cdot \int_{r_0}^{R_0} r \cdot \sqrt{\omega_0^2 \cdot r^2 + v_0^2}\ dr = \\ = \frac{z \cdot \rho \cdot \omega_0}{2} \cdot C_L(i_x) \cdot v_0 \cdot b_x \cdot \frac{\left(\omega_0^2 \cdot R_0^2 + v_0^2\right)^{3/2} - \left(\omega_0^2 \cdot r_0^2 + v_0^2\right)^{3/2}}{3 \cdot \omega_0^2} \tag{12}$$

Here $i_x = i\ (r_x)$ and $b_x = b\ (r_x)$. This solution is calculated for the centre blade values and also for the above mentioned values for air density, wind velocity and runner speed of rotation in Table 5:

TABLE 5

z	R_0	r_0	r_x	b_x	i_x	P_6	Observations
-	[m]	[m]	[m]	[m]	[^{0}]	[W]	Turbine
3	3.5	1.5	2.5	0.525	7.54	14134.91	Ciugud - B
5	2.5	0.8	1.65	0.300015	8.18944395	5637.467	Ciugud - A
5	2.5	1.1	1.8	0.28335	8.46544	5104.375	Ciugud - A

Numeric integration of rel. (6) with different number of intervals, "N", along the runner blade shows the asymptotic limit of the power for Ciugud - B in Table 6.

TABLE 6

N	-	10	50	100	500	1000	5000	10000
P_7	[W]	9203.521	9659.047	9716.872	9763.28	9769.089	9773.746	9774.33

Numerical integration represented in Table 6 gives:

$$P_7 \cong 9773 \quad kW \tag{13}$$

Wind power plant Ciugud with the rated values of the parameters and the known dependence of the airfoils chord in function of runner radius and the angle of attack in function of runner radius:

$$i(r) = 0.2555 \cdot r^2 - 2.4207 \cdot r + 11.99488 \tag{14}$$

For a more precise dependence of the angle of attack in function of blade radius:

$$i(r) = -12.62158 + 54.51764 \cdot r - 50.827 \cdot r^2 + 22.25471 \cdot r^3 - 4.71721 \cdot r^4 + 0.3900138 \cdot r^5 \tag{15}$$

The power, " P_8", results for two extensions of the blade radius are presented in Table 7:

TABLE 7

N	i	R_0	r_0	P_8	Obs.
-	[^{0}]	[m]	[m]	[W]	Ciugud - B
5000	6.696	3.5	1.5	9771.354	Without hub zone extension
5000	6.696	3.5	0.8	11372.11	with hub zone extension

Through numerical integration in a QBasic program (PUTEREA.BAS) with:

- The angle of attack in function of blade radius by the relation (15);

- The lift coefficient of the profile in function of blade radius by the relation (9);

- The drag coefficient in function of blade radius by the relation (10);

- The chord of the bade airfoils in function of blade radius by relation (7) respectively (8).

The runners Ciugud – A and Ciugud – B have the blade airfoils from the same NACA family and the lift and drag coefficients have the same dependency from the angle of attack by different radiuses. The power "P_9" integration results using the relation (6) solved numerical are given in Table 8.

TABLE 8

ρ = 1.2 kg / m^3 ; n_0 = 120 rev / min ; v_0 = 8.5 m / s ; N = 5000;						
z	**R_0**	**r_0**	**$b(R_0)$**	**$b(r_0)$**	**P_9**	**Observations**
-	**[m]**	**[m]**	**[m]**	**[m]**	**[W]**	**-**
3	3.5	1.5	0.355	0.695	9800.96	Ciugud - A
		0.8	0.355	0.814	11378.05	
5	2.5	1.5	0.20558	0.31668	3034.538	Ciugud - B
		1.1	0.20558	0.36112	3850.07	

3. Power coefficient – solidity performances

Knowing the power coefficient in function of the number of wind runner blades from the literature [4],[5] as in Fig. 1:

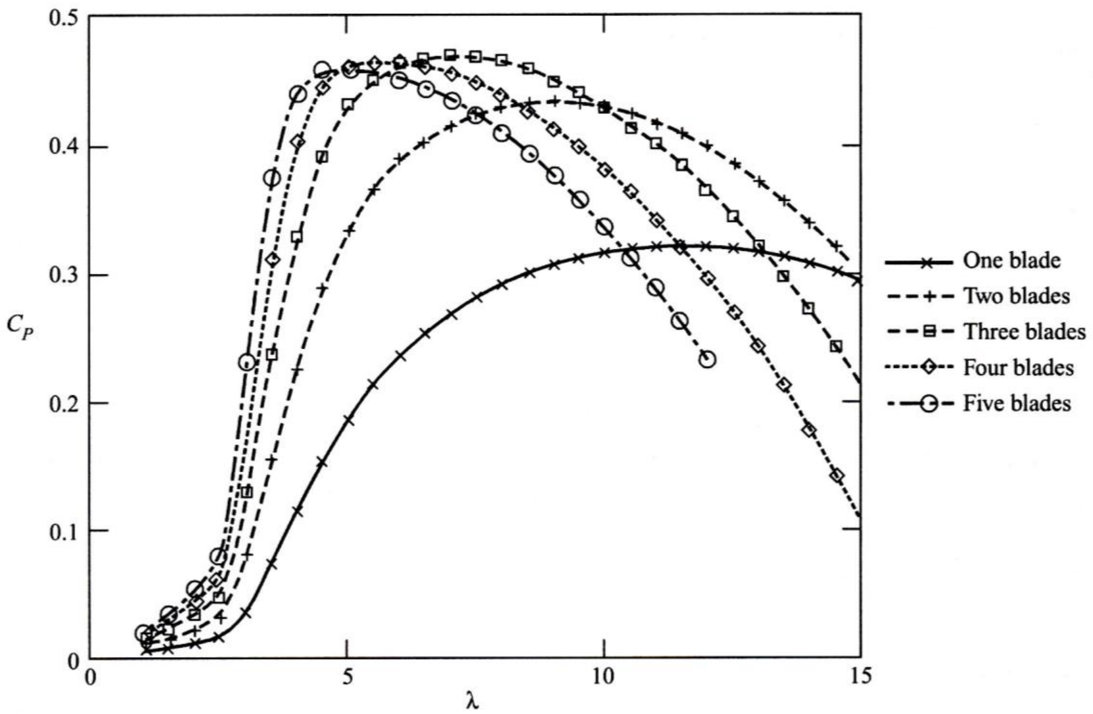

Fig. 1. Power coefficient in function of the number of runner blades of an axial wind turbine

The extracted data from [1] for 18.02.2016 and 30.12.2015 gives the dependence from Fig. 2.

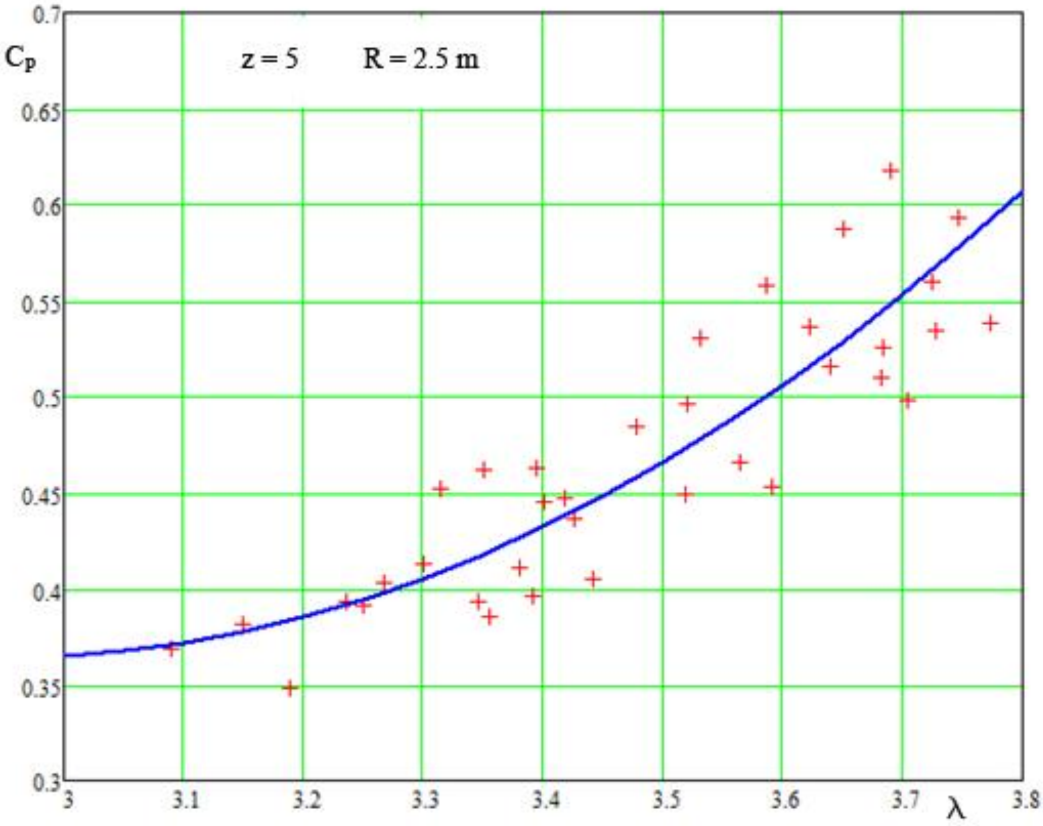

Fig. 2. Power coefficient in function of rapidity for Ciugud - A wind turbine

This correlation of the power with solidity C_P – λ, Fig.2, compared with above given Fig. 1 has the following peculiarities:
- The rapidity of the Ciugud Wind power plant is smaller than the rapidity from literature for the axial wind runner with same number of blades.
- The values of the power coefficients are greater than the values from Fig. 1.

The conclusion from Fig. 2 shows that there are some problems with estimation, for the same rapidity, of the speed of rotation for calculus of solidity.

For the power coefficient I have no explanations.

A better approximation is: $C_P = 62.61622 - 54.25966 \cdot \lambda + 15.636 \cdot \lambda^2 - 1.487106 \cdot \lambda^3$

With $\varepsilon_m = 0.025968$ and $\sum \varepsilon_i^2 = 0.01332$ all for sign "o" in Fig. 2.

Another approximation is $C_P = 32.21472 - 27.86668 \cdot \lambda + 8.04936 \cdot \lambda^2 - 0.765309 \cdot \lambda^3$

With $\varepsilon_m = 0.023025$ and $\sum \varepsilon_i^2 = 0.021515$ all for sign "+" in Fig. 2.

A better approximation for Fig. 3 is: $C_P = -0.10775 + 0.05867376 \cdot \lambda$

With $\varepsilon_m = 0.0020928$ and $\sum \varepsilon_i^2 = 0.000058631$

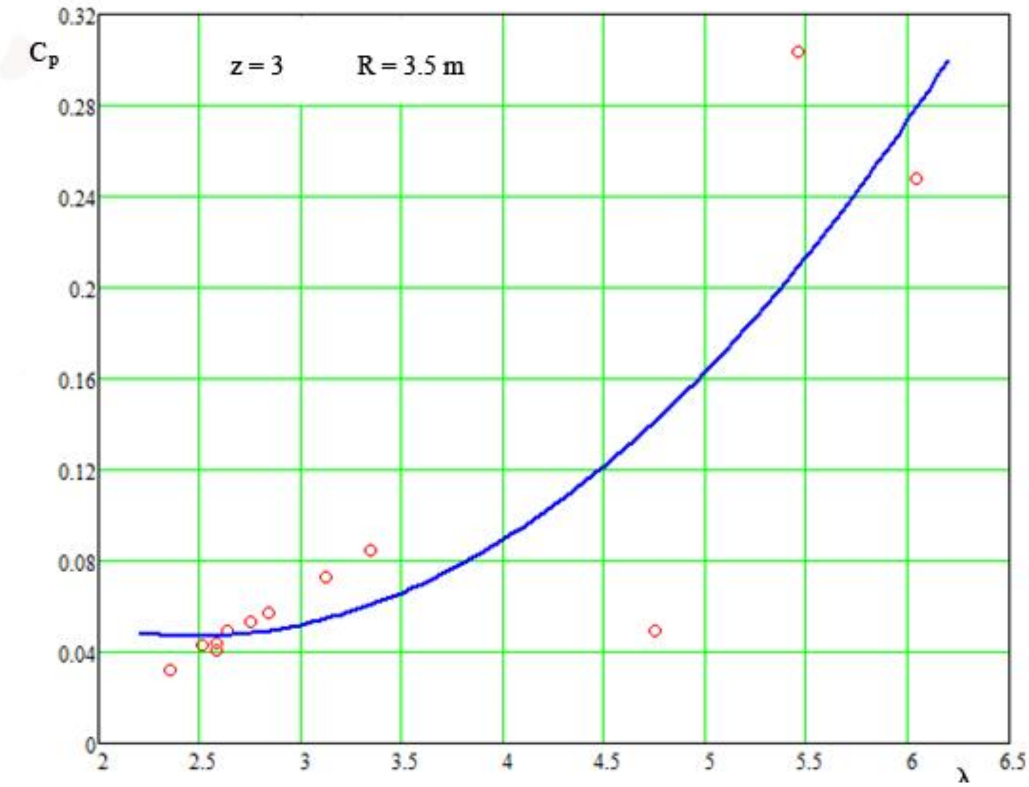

Fig. 3. Power coefficient in function of rapidity for Ciugud – B wind turbine

Power coefficient in function of rapidity for Ciugud - B wind turbine is presented in Fig. 3. The rapidity intervals for Ciugud – A (z = 5) namely in Fig. 2 , λ = 3…4 corresponds with the data from [1] for z = 3…6 , λ = 3 and for Ciugud – B (z = 3) Fig. 3 , λ = 2.5…3.5 corresponds with the data from [1] z =2…4 , λ = 4. The small discrepancy can be explained because the measured data are in a zone of increasing values of the power coefficient in Fig. 2 and Fig.3 and not in the maximum of C_P values.

5. Blade extension and angle of attack function

Wind turbine runner blade for Ciugud – B from type R_0 = 3.5 m to hub r_0 = 1.5 m and for runner blade extension Ciugud – B from type R_0 = 3.5 m to hub r_0 = 0.8 m power is calculated.

The airfoils angle of attack in 28 sections of the runner blade (of Ciugud – B) between tip and hub are approximated polynomial.

Angle of attack "i" in function of runner blade radius "r" is approximated through a:

- Two degree relation (Pol. 2):

$$i = 9.14033 + 0.0190224 \cdot r - 0.2313245 \cdot r^2 \tag{14}$$

With mean error: ε_m = 0.2301 and square error sum $\sum \varepsilon^2$ = 2.305.

- Three degree relation (Pol. 3):

$$i = 5.099766 + 7.08846 \cdot r - 3.823122 \cdot r^2 + 0.5568679 \cdot r^3 \tag{15}$$

With mean error: ε_m = 0.13349 and square error sum $\sum \varepsilon^2$ = 0.83193.

- Four degree relation (Pol. 4):

$$i = -2.31821 + 24.1731 \cdot r - 17.49545 \cdot r^2 + 5.068098 \cdot r^3 - 0.5245616 \cdot r^4 \quad (16)$$

With mean error: ε_m = 0.070217 and square error sum $\sum \varepsilon^2$ = 0.20957.

- Five degree relation (Pol. 5):

$$i = -12.62158 + 54.51764 \cdot r - 50.827 \cdot r^2 + 22.25471 \cdot r^3 - 4.71721 \cdot r^4 + 0.3900138 \cdot r^5 \quad (17)$$

With mean error: ε_m = 0.03237 and square error sum $\sum \varepsilon^2$ = 0.041344.

The comparative results are mentioned in Table 9 with respect of numerical approximation of the extracted power, P_{10}.

TABLE 9

Wind turbine	i	R_0	r_0	P_{10}	Obs.
-	[^{0}[]	[m]	[m]	[W]	
Ciugud - B	Pol. 2	3.5	1.5	9804.595	
	Pol.3			9844.215	
	Pol.4			10429.47	
	Pol. 5			9775.817	
Ciugud - B	Pol. 2	3.5	0.8	11382.93	
	Pol.3			11440.23	
	Pol.4			12078.25	
	Pol. 5			11378.13	

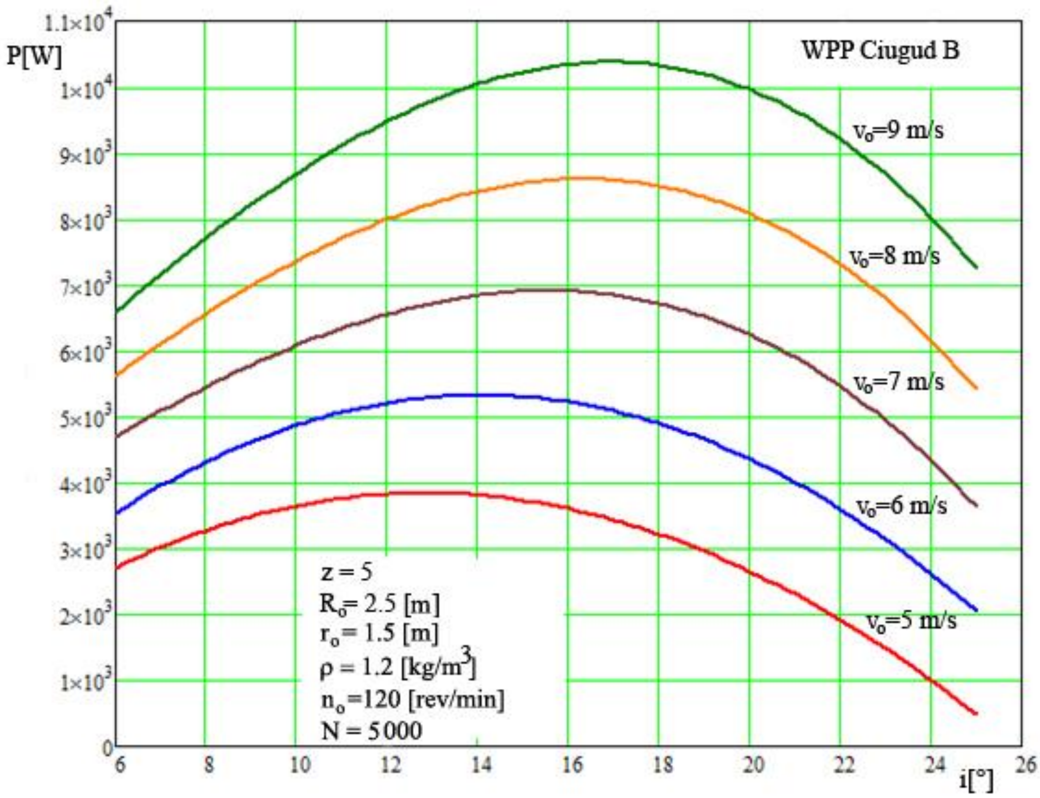

Fig. 4. Extracted power in function of the attack angle by different wind velocities

Runner blades airfoils angle of attack can be modified through blade rotation in respect of blade axis. Because the wind velocity and speed of rotation of the runner remain constant the sum of the angles of attack and stagger angle of the blade remain constant. So the power obtained may be modified. Increasing power is in the same time more close to flutter and the assurance of the operation is decreased. In Fig. 4 the power obtained from Ciugud – B in function of the angle of attack by different wind velocities using the model in QBasic (EXTREM.BAS) for numerical integration of relation (6).
The maximum power and the correspondent angle of incidence gives the degree of protection against stall of the profile respectively blades of the turbine runner.

4. Conclusions

Comparative analysis of the results for different mathematical models of power calculations made to the wind power plant Ciugud establish:

1) The maximum theoretical powers value, for Ciugud – A, $P_3 \approx 4287$ W and for Ciugud – B, $P_3 \approx 8403$ kW, are greater than the designed values of P = 3 kW and respectively P = 5 kW.

2) The maximum aerodynamic power statistical and similitude data are around the designed value as it is seen from Table 2 and Table 3.

3) The analytical power $P_6 \approx 5$ kW and respectively $P_6 = 14$ kW - taking into consideration a two-dimensional model and neglecting a lot of factors and influences – is much greater than the rated values. The simplification to obtain an analytical integral took its toll.

4) The numerical integral is much closer to the reality. The value of power $P_7 = 9794.644$ W obtained after a satisfactory number of 5000 intervals is adequate.

5) The measured parameters of power and rapidity are in a narrow band of values and so the conclusions are hazardous.

6) The blade extension of the runner has a major importance for the extracted power.

7) Higher polynomial approximation of the angle of attack in function of the blade radius introduced little influence on the calculated power.

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