Discreet Flow Distribution in Central Pivot

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Abstract: *In the search of a Central Pivot's best operational performance, the equipment's electric and mechanical characteristics, plus the realistic knowledge of the hydraulic relationships to which the "system" as a whole is submitted, is a must for its proper sizing in function of its components adequate choice. The presence of different sprinklers, featuring diverse diameters and hydraulic characteristics, pressure regulators and the possibility of the presence of a larger sprinkler with a booster on the network, allied to the necessity of attending the physical-natural restrictions which wrap-up agriculture production technology, properly express the arduous task of developing irrigation and drainage projects seeking a harmonic arrangement between a uniform application and the efficient use of water catchment's reserves.*
Leaning towards the discussion of this subject and seeing the importance of the presence of automatic irrigation systems on a large scale agricultural productivity, the present paper presents its basic considerations under the mathematical modeling scope of the sprinkler's flow distribution along the Central Pivot's line, so as to harmonize both their continuous and discrete flows, aiming at a better aspersed water uniformity application.

Keywords: *Discrete distribution, discrete flow.*

1. Introduction

The Central Pivot is a mechanized aspersion system, idealized by Fank Zybach (EUA) in the '70s, whose main fascination lies in it being an irrigation equipment endowed with self-movement.
In the late '70s, at the height of the "green revolution" proposed by the economic strength of the Brazilian military regime, the merging of the equipment's operational versatility with the possibility of service to larger areas, sealed for the first time the interest of national agriculture in 'automated" and "dynamic" irrigation, a feature extremely attractive in a country with such continental dimensions and tropical climate as Brazil. [1]
Effectively, this is an equipment made up of "an irrigational radial line bearing a fix extremity, from where it is supplied with water and electric energy and around which it describes a spinning circle, having as a main feature, the fact of moving itself while irrigating [1].
On the other hand, although bearing sophisticated performance characteristics, once hydraulically poorly dimensioned, those devices mostly present a high water and electricity consumption, resulting in a sharp increase in agricultural production costs, enabling the occurrence of runoff and consequent fertile soil erosion, affecting definitely the cultivated area by the indiscriminate use of high intensity applications for long periods, which usually occurs due to technical conception failures. [3]
This framework, however, is a practice no longer allowed nowadays, where the shortage of water in the original watersheds, justified by the necessity of raising up new areas of cultivation and the ongoing climate irregularity, caused by years of reprehensible environmental practices is felt in a Brazil that awakens to a new consciousness based on three points of interest: the competitive use of water, preservation of natural resources and concern about the immediate need for sustainable development of irrigated agriculture. [1]

2. Applying intensity x continuous distribution

In pipeline outflows with multiple exits, known as those with distribution in motion, inherent to the Central Pivot irrigation equipment, the flow that goes through a stretch of the pivot generic line is, in a first moment, treated mathematically as a variable which varies continuously along the outflow pressure. [8]
This hypothesis is assumed in this first step because of the interest on knowing the relationship between required flow and wetted circular area with different lengths of line, "R" and "r"respectively. Thus, the intensity of sprinkled "rain" "I" in a given area, Figure 1, is described analytically for the two radius lengths as:

$$\frac{Q_0}{\pi R^2} = \frac{Q}{\pi (R^2 - r^2)} = I \quad (1)$$

or rewritten evidencing the residual flow in the "$R-r$" length stretch.

$$Q = Q_0 \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (2)$$

being:

Q - the flow that meets the downstream stretch of the corresponding length "$R-r$";
Q_0 - the entrance flow in the equipment (required by the system);
R - the radius effectively moistened;
r - the generic radius.

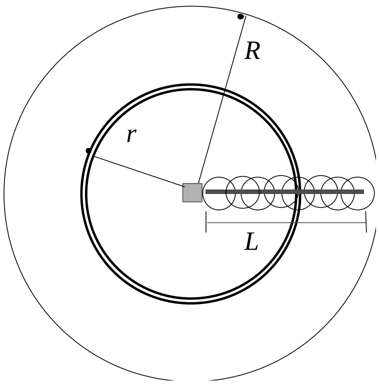


Fig. 1. Area irrigated by a generic Pivot

$$Q_0 \left(\frac{r^2}{R^2} \right) = Q_0 - Q \quad (3)$$

The principle of mass conservation, applied to flows with multiple derivations, Figure 2, ensures that the sum of the sprinklers flow to a position "r" of the line, should meet the relation:

$$\sum q_i = Q_0 - Q \quad (4)$$

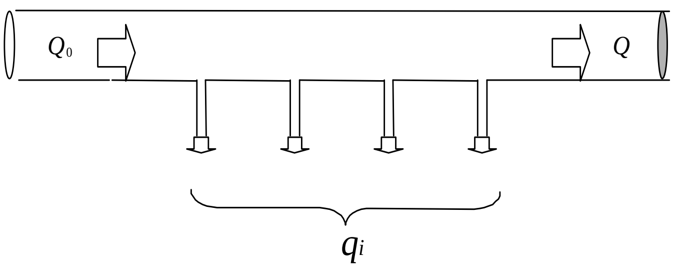


Fig. 2. A Central Pivot generic stretch

3. Continuous distribution x discreet distribution

The Equation 2, which models the flow that will sprinkle the final stretch of the downstream cultivation area, corresponding to the circular crown's difference radius "$R-r$" carries in its wake the hypothesis of continuous water distribution along the line. In practice this is not the case, being effectively a discrete distribution of water through equally spaced nozzles. [2]
This difference between the continuous theoretical model, Eq 2, and the effective functioning of the discrete distribution of water in the actual performance of a Central Pivot must be properly bypassed, at the risk do not following the principle of mass conservation, which bears on errors in the necessary numerical calculation results for a Central Pivot careful design. [6]
The compatibility between these ways of conceiving the distribution can be obtained by accepting that the discrete flow "q_i" released by a sprinkler located in a given generic position "i" must water the ring area bounded by the sprinklers located between positions "i" and "$i+1$", distant from "Δx".This reasoning requires that the flow released by the last sprinkler (downstream) will water, in addition, a circular crown radius "Δx", this crown, located beyond the effective length "L" of the Central Pivot's line.
Considering the "Q_i" flow in the position "i" of the Pivot line, flow which should meet all the sprinklers demand of that "i" position until the downstream's end plus any further demand from a large sprinkler (with booster) corresponding to a "Q_c", flow designed to water the Crown between "L and R", where "L"is the effective pipe length and "R" is the radius of the total watered area, "$R-r$", one can therefore rewrite the water distribution model as:

$$Q_i - Q_c = I\ \pi \left[(L+\Delta r)^2\right] \tag{5}$$

$$Q_i - Q_c = I\ \pi \left[(L+\Delta r)^2 - r_i^2\right] \tag{6}$$

This expression Eq. 6, when written for the first line sprinkler, "x_1", takes the form:

$$Q_1 - Q_c = I\ \pi \left[(L+\Delta r)^2 - r_1^2\right] \tag{7}$$

With Eqs 6 & 7, formulated to express the moving flow (discreet outputs) along the line of a pivot, one can write the application intensity bearing in mind the hypothesis of continuous outputs as:

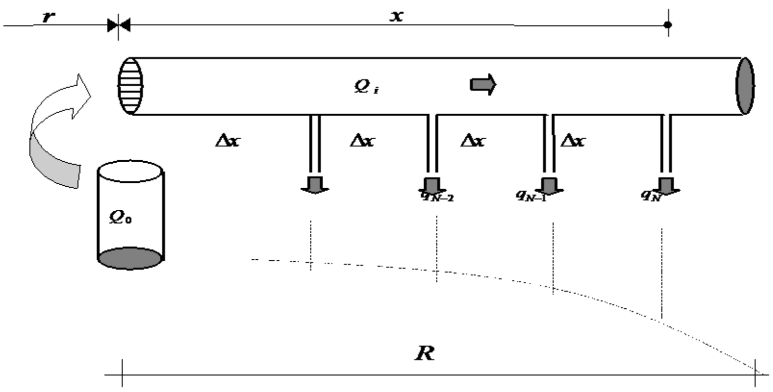


Fig. 3. A generic Central Pivot stretch (identification of flows in motion)

$$\frac{Q_i - Q_c}{Q_1 - Q_c} = \frac{(L+\Delta r)^2 - r_i^2}{(L+\Delta r)^2 - r_1^2} \tag{8}$$

whose line flow, made explicit for a generic sprinkler "i",can be written as:

$$Q_i - Q_c = \left[\frac{(L+\Delta r)^2 - r_i^2}{(L+\Delta r)^2 - r_1^2}\right](Q_1 - Q_c) \tag{9}$$

and to the next downstream sprinkler "$i+1$", as:

$$Q_{i+1} - Q_c = \left[\frac{(L+\Delta r)^2 - r_{i+1}^2}{(L+\Delta r)^2 - r_1^2} \right] (Q_1 - Q_c) \tag{10}$$

Thus, identifying the flows present in a pivot's main line, at the borders of a generic sprinkler "i", the principle of mass conservation ensures that this sprinkler's flow is expressed by $q_i = Q_i - Q_{i+1}$, Figure 5, where the elimination of "Q_i" and "Q_{i+1}" occurs with the interaction of Eqs 11 and 12 respectively. This facilitates the obtaining of a "q_i" flow rate law, expressed by:

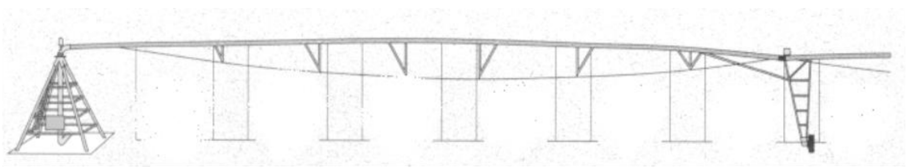
$$q_i = \left[\frac{r_{i+1}^2 - r_i^2}{(L+\Delta r)^2 - r_1^2} \right] (Q_1 - Q_c) \tag{11}$$

By expanding this expression's numerator, Eq 13, one obtains:

$$r_{i+1}^2 - r_i^2 = (r_i + \Delta r)^2 - r_i^2 = 2\, r_i^2 \Delta r + \Delta r^2 \tag{12}$$

With an expanded numerator, we can write Eq 16 as:

$$q_i = \left[\frac{2x_i \Delta x + \Delta x^2}{(L+\Delta x)^2 - x_1^2} \right] (Q_1 - Q_c) \tag{13}$$



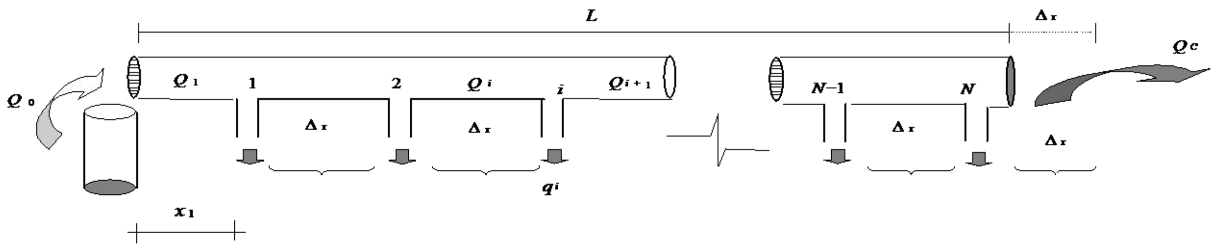


Fig. 4. A generic stretch of a Central Pivot (continuous treatment of withdrawals under way)

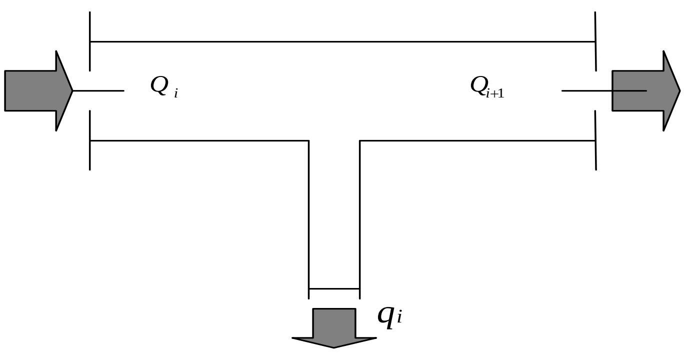


Fig. 5. Schematic representation of a generic sprinkler "i"

Thus, working out the algebraic relations between "L" and "r_i" it gives:

$$L = r_i + (N - i)\,\Delta r \tag{14}$$

$$r_i = (L - (N - i)\,\Delta r) \tag{15}$$

Where the expression for the "q_i" flow can be written as:

$$q_i = \left[\frac{2(L-(N-i)\,\Delta r)\,\Delta r + \Delta r^2}{(L+\Delta r)^2 - r_1^2}\right](Q_1 - Q_c) \tag{16}$$

or still further as:

$$q_i = \left[\frac{2L\Delta r - (2(N-i)-1)\,\Delta r^2}{(L+\Delta r)^2 - r_1^2}\right](Q_1 - Q_c) \tag{17}$$

4. Conclusions

In possession of the relationships shown in Eqs 14 and 15, and developing the denominator of Equation 17 in the attempt of obtaining an expression that is involved only with the magnitudes "L", "N" and "Δr",and it is possible to eliminate the term "r_1^2" in this part of the equation, writing:

$$(L+\Delta r)^2 - r_1^2 = \left(L^2 + 2L\Delta r + \Delta r^2\right) - r_1^2 \tag{18}$$

$$= \left(L^2 - r_1^2\right) + 2L\Delta r + \Delta r^2$$

$$= \left(2L(N-1)\Delta r - (N-1)^2\Delta r^2\right) + 2L\Delta r + \Delta r^2$$

$$= 2\,L\,N\,\Delta r - 2\,L\,\Delta r - N^2\Delta r^2 + 2\,N\,\Delta r^2 - \Delta r^2 + 2\,L\,\Delta r + \Delta r^2$$

$$= 2\,L\,N\,\Delta r - N^2\Delta r^2 + 2\,N\,\Delta r^2 \tag{19}$$

Finally, rewriting Eq. 17 with the denominator expression presented by Eq. 19, we have modeled the final equation for calculating the flow rate of a generic sprinkler, "i", in a pivot's line, described as:

$$\boxed{q_i = \left[\frac{2\,L\,\Delta r - 2\,(N-i)\,\Delta r^2 + \Delta r^2}{2\,L\,N\,\Delta r - N^2\Delta r^2 + 2\,N\,\Delta r^2}\right](Q_1 - Q_c)} \tag{20}$$

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