Operating Equations of a Rotating Machine that Drives Fluids

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Abstract: *The paper contains originality elements regarding both the constructive solution of the rotating machine and its functional equations. There are theoretical aspects regarding the fluid handling with this new type of rotating machine with profiled rotors. The end of the paper presents the operating equations and the advantages of using this new type of rotating machine.*

Keywords: *Rotating machine, profiled rotors, operating equations*

1. Introduction

By its content, the paper aims to present a new type of rotating machine with profiled rotors that can be practically used in several versions:
- As a working machine (pump, fan and blower); in this case the fluid pressure at discharge (p_2) is higher than at suction (p_1).
- As a force machine (steam engine or combustion gases, hydrostatic motor); in this case $p_1 > p_2$.
The use of rotating working machines is focused on two directions [1]:
a) Transportation of fluids from one place to another;
b) Increasing fluid pressure to generate the necessary forces for different hydraulic drives.
The constructive solution presented in this paper refers only to a).
Rotating working machines have the following advantages [2]:
• Transfers the received engines torque to the shaft with minimal losses in potential fluid pressure; these machines do not have alternate rectilinear motion, there are no valves;
• It has a higher reliability in operation; do not require maintenance over a relatively long period of time;
• Certain constructive solutions can carry fluids with impurities, rheological fluids.
The paper reveals new theoretical aspects regarding the fluid circulation with a new type of rotating machine with profiled rotors, its performances were validated by experimental researches.
It is stated that there are fundamental equations of the operation of hydraulic and thermal machines but it should not be confused with the operating equations.

2. Presentation of the rotating machine with profiled rotors

2.1 The sketch and operation principle of the rotating machine

In the paper, the term "machine" is used in the sense that it can be used as a fan, a low pressure compressor and a pump for pumping pure liquids or suspensions.
The sketch of the rotating machine and the operating principle are shown in Figure 1. The machine (fig.1) has two identical profiled rotors (2, 5) of a special shape which rotate with the same speed within a case (1, 4).

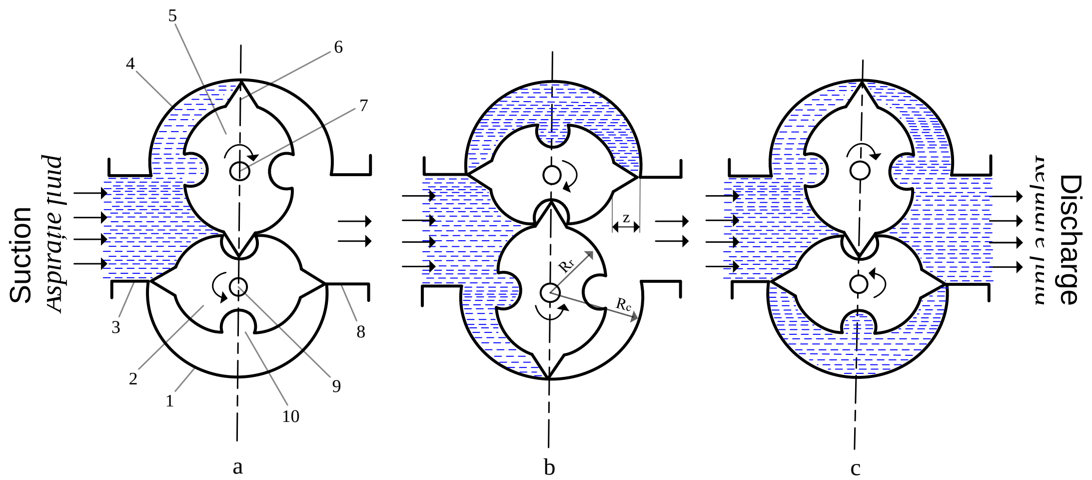

Fig. 1. The rotors position after a 90° rotation
1- lower case; 2- lower rotor; 3- suction chamber; 4- upper case; 5- upper rotor; 6- rotating piston; 7- driven shaft; 8- discharge chamber; 9- driving shaft; 10- cavity in which the upper rotor piston enters

The aspirated fluid (fig. 1. a) is transported to the discharge and after a 90° rotation of both rotors, the situation in Figure 1. b and in Figure 1. c is reached.
The synchronization of the rotation of the two rotors (2 and 5) is ensured by two gears of the same division diameter mounted outside the machine.
In Figure 1, the two rotors have the same radius (R_r), the cylindrical case has the radius (R_c), and the triangular-shaped rotating piston has the height (z). From figure (1) one can see that:

$$R_c = R_r + z[m] \tag{1}$$

2.2 The calculation of the rotating machine circulated fluid flow rate

It is noted with V_u (figure 1) the volume between the rotating pistons, the side surface of the case and the side surface of the lower rotor (2) of length l. On a 360 ° rotation of the shaft (9) two such volumes will be transported from suction to discharge:

$$2 \cdot V_u = 2\left(\frac{\pi R_c^2}{2} - \frac{\pi R_r^2}{2}\right) \cdot l \tag{2}$$

Replacing the relation (1) in (2) one can obtain:

$$V_u = \pi l z \cdot (z + 2R_r)\ [m^3 / rot] \tag{3}$$

The volumetric flow rate of the fluid displaced by the machine will be:

$$\dot{V} = \pi l z \cdot (z + 2R_r)\upsilon \left[m^3 / s\right] \tag{4}$$

in which υ - the rotation speed of the machine [rot / s]
replacing:

$$\upsilon = \frac{n}{60} \tag{5}$$

where n is the machine speed [rpm].
From the relation (4) one can obtain:

$$\dot{V} = \pi l z \cdot (z + 2R_r)\frac{n}{60} \left[m^3 / s\right] \tag{6}$$

Since the working machine has two identical rotors, the fluid flow of the machine will be:

$$\dot{V}_m = 2\dot{V} = \pi l z \cdot \left(z + 2R_r\right)\frac{n}{60}\left[m^3/s\right] \tag{7}$$

From the relation (7) one can observe that for a constructive solution given by l, z, R_r, the volumetric flow of the fluid varies linearly with the machine speed.
This rotating machine was built and tested in the laboratory of the Department of Thermotechnics, Engines, Heat & Refrigeration Equipment of POLITEHNICA University of Bucharest [3] [4].
If in relation (7) the following are introduced:
l = 0.05 m; z = 0.03 m; R_r = 0.05 m
n = 100, 200, 300, 400, 500 rpm, the data in Table 1 are obtained.

Table 1: Values of $\dot{V}_m$ = f (n)

n [rot/min]	100	200	300	400	500
$\dot{V}\cdot 10^{-3}$ [m³/s]	2.04	4.08	6.12	8.16	10.2

Based on the data in Table 1, the diagram in Figure 2 was constructed.

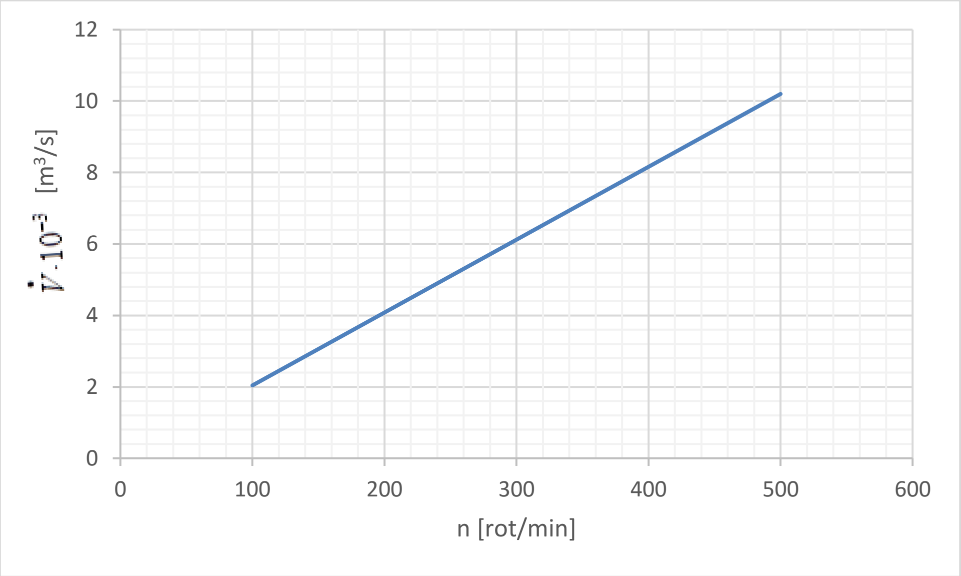

Fig. 2. The function $\dot{V}_m$ = f (n)

From graph 2 one can observe that the function $\dot{V}_m$ = f (n) is a linear function.

For a given constructive solution, from relation (7) it is observed that $\dot{V}_m = ct \cdot n\left[m^3/s\right]$, relation in which n is "given input", and $\dot{V}_m$ is "output date".

2.3 Calculating the theoretical power of the rotating machine

It is known from the literature [5] [6] [7] that the theoretical power to drive a rotating working machine is given by the relation:

$$P_m = \dot{V}_m \cdot \Delta p \left[W\right] \tag{8}$$

in which:
$\dot{V}_m$ - the volumetric flow rate [m³ / s]
Δp- machine pressure increase between suction and discharge [N / m²]
With relation (7), relation (8) becomes:

$$P_m = \pi l z \cdot \left(z + 2R_r\right)\frac{n}{60}\cdot \Delta p \tag{9}$$

Its value can be replaced as [8] [9]:

$$\Delta p = \rho_{H_2O} \cdot g \cdot H \left[N/m^2 \right] \quad (10)$$

where H is the pumping height [mH_2O]:

$$P_m = \pi l z \cdot (z + 2R_r) \frac{n}{60} \cdot \rho_{H_2O} \cdot g \cdot H \quad (11)$$

or:

$$P_m = \dot{V}_m \cdot \rho_{H_2O} \cdot g \cdot H = \dot{V} \cdot \Delta p \left[W \right] \quad (12)$$

For H = 2 m;

$$\Delta p = 10^3 \cdot 9.81 \cdot 2 = 19620\, N/m^2 \quad (13)$$

$$P_m = \dot{V}_m \cdot \Delta p = 2.04 \cdot 10^{-3} \cdot 19620 = 40.20\, W \quad (14)$$

Using the relations (10) and (12) for the same machine solution, the data in Table 2 are shown.

Table 2: Values of P_m =f(n), P_m =($\dot{V}$),P_m=f(H)

n [rot/min]	$\dot{V} \cdot 10^{-3}$ [m³/s]	H [m]	Δp [N/m²]	P_m [W]
100	2.04	2	19.620	40.20
200	4.08	4	39.240	160.09
300	6.12	6	58.860	360.22
400	8.16	8	78.480	640.39
500	10.2	10	98.100	1000.62

Based on the data in Table 2, the graphs were represented graphically:
P_m = f (n) in Figure 3;
P_m = f ($\dot{V}$) in Figure 4;
P_m = f (H) in Figure 5.

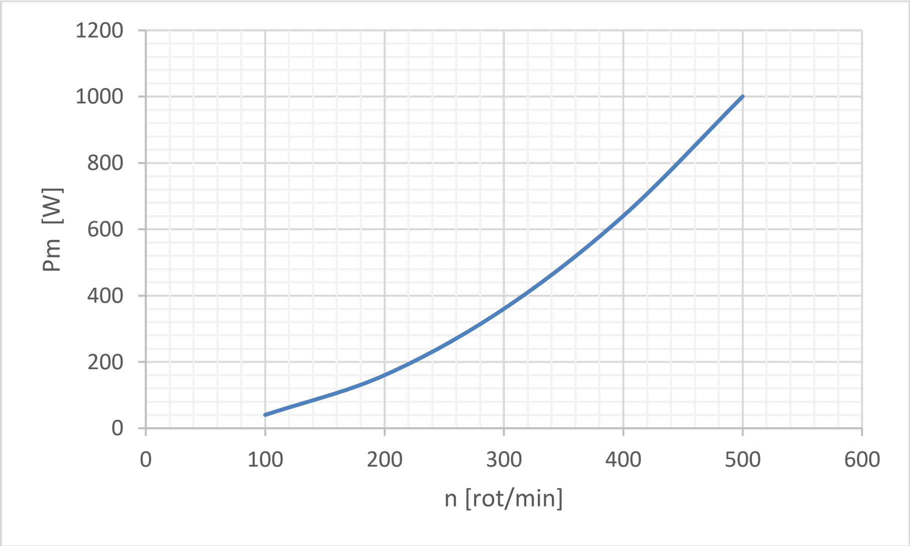

Fig. 3. The function P_m =f(n)

From Figure 3 one can observe that the function P_m =f(n) is not linear.

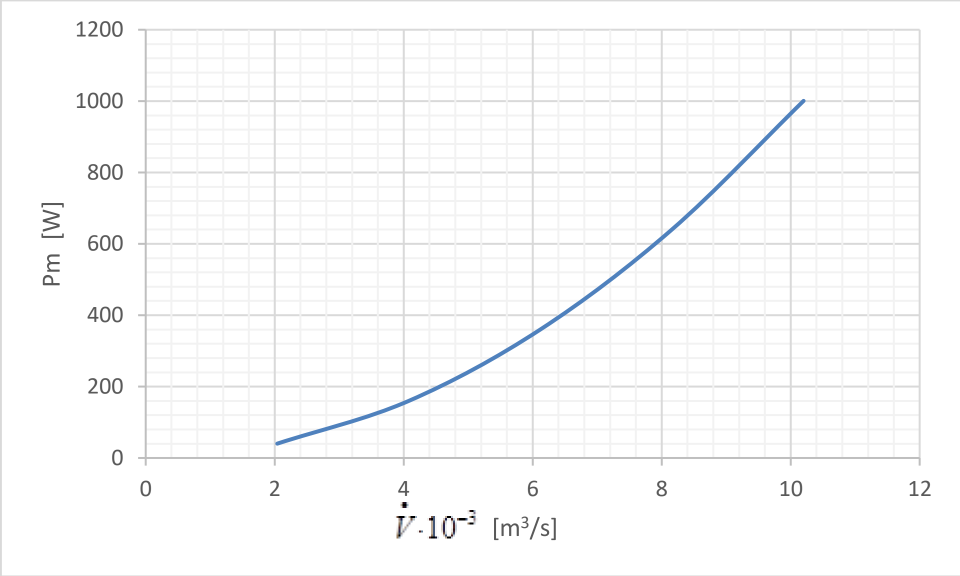

Fig. 4. The function $P_m = f(\dot{V})$

From Figure 4 one can observe that the function $P_m = f(\dot{V})$ is not linear.

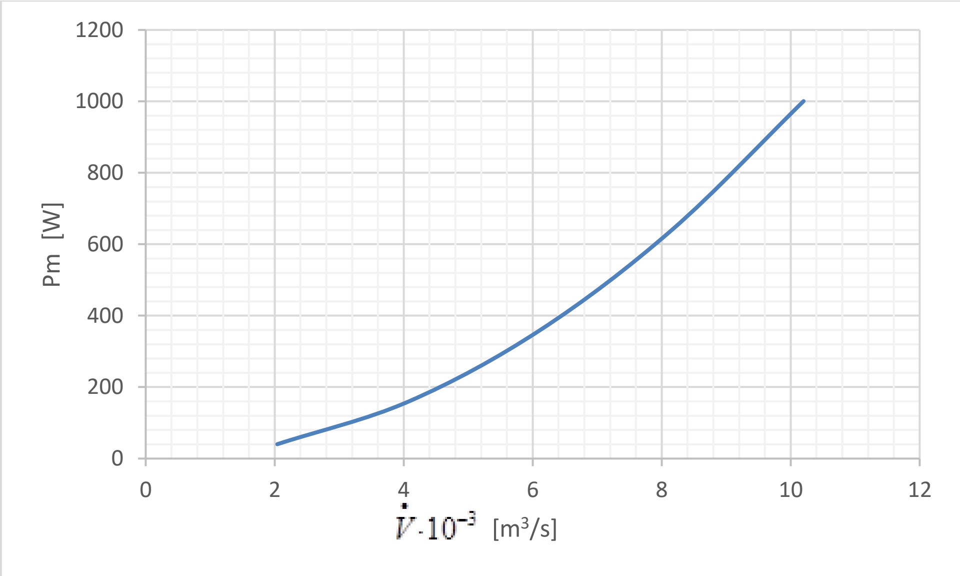

Fig. 5. The function $P_m = f(H)$

From Figure 5 one can observe that the function $P_m = f(H)$ is not linear.
Increasing pumping height (H) will obviously increase the theoretical power required to drive the rotating machine.

3. Operation equations

3.1. Definitions, operation equations of technical installations

In technique there are many operating equations, for example for [10] [11] [12]:
a) Electric transformers;
b) Power transmission lines;
c) Synchronous electric machines;
d) Bipolar transistors;

e) Three-phase asynchronous machines, etc.
The operating equations are the equations linking the inputs to the installations with the outputs of the installations. The operating equations must be equal to the number of unknowns.
From the above equation categories (a ... e) the paper refers briefly to class a).
To determine the operating equations of the single-phase transformer is considered as an iron-free transformer (Figure 6).

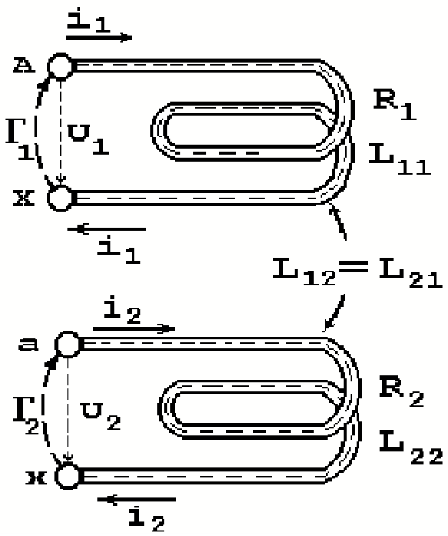

Fig. 6. Electric transformer with magnetic coupling without iron core

To establish the voltages equations, Kirchhoff's second theorem is applied to the contours Γ_1 and Γ_2; R_1 and R_2 are the resistors of the two windings (primary + secondary) and L_{11} and L_{22} are the inductances.
The mutual inductance between the two windings fulfils the condition: L_{12}. u_2 is the voltage at the secondary winding terminals. Under these conditions, the voltage equations for the two circuits are [10]:

$$u_1 = R_{1i1} + \frac{d}{dt}\left(L_{11i1} + L_{12i2}\right) \tag{15}$$

$$u_2 = R_{2i2} + \frac{d}{dt}\left(L_{11i2} + L_{21i1}\right) \tag{16}$$

These equations contain three unknowns: i_1, i_2, u_2, because the other sizes are known. To determine the unknown, the third equation for the voltage u_2 is written according to the impedance load parameters:

$$u_2 = R_{i_2} + L\frac{di_2}{dt} + \frac{1}{C}\int_0^t i_2\,dt \tag{17}$$

The three equations (15), (16), (17) describe the operation of the transformer in any mode; there is a closed system of 3 equations with 3 unknowns. It is noted that these three equations link the input quantities R_1, R_2, i_1, L_{11}, L_{22} to the output i_2, u_2.

3.2. Analysis of the operating equations of the rotating machine for fluid flow

It is considered that the machine architecture is specified, i.e. the sizes: l, z, R_r, R_c are known. It is formed a system of two equations:
- The first equation specifies the volumetric flow rate of the rotating machine;
- The second equation determines the theoretical driving power of the machine.

$$\dot{V} = \pi l z \cdot \left(z + 2R_r\right)\frac{n}{30}\left[m^3/s\right] \tag{18}$$

$$P = \dot{V} \cdot \Delta p = \dot{V}\left(p_2 - p_1\right) = \pi l z \cdot \left(z + 2R_r\right) \cdot \frac{n}{30}\left(p_2 - p_1\right)\left[W\right] \tag{19}$$

In this system of two equations the following are known: l, z, R_r, n, p_1. The unknowns are: $\dot{V}$ and p_2. So this system, for a given constructive solution, links the input sizes: n, p_1, and the outputs and p_2.
There are three cases: I, II, III.
I) l, z, R_r, n known and $\dot{V}$ [m^3 / s] results.
The calculation relation was previously stated:

$$\dot{V} = \pi l z \cdot \left(z + 2R_r\right)\frac{n}{30}\left[m^3/s\right]$$

and the function $\dot{V}_m$ = f (n) is represented in figure 2.

For these operating equation, the sizes l, z, R_r [m], n [rot / min] are input quantities and $\dot{V}$ [m^3 / s] is the output of the rotating machine. Changing the speed (n) leads to the change in volumetric flow rate ($\dot{V}$). The change of the rotation speed of the rotating machine is made by changing the speed of the asynchronous electric motor which can be done in 3 ways:
- Frequency variation (f_1) of the supply voltage;
- Changing the number of pairs of poles (p);
- Changing the slip (s).
The calculation relation is [13]:

$$n_2 = n_1(1-s) = \frac{f_1}{p}(1-s) \tag{20}$$

In the experimental investigations carried out, f_1 was modified, therefore n_2 was modified.
II) The known input sizes are: p_1, H and n and from the relation:

$$p_2 = p_1 + \Delta p = p_1 + \rho_{H_2O} \cdot g \cdot H\left[N/m^2\right]$$

the values of p_2 results.
Some data (H, n) of Table 2 are resumed, and the values of p_2 with the relation (20) results (shown in Table 3).

n [rot/min]	100	200	300	400	500
H [m H_2O]	2	4	6	8	10
Δp[N/m²]	19.620	39.240	58.860	78.480	98.100
p_1 [N/m²]	101325	101325	101325	101325	101325
p_2 [N/m²]	120945	140565	160185	179805	199425

Based on the data in Table 3, the function: p_2 = f (H) is graphically represented in Figure 7.

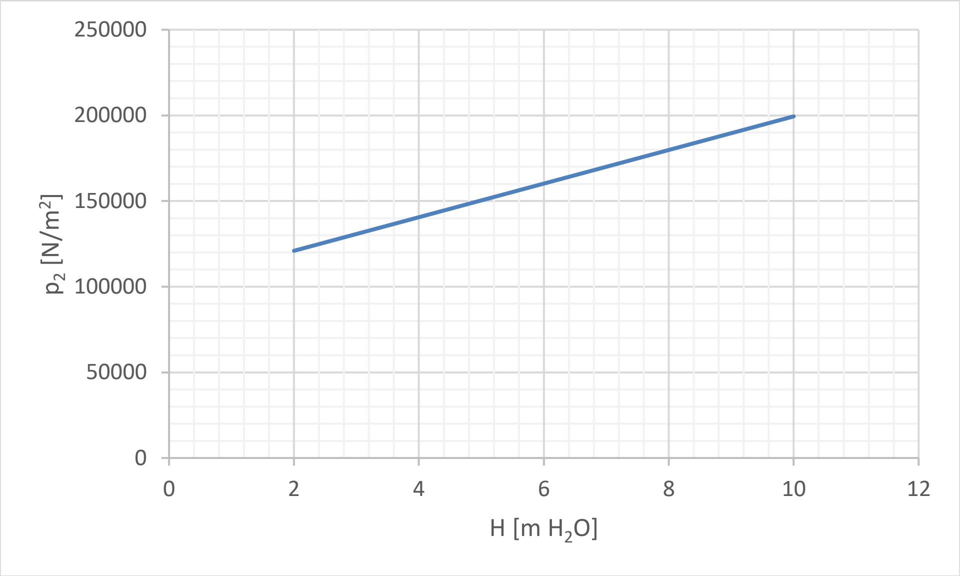

Fig. 7. The function: $p_2 = f(H)$

In case II, as input data: n and H and as output data p_2 is obtained.
The relation (19) can be written:

$$p_2 = p_1 + \frac{30 \cdot P}{\pi l z \cdot (z + 2R_r) \cdot n} \tag{21}$$

For a constructive solution with the sizes l, z, R_r and p1, considered constant:

$$p_2 = ct + ct\frac{P}{n} \tag{22}$$

From this relation one can observe that:
a) For a constant speed (n = ct) the pressure of the discharge fluid increases linearly with the power (P) supplied by the driving motor;
b) If P is maintained constant and the machine speed increases, the discharge pressure value (p_2) will decrease.
III) In the equation system (18 + 19) P is considered as unknown instead of $\dot{V}$, thus:

$$P = \pi l z (z + 2R_r)\frac{n}{30}\Delta p \tag{23}$$

By replacing: l = 0.05 m, z = 0.03 m, R_r = 0.05 m, one can obtain:

$$P = 2.04 \cdot 10^{-5} \cdot n(p_2 - p_1) \tag{24}$$

If n = 100 rpm and $\Delta p = 2 \cdot 10^5\ N/m^2$:

$$P = 2.04 \cdot 10^{-5} \cdot 100 \cdot 2 \cdot 10^5 = 408\ W$$

If n = 100, 200, 300, 400, 500 rpm, with the relation (24) the data in Table 4 is obtained.

Table 4: Values of P =f(n) for $\Delta p = 2 \cdot 10^5\ N/m^2$

n [rot/min]	100	200	300	400	500
Δp[N/m²]	$2 \cdot 10^5$	$2 \cdot 10^5$	$2 \cdot 10^5$	$2 \cdot 10^5$	$2 \cdot 10^5$
P [W]	408	816	1224	1632	2040

Performing the calculations for $\Delta p = 4,6,8,10 \cdot 10^5\ N/m^2$ the graphs in Figure 8 is obtained.

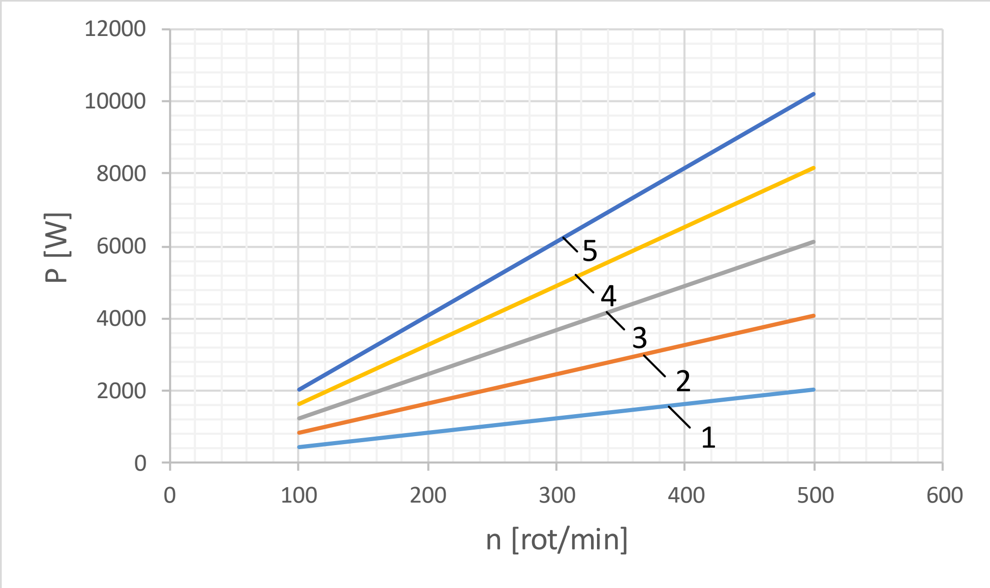

Fig. 8. The function: P = f (n) for different values of Δp

1 - $\Delta p = 2 \cdot 10^5\ N/m^2$, 2 - $\Delta p = 4 \cdot 10^5\ N/m^2$, 3 - $\Delta p = 6 \cdot 10^5\ N/m^2$, 4 - $\Delta p = 8 \cdot 10^5\ N/m^2$,
5 - $\Delta p = 10 \cdot 10^5\ N/m^2$

From Figure 8 one can notice that the shapes of the graphs are linear if Δp is constant.
Following the analysis of the operating equations of the rotating machine, the data presented in Table 5 were obtained.

Table 5: The link between the input and the output for the rotating machine

	Input data	The operation equation	Output data	Graphical representation
I	l, z, R_r [m], n [rot/min]	$\dot{V} = \pi lz \cdot (z + 2R_r)\frac{n}{30}\left[m^3/s\right]$	$\dot{V}\left[m^3/s\right]$	Fig. 2
II	p_1 [N/m^2], H [m H_2O]	$p_2 = p_1 + \Delta p = p_1 + \rho_{H_2O} \cdot g \cdot H\left[N/m^2\right]$	$p_2\left[N/m^2\right]$	Fig. 7
III	L, z, R_r [m], n [rot/min], Δp [N/m^2]	$P = 2.04 \cdot 10^{-5} \cdot n(\Delta p)$	P [W]	Fig. 8

Other forms of connection can be established between the different input and output sizes, such as P = f ($\dot{V}$), but the essential ones are shown in Table 5 [14] [15].

4. Conclusions

Rotating working machines has the following advantages:
a) Transforms the received motor torque on the shaft with minimal losses into potential pressure of the fluid;
b) These machines can carry fluids with impurities, rheological fluids and, in general, polyphase fluids;
c) It has increased reliability in operation; do not require maintenance over a relatively long period of time.
d) The above operating equations link the input sizes of the plant or process to the outputs one of the installation;
e) With this equation one can quickly determine the influence of the input quantities on the output quantities or vice versa;
f) This paper provides an important amount of specialized knowledge in the field of rotating volumetric machines with profiled rotors, machines for the transport of polyphase fluids.

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