Establishing the Mathematical Relation between the Rotor Radius and the Height of the Rotating Piston for a Rotating Machine with Profiled Rotors

PhD Student **Mariana Mirela STOICAN (PRISECARU)[1]**, Prof. Dr. Eng. **Nicolae BĂRAN[1]**,
PhD Student **Almaslamani Ammar Fadhil SHNAWA[1]**

[1] University Politehnica of Bucharest, mirela.prisecaru@yahoo.com

Abstract: *Mathematically, the paper establishes a link between the constructive elements, the rotor radius and the height of the rotating piston, which is part of the construction of a machine with profiled rotors that transports fluids.*
For a certain constructive solution, the maximum height of the rotating piston is set; the influence of the height of the piston on the flow rate and the driving power of the machine are highlighted.
A constructive solution which allows the maximum power of the machine is presented.

Keywords: *Rotating machine, profiled rotor, rotating piston*

1. Introduction

This paper is part of the category of scientific paper that addresses the research field of rotating machines that transports fluids.
A type of rotating working machine with profiled rotors is presented; it can work as [1], [2]:

- a fan, for driving different gas mixtures with or without suspensions;
- a low pressure compressor;
- a rotating volumetric pump for the conveyance of any type of liquid or gas fluid, namely:
  - general fluids: water, air, steam, etc.
  - multiphase fluids: water + air, water + sand, water + ash etc.
  - viscous fluids: oil, diesel, petroleum, etc.

The advantage of the rotating working machine is that the entire torque received from the drive motor is used to transport the fluid.
There are no reciprocating parts; there are no large friction between the movable and the fixed part of the rotating machine [3].
Generally, the machines are divided into two categories [1]:

I. Power machines that produce energy; they transform a certain form of energy into mechanical energy.

II. Working machines, which consume mechanical energy and produce another form of energy.

Fans, compressors, pumps are part of the category of machines that consume mechanical energy to create the potential increase of potential pressure energy of fluids (liquids or gases, vapors).
This paper offers new researches directions in the field of fluid transport by rotating machines.

2. The operating principle and the constructive solution of the rotating machine

The fluid sucked into the suction connection (1) (figure 1) is transported by a rotational movement by the pistons (4) to the discharge connection (7); the fluid may be polyphase (air + water, water + sand, water + steam, water + ash, etc.) or may be viscous, or with high density.
The machine has two profiled rotors that rotate counterclockwise inside some housing (figure 1). The synchronous rotation of the two rotors (3, 8) is ensured by means of two gear wheels which form a cylindrical gear with straight teeth, gear located outside the machine.

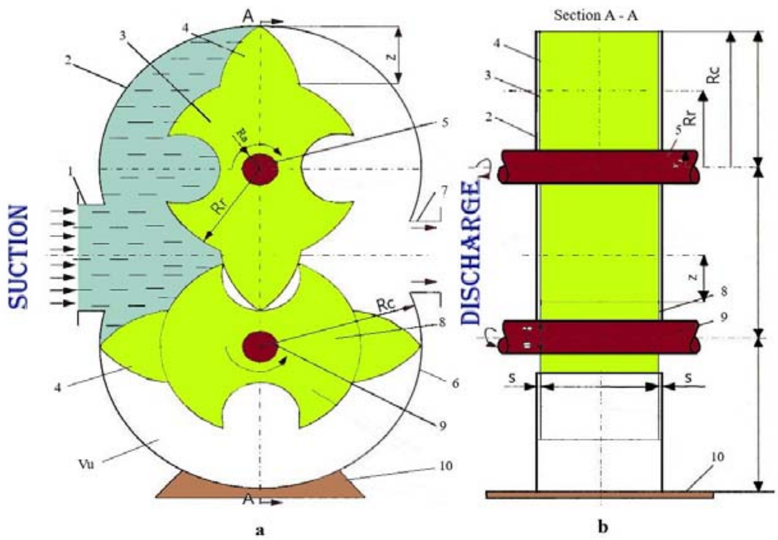

Fig. 1. Cross section (a) and longitudinal section (b) through the rotating machine
1 - suction connection; 2 - upper housing; 3 - upper rotor; 4 - rotating piston; 5 - driven shaft; 6 - lower housing; 7 - fluid discharge connection; 8 - lower rotor; 9 - driving shaft; 10 – support

In figure 1 was noted:
R_c - housing radius; R_r - rotor radius; R_a - shaft radius; z - rotating piston height;
V_u - the useful volume between two successive pistons and the inner wall of the housing.

3. Deduction of the calculation formula for the flow rate transported by the machine and for the driving power of the machine

At a complete rotation of the shafts (5.9), two such volumes (V_u) will be transported from the suction to the discharge.

$$\dot{V}_u = 2\left(\frac{\pi R_c^2}{2} - \frac{\pi Rr^2}{2}\right)\cdot l \quad \left[m^3/rot\right] \tag{1}$$

The housing radius (R_c) is the sum of the rotor radius (R_r) and the height of the piston (z) (figure 1).

$$R_c = R_r + z \qquad [m] \tag{2}$$

From relation (1) and (2) results:

$$\dot{V}_u = \pi \cdot l \cdot \mathrm{z}(\mathrm{z} + 2\,\mathrm{R_r}) \quad \left[m^3/rot\right] \tag{3}$$

The volumetric fluid flow rate flowed by a single rotor of length l [m] and speed n_r [rot / min] will be:

$$\dot{V}_u = \pi \cdot l \cdot z\left(z + 2R_r\right)\cdot\frac{n_r}{60} \quad \left[m^3/s\right] \tag{4}$$

The rotating machine has two identical rotors, so the fluid flow rate will be:

$$\dot{V}_m = 2\dot{V}_m = \pi l z\left(z + 2R_r\right)\cdot\frac{n_r}{30} \quad \left[m^3/s\right] \tag{5}$$

From relation (5) one can observe that the fluid flow rate conveyed by the machine varies according to the following parameters:

• Geometrical parameters: l - rotor length [m]; R_r - rotor radius [m]; z - rotating piston height [m].

• Functional parameters: n_r - speed of the rotating machine [rot / min].

4. Establishing the mathematical relation between the rotor radius and the height of the rotating piston

It is considered one piston (4) fixed to the lower rotor (figure 2).

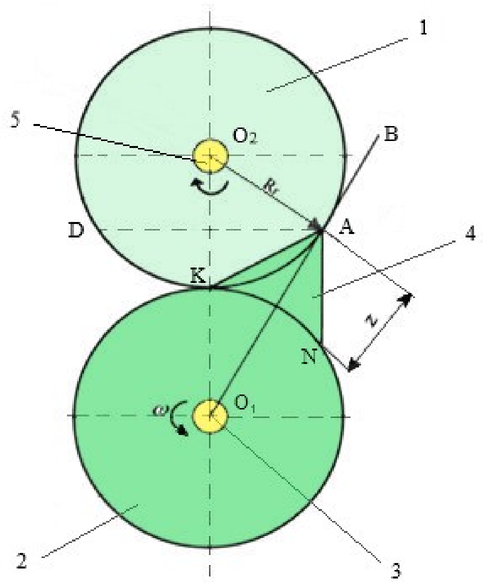

Fig. 2. Calculation notations
1 - upper rotor; 2 - lower rotor; 3 - driving shaft; 5 - driven shaft;
4 - rotating piston of triangular shape

The rotor radius (1) is extended by a length (z) and thus the line O1B reaches the rotor (2) at point A. Theoretically, when point K reaches point D, point A reaches K, respectively point N reaches K, because the length of the circle arcs AK, KD and KN is the same. When the piston (5) exits the gap created in the rotor (2), points A and N reach point K; the sealing between the two rotors being ensured by the direct contact between the lateral surfaces of the rotors.
From the right triangle O1O2A results:

$$O_1O_2^2 = AO_2^2 + AO_1^2 \tag{6}$$

$$(2R_r)^2 = R_r^2 + (R_r + z)^2 \tag{7}$$

relation that becomes:

$$z^2 + 2R_r z - 2R_r^2 = 0 \tag{8}$$

$$z_{1,2} = \frac{-2R_r \pm \sqrt{4R_r^2 + 8R_r^2}}{2} \tag{9}$$

Performing the calculations, from relation (9) results $z_1 = 0.732R_r$, $z_2 = -2.73R_r$

The relation (8) specifies the correlation between the height of the piston (z) and the rotor radius (R_r); thus, the housing radius will be:

$$R_c = R_r + z = R_r + 0.732R_r = 1.732R_r \ \ [m] \tag{10}$$

or:

$$R_c = \frac{z}{0.732} + z = 2.366z \quad [m] \tag{11}$$

The relations (8), (10) and (11) give the mathematical connection between R_r, z and R_c.
From the mathematical analysis [6] it is known that between two roots of the function there is a maximum or minimum point of the function; for this purpose, the relation (3) is derived according to z and one can obtain:

$$2z + 2R_r = 0 \tag{12}$$

$$z = -R_r \tag{13}$$

The function f (z) (figure 3) is graphically represented, choosing for R_r the value of 0.05 m which was adopted when constructing a prototype in the laboratories of the Department of Thermotechnics, Engines, Thermal and Refrigeration Equipment's of University Politehnica of Bucharest.
From relation (3) the data from table 1 are obtained.

Table 1: Values for z and f (z)

$z \cdot 10^{-2}$ [m]	-15	-10	-5	0	5
$f(z) \cdot 10^{-3}$ [m]	25	-50	-75	-50	25

Respectively, one can obtain:

$$z_1 = 0.732 R_r = 0.732 \cdot 0.05 = 0.0366 \ [m]$$

$$z_2 = -2.73 R_r = -2.73 \cdot 0.05 = -0.1366 \ [m]$$

The intersection of the function with the oz axis is given by the points: A (0.366; 0), B (- 0.1366; 0).
The end point of the function is C (- 0.0075; - 0.05)
With the data in table 1 and having the coordinates of the intersection points of the function with the oz axis, the function f (z) is plotted in figure 3.

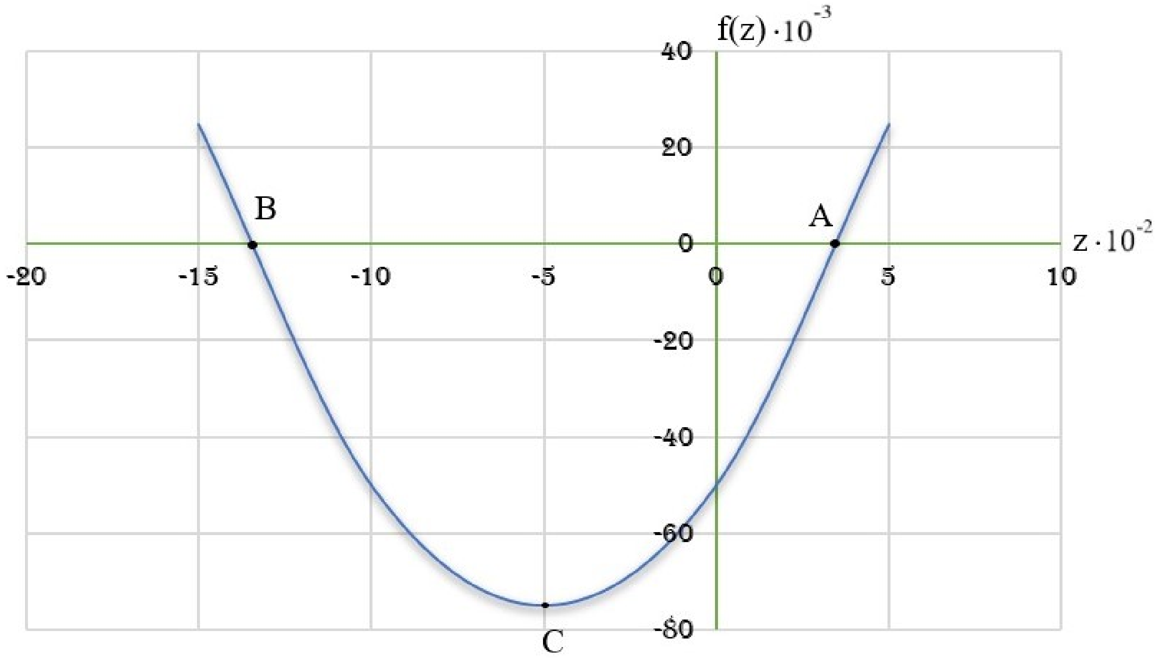

Fig. 3. Graphical representation of the function f (z)

From figure 3 one can observe that the function f (z) has the extreme point for z = - R_r = - 5 cm; the same thing is also given by the relation (8): $P^{/} = f\left(z\right)$ from which the same value results: z = - R_r.
In conclusion, the driving power of the machine is maximum when z = R_r technically acceptable result [7].

5. The constructive solution of the machine when the height of the piston tends towards the rotor radius

The constructive solution presented in figure 4 has the following particularities:
a) The piston height is smaller by 1-2 mm than the rotor radius;
b) The shaft for each rotor does not penetrate inside the rotor, thus z → R_r;
c) The shaft drives the lower rotor through a flange fixed with rotor screws; the flange rotates inside the side wall of the housing (figure 4.b);
d) In the figure 4 the side wall of the housing on the right side is not drawn, which is similar to the wall on the left side; in this way, in view (a) the two rotors are observed.
The elaboration of this constructive solution aims to validate the previously established conclusion, namely that the value of z must tend to R_r.

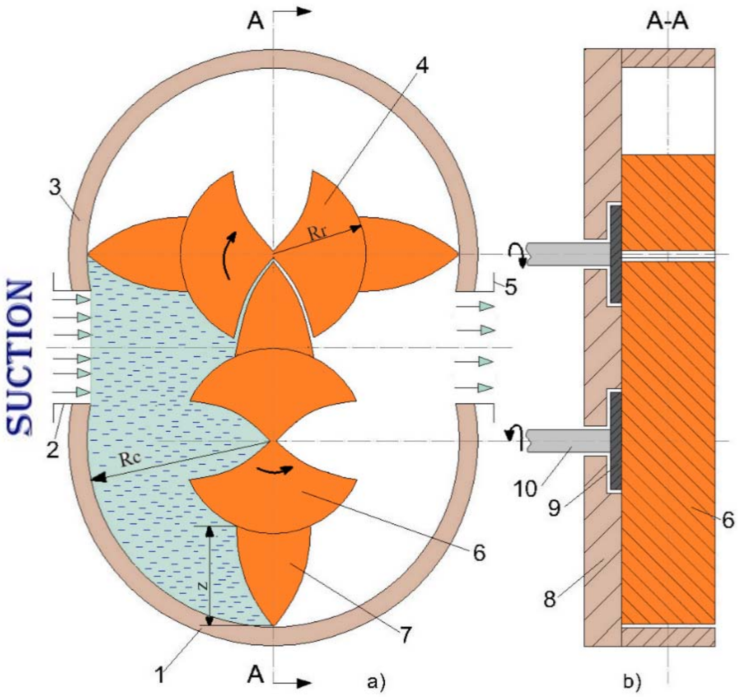

Fig. 4. View (a) and longitudinal section (b) through the rotating machine
a: 1 - lower housing; 2 - fluid suction connection; 3 - upper housing; 4 - upper rotor;
5 - fluid discharge connection; 6 - lower rotor; 7 - rotating piston;
b: 8 - housing side wall (left); 9 - flanges fixed by the rotor; 10 - shaft fixed by the flanges.

Previous researches in the field of rotating machines [8], [9], [10] continue with this new constructive solution where the value of z tends towards R_r.

6. Conclusions

a) Both by deriving the power formula of the machine and by graphically representing the function f (z) = 0 resulted:
- If the rotating machine acts as a working machine (pump, compressor) the required drive power from the outside is maximum if z = R_r; of course, and the flow rate transported by the machines is maximum.

- If the rotating machine acts as a motor machine (steam engine, pneumatic motor), the power developed by it is maximum when $z \to R_r$.

b) In figure 4 a constructive solution of the machine was presented, a solution that allows z to tends towards R_r.

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