THEORETICAL CONSIDERATIONS REGARDING THE MECHANISM FOR ADJUSTING THE CAPACITY OF THE PUMS WITH RADIAL PISTONS

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Abstract

The pumps with radial pistons -ppr- have been little studied in the scientific engineering environment from Romania. Due to the fact that these pumps are very important for the field of hydraulic and pneumatic drives, IHP has set as one of its objectives, the exhaustive study of this issue and the finding of some innovative solutions which to lead to higher performances and a diversification of the applications in which are used pumps with radial pistons.

This article presents the main theoretical elements of the dynamics of the mechanism for adjusting the capacity of the pumps with radial pistons.

1. Introduction

The variation of the flow of the pumps with radial pistons is performed by a positioning servomechanism which modifies the capacity (geometrical volume) of the pump [1], [4]. For explaining explicitly its operation below is briefly described the construction and operation of the pumps with radial pistons.

The pumps with radial pistons-fig.1 consist of the stator ring 1 in whose interior is wheeling the rotor 2 that has, radially positioned, a certain number of cylindrical cavities 3 in which may displace a corresponding number of small pistons 4. The fluid is aspired from the circuit by the aspiration C_a in the aspiration chamber A from where by pumping is carried in the discharge chamber R, respectively in the discharge circuit C_r.

During the wheeling of the rotor the piston heads press on the interior wall of the stator ring, due to the centrifugal force and a guiding system. The axis I – I of the rotor is displaced with distance e facing the axis II – II of the stator. At a complete rotation of the rotor each small piston makes a displacement on a radial direction towards the exterior, during the crossing of the arc length aa and a displacement towards the interior during crosssing the arc length a'a. The radial pistons make during the rotor wheeling a relatively alternative rectilinear move to the rotor.

During the displacement towards the exterior the piston cylinders are in connection with the aspiration chamber A, performing the aspiration.

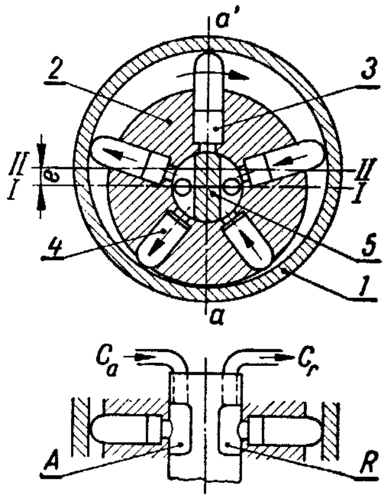

Fig. 1

During the displacement towards the interior the cylinders are in connection with the discharge chamber R, performing the discharge. The pumping becomes possible due to the relatively excentric position of the rotor towards the stator ring. The flow of the pumps with radial pistons is proportional with a constructive constant C and with two variable parameters, eccentricity *e* and turation *n* of the pump.

$$Q = C \cdot e \cdot n$$

The result *„C · e”* is called capacity or geometric vollume and it is expressed $V_g = C \cdot e$.

The variation of capacity it is obtained by the variation of eccentricity between the two set limits + *e* şi – *e*.

By the eccentricity variation varies the relative position of the rotor to the stator ring and as a consequence the displacement speed of the small pistons in the cylinders. At null eccentricity the flow is also null.

2. The forces developed by the pistons of the pumps with radial pistons

The symbols used and their meaning

$\mathbf{F_p}$ axial force given by the piston

$\mathbf{F_M}$ the $\mathbf{F_p}$ projection on the normal *n* at the sliding ring in the contact point

$\mathbf{F_x}$, $\mathbf{F_y}$ the projections of force F_M on the system of axis X O Y

ρ instant radius units the rotation center O_1 with the point of contact on the sliding ring; **e** the eccentricity of the pump;

β the angle between the direction of the piston axis and the normal at the cirumference of the sliding ring in the point of contact of the piston axis; **R** the radius of the sliding ring; $\mathbf{d_p}$ the dyameter of the piston; $\overline{\alpha}$ the current angle of rotation of the piston K – 1; **z** the number of pistons; $\mathbf{n_a}$ the number of active pistons in discharge; $\mathbf{l_p}$ the length of the piston; $\bar{b}_m$ the minimum contact length rotor piston; **L** the hypothetical contact length sliding ring – case; **j** the gap case-sliding ring; $\bar{b}(\alpha)$ the instant contact width sliding ring-case; $\bar{b}$ the average contact length rotor-piston; η dynamic viscosity $\left(0{,}032\,\frac{kg}{m\cdot s}\right)$; **a,b** the halfaxis of the ellipse; **2** φ the angle between 2 pistons; $\mathbf{R_E}$ the exterior radius of the stator ring; $\mathbf{R_M}$ the average radius of the stator ring; **E** elasticity module $\left(2{,}1\cdot10^{11}\,\frac{kg}{m\cdot s^2}\right)$; $\bar{x}$ the halfwidth of the segment of intersection ellipse circle; **F** the resultant of forces; **f** distortion from the circular shape under the action of the force (hypothesis rigid case); $\mathbf{t_H}$, $\mathbf{t_h}$ tolerances at adjustment corresponding to the dimension R_E; **m** total mass; $\mathbf{T_C}$ time constant of the mechanical system; $\mathbf{K_E}$ the slope of the characteristic $\Delta p = f(i)$; $\mathbf{a_0}$, $\mathbf{a_1}$ modules area mass; **I** inertial moment

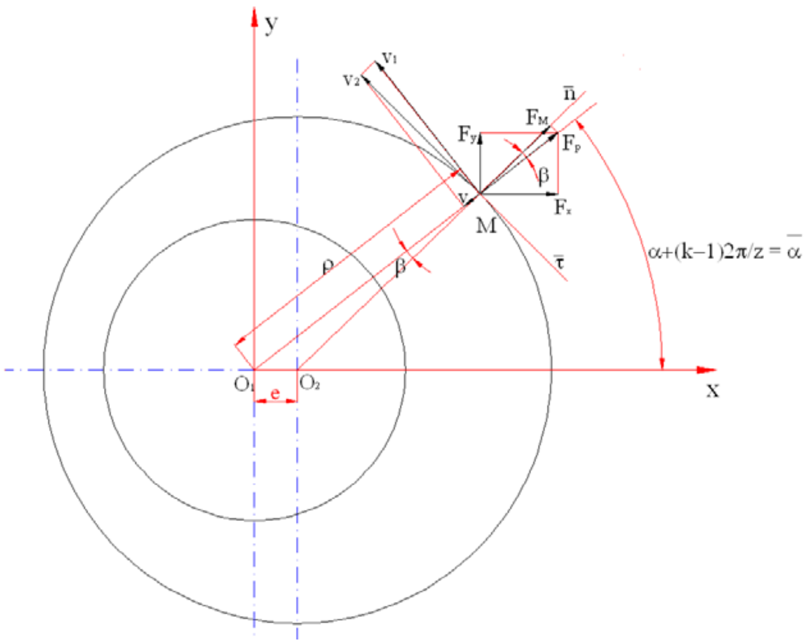

Fig. 2

a) The kinematics of the system

$O_2M = R;\quad O_1M = \rho$; It is applied the theorem of sinuses and it is obtained:

$$\frac{e}{\sin\beta} = \frac{R}{\sin\bar{\alpha}} = \frac{\rho}{\sin\left(\pi-\beta-\bar{\alpha}\right)};\ \sin\left[\pi-\left(\beta+\bar{\alpha}\right)\right] = -\cos\pi\,\sin\left(\beta+\bar{\alpha}\right) = \sin\left(\beta+\bar{\alpha}\right)$$

$$\frac{e}{\sin\beta} = \frac{R}{\sin\bar{\alpha}}\quad;\qquad \sin\beta = \frac{e}{R}\sin\bar{\alpha}$$

At the axial components of the forces F_p with which the pistons operate on the sliding ring of the pump – force F_x oposses force F_x developed by the linear hydraulic motor – with piston of higher diameter of the system, (fig. 7) force which positions the ring in any point of the eccentricity – e.

$F_M = F_p \cos\beta$ The force of one piston: $F_x^1 = F_M \cos(\bar{\alpha}+\beta) = F_p \cos\beta \cos(\bar{\alpha}+\beta)$

where: $\cos(\bar{\alpha}+\beta) = \cos\bar{\alpha}\cos\beta - \sin\bar{\alpha}\sin\beta = \cos\bar{\alpha}\sqrt{1-\frac{e^2}{R^2}\sin^2\bar{\alpha}} - \left(\frac{e}{R}\sin\bar{\alpha}\right)\sin\bar{\alpha}$

Developing in binominal series and taking into account that $(\sin\bar{\alpha})_{max} = 1$, and the relation

$$\frac{e}{R} \cong 10^{-1}; \quad 1-\left(\frac{e^2}{R^2}\sin^2\bar{\alpha}\right)^{\frac{1}{2}} = 1-\frac{1}{2}\cdot\frac{e^2}{R^2}\sin^2\bar{\alpha} - \frac{1}{8}\frac{e^4}{R^4}\sin^4\bar{\alpha} \cong 1-\frac{e^2}{2R^2}\sin^2\bar{\alpha}; \quad \frac{e^4}{R^4} \cong 10^{-4} \cong 0$$

b) The precise formula of the unitary forces on the two directions x and y, is:

$$F_M = F_p \cos\beta \begin{cases} F_x^1 = F_M \cos(\bar{\alpha}+\beta) = F_p \cos\beta \cos(\bar{\alpha}+\beta) \\ F_y^1 = F_M \sin(\bar{\alpha}+\beta) F_p \cos\beta \sin(\bar{\alpha}+\beta) \end{cases}$$

if $\beta \cong 0$, it results:

c) The approximate formula of the unitary forces of a piston become:

Axial force : $F_x^1 = F_p \cos\bar{\alpha}$ (in opposition with the force of the positioning mechanism);

Normal force : $F_y^1 = F_p \cos\bar{\alpha}$ (normal force which determines the friction of the sliding ring).

d) The force in the piston

The force developed by the pistons of the pump is given by the pressure of the hydraulic oil from which is decreased the force caused by friction between the pistons and the rotor of the pump:

$$F_p = \frac{\pi d_p^2}{4} p - \pi d_p \left(\frac{j}{2} p - \frac{\eta \bar{b}}{j} V\right); \quad Cu: \rho \cong R - e\cos\bar{\alpha} \; si \text{ viteza}: \; V = \dot{\rho} = +e\omega\sin\bar{\alpha}$$

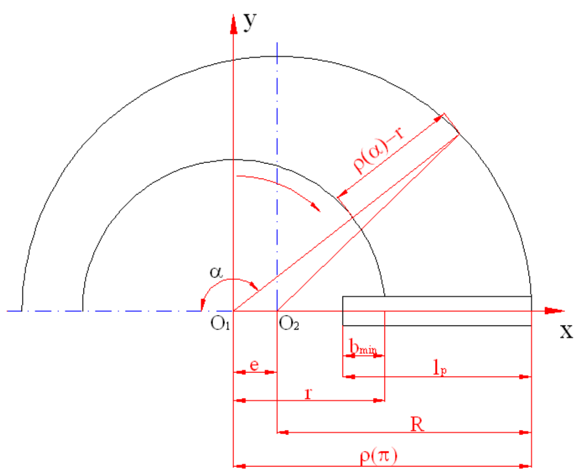

Fig. 3

From fig. 3 it results:

$$\begin{cases} l_p = \bar{b}_{\min} + \rho(\pi) - r \\ l_p = b(\alpha) + \rho(\alpha) - r \\ b(\alpha) + \rho(\alpha) = b_{\min} + \rho(\pi) \end{cases}$$

$$b(\alpha) = b_{\min} + \rho(\pi) - \rho(\alpha) \rightarrow \begin{cases} \rho(\pi) = R + e \\ \rho(\alpha) = R - e\cos\alpha \end{cases}$$

$$b(\alpha) = b_{\min} + (R + e) - (R - e\cos\alpha)$$
$$b(\alpha) = b_{\min} + e + e\cos\alpha$$

$$\overline{b}(\alpha) = \frac{1}{\pi - 0}\int_0^{\pi} b(x)\,d\alpha = \frac{1}{\pi}\left[(b_{\min} + e)\alpha + e\sin\alpha\right]\Big|_0^{\pi} =$$

$$= \frac{1}{\pi}\left[(\bar{b}_{\min} + e)\pi\right] = b_{\min} + e;\ Deci : \bar{b} = b_{\min} + e$$

Replacing the expression of speed in the formula of the force developed by the pistons of the pump it is obtained:

$$F_p = \frac{\pi d_p^2}{4}\cdot p - \frac{\pi j d_p}{2} p + \frac{\pi \eta \bar{b} d_p}{j} e\omega\sin\alpha = F_p = \frac{\pi d_p}{2}\left(\frac{d_p}{2} - j\right)p + \frac{\pi\eta\bar{b} e\omega d_p}{j}\sin\alpha$$

3. The force required at the positioning mechanism

The positioning mechanism must conquer the sum of the forces F_x which represent the horizontal components of the force of the active pistons (being under pressure in the discharge chamber) :

Noting with RF_x this resultant su mis obtained:

$$RF_x = F_p\sum_{K=1}^{na}\cos\alpha = F_p\sum_{K=1}^{na}\cos\left[\alpha + (k+1)\frac{2\pi}{z}\right] == F_p\left[\sum_{K=1}^{na}\left(\cos\alpha\cos(K-1)\frac{2\pi}{z} - \sin\alpha\sin(K-1)\frac{2\pi}{z}\right) = \right.$$

$$\left. = F_p\left[\cos\alpha\sum_{K=1}^{na}\cos(K-1)\frac{2\pi}{z} - \sin\alpha\sum_{K=1}^{na}\sin(K-1)\frac{2\pi}{z}\right] = F_p\left[\cos\frac{2\pi}{z}\cos\alpha - \frac{\cos\frac{\pi}{z}}{\sin\frac{\pi}{z}}\sin\alpha\right]\right.$$

a) The normal force which determines the friction of the slide

The unitary force on axis y has the value: $F_y^1 = F_p\sin\left[\alpha + (K-1)\frac{2\pi}{z}\right]$

The resultant of all pistons from the discharge chamber pressure chamber ÷ RF_y is:

$$R\,F_y = F_p \sum_{K=1}^{na}\left(\sin\alpha\cos(K-1)\frac{2\pi}{z} + \cos\alpha\sin(K-1)\frac{2\pi}{z}\right) =$$

$= F_p\left(\sin\alpha\sum_{K=1}^{na}\cos(K-1)\frac{2\pi}{z} + \cos\alpha\sum_{K=1}^{na}\sin(K-1)\frac{2\pi}{z}\right)$; First and second sum, respectively S_1 and S_2 being $S_1 = \cos\frac{2\pi}{z}$; $S_2 = \frac{\cos\frac{\pi}{z}}{\sin\frac{\pi}{z}}$ In final: $RF_y = \left(\cos\frac{2\pi}{z}\sin\alpha + \frac{\cos\frac{\pi}{z}}{\sin\frac{\pi}{z}}\cos\alpha\right)$

b) sliding ring - housing friction

In correspondence with the notations from fig. 4

If : $K_v = \frac{\bar{b}\,L\,\eta}{2\,j}$ is coefficient of viscous friction,

Then existing two forces of the kind it results :

$F_f = \frac{\bar{b}\,L\,\eta}{j}\dot{x}$, where $\dot{x}$ is the speed of the ring

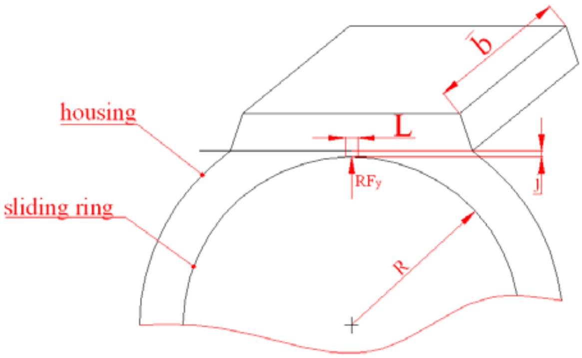

Fig. 4

c) The length of the contact surface: L

L is deduced from the distortion of the ring under the action of all forces by determining the points of intersection between the initial circle and the ellipse resulting due to the charge with the respective forces.

The equations of the ellipse and circle from fig. 5 are :

$$\begin{cases}\dfrac{x^2}{(R_E+f)^2} + \dfrac{y^2}{(R_E-f)^2} = 1\\ x^2 + y^2 = R^2\end{cases}$$

As $2\varphi = \frac{2\pi}{z}$; z-number of pistons

Deformation: $f = -\frac{F\,R_M^3}{3\,EI}\left[\frac{2}{\varphi} - \frac{1}{\sin\varphi} - \frac{\varphi\cos\varphi}{\sin^2\varphi}\right]$

In which the resistence module:

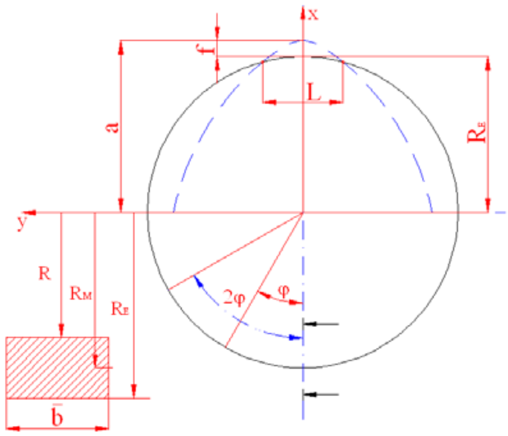

Fig. 5

$$I = b\,R_M^2\left(R_M \ln\frac{2R+h}{2R-h} - h\right) \quad \text{with expressions} \rightarrow \begin{cases} \mathrm{b-width} \\ \mathrm{R_M = \dfrac{R_E + R}{2}} \\ \mathrm{h = R_E - R} \end{cases}$$

The maximum force is the sum of the resultants:

$$F_{\max} = \sqrt{\left(RF_x\right)_{\max}^2 + \left(RF_y\right)_{\max}^2}$$

For the position from fig. 6.7 may be written :

$$tg\ \alpha_{\max} = \frac{\left(RF_y\right)_{\max}}{\left(RF_y\right)_{\max}} \qquad \varphi = \frac{\pi}{z}$$

$$F = F_{\max}\sin\alpha_{\max}$$

It is solved the system (the intersection between circle and ellipse) and are obtained :

$$x = \sqrt{R^2 - y^2}; \qquad \frac{R^2 - y^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad in\ care \begin{cases} a = R + \\ b = R - \end{cases}$$

$$\bar{y} = +\sqrt{\frac{1 - \dfrac{R^2}{b^2}}{\dfrac{1}{b^2} - \dfrac{1}{a^2}}} \qquad \begin{matrix} b > R \\ a > b \end{matrix}$$

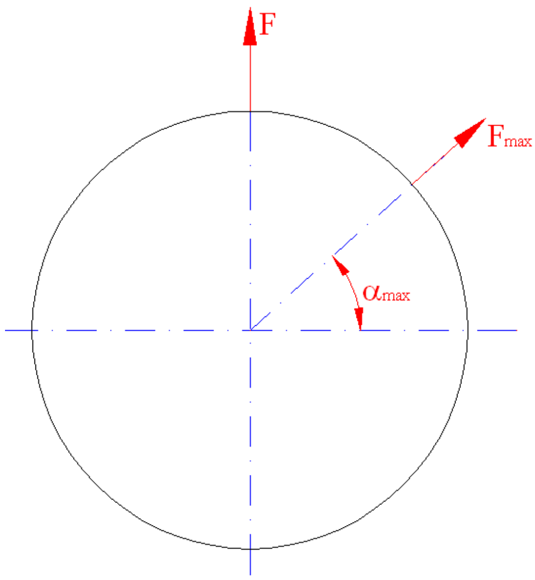

Fig. 6

It results that the length L is: $L \le 2\bar{y}$

d) The gap: j corresponds to the average game of the fit $\dfrac{H7}{h7}$

$$j = \frac{t_H - \left(-t_h\right)}{2} \qquad t_H; t_h \quad \text{tolerances at fit}$$

May be calculated now:

$$\mathrm{K_V = \frac{\bar{b}\,L\,\eta}{j}},\ \mathrm{cu} \qquad \begin{cases} \mathrm{L, j} & \text{determined} \\ \mathrm{\bar{b}} & \text{the ring width} \\ \mathrm{\eta = 0{,}0342\,\dfrac{kg}{m \cdot s}} & \text{(dynamic viscosity)} \end{cases}$$

In the end the equation of the friction force: $F_f = K_V \cdot \dot{x}$, becomes definite entirely.

4. The balance of the forces from the positioning mechanism

In the correspondence with the notations from fig. 7 may be written:

$F_i + F_f + F_e + RF_x = A(p - \Delta p) - a\ p$, where "A" is the area of the bigger piston and a" of the small piston, Δ_p being the pressure drop on the valve :

The inertial force:

$F_i = (M_p + m_p + M_i)\cdot \ddot{x}$, with

M_p: the mass of the bigger piston;

M_p: the mass of the smaller piston;

M_i: the mass of the sliding ring

The elastic force:

$F_e = (K_1 - K_2)\ x$,

K_1 and K_2 – the rigidity of the arcs

The viscous friction force:

$F_f = K_V\ \dot{x}$

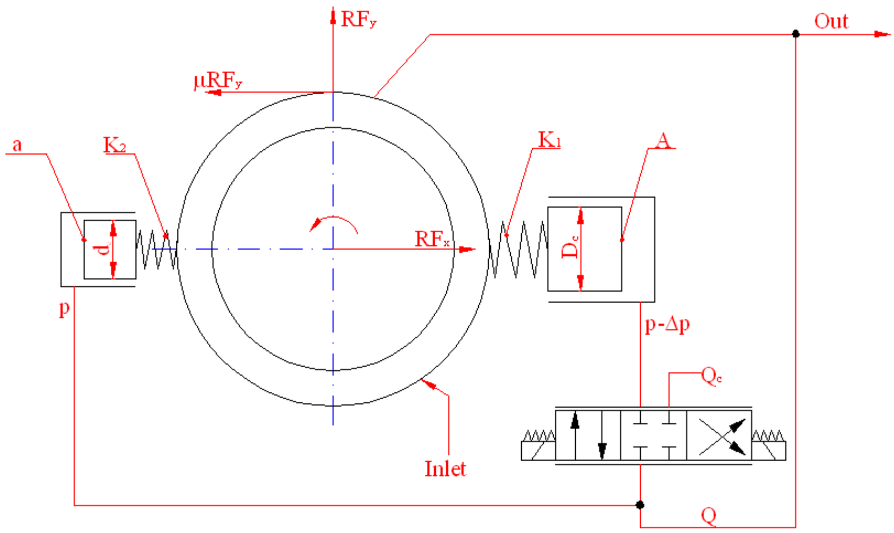

Fig. 7

The resultant of the axial forces RF_x is:

$$RF_x = \frac{\pi d_p^2}{\varphi}\cdot p \cdot \left[\cos\frac{2\pi}{z}\cos\alpha - \frac{\cos\frac{\pi}{z}}{\sin\frac{\pi}{z}}\sin\alpha\right] \qquad \text{where: } \alpha = \omega t$$

If it is noted : $f(t) = \frac{\pi d_p^2}{\varphi}\left[\cos\frac{2\pi}{z}\cos\alpha - \frac{\cos\frac{\pi}{z}}{\sin\frac{\pi}{z}}\sin\alpha\right]$, results: $RF_x = p\ f\ (t)$

In the end the equation of balance has the following structure:

$$\begin{cases} m\ddot{x} + K_V\dot{x} + (K_1 - K_2)\,x = [(A - a) - f(t)]\,p - A\cdot\Delta p \\ x(0) = \dot{x}(0) = 0 \\ \Delta p = K_E i \quad ; \quad \alpha = \mathrm{arc\ tg}\ K_E ; K_E \quad \text{characteristic slope} \\ \Delta p = \frac{1}{4}p \end{cases}$$

5. Remarks and conclusions

The determination of the constructive operational parameters and the dimensioning of the servomechanism which adjusts the capacity of the pumps with radial pistons lead us to complex mathematical relations and at non linear diferential equations whose solving is made using numerical calculation systems.

The electrohydraulic adjustment systems [2] are very complex systems where take place phenomena associated to the flowing of fluids from the field of volumetric hydraulic machines and also phenomena specific for the processes of automatic adjustment. Due to the complexity of these phenomena the finding of most adequate solutions in designing and realizing the mit is made iteratively. Reaching such performances [3] implies the use of the methods of mathematical modelling and numerical simulation of these systems.

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