

## ADAPTIVE ROBUST CONTROL APPLICABLE ON VARIABLE PUMPS

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**Abstract:** *The focus of the paper is on the nonlinear model based control of systems with unknown parameters and uncertain nonlinearities. The objective is to maximize the achievable performance of a controlled system and to obtain accurate parameter estimates. This is achieved by integrating the excellent output tracking performance achieved by the direct adaptive robust control with the good parameter estimation process of indirect adaptive designs. The paper contains a brief description of the nonlinear control algorithm and concludes with the results that demonstrate the high quality of the nonlinear controller*

**Keywords:** *nonlinear system, nonlinear control, uncertain nonlinearities.*

### 1. Introduction

Essential trends, manifested today in hydraulic machines construction are those of flexibility and automation, meaning to increase their level of intelligence and adaptation to possible disturbances. [1]

Variable displacement pumps allow easy control of system parameters (pressure, flow, power, or combinations between them). Their technical characteristics make them become the best option for most applications from machine tools to mobile devices. [1]

Companies with a tradition in manufacturing pumps and motors with axial piston and variable displacement, produce this machineries for high automation systems (Rexroth, Bosch, Vickers, Parker.) the existing data in the literature on constructive solutions are few.

The biggest producers of hydraulic machines are opting for mechano-hydraulic control structures, that allow their use in circuits regulating pressure, flow and power independently one of another, each parameter control requiring a different type of constructive control structure .

For many years, the research in control of hydraulic machines, was focused on linear control, this was mostly due to the simplicity and ease of implementation of these methods.

Besides the nonlinear nature of the dynamic behavior, hydraulic systems have a lot of parametric uncertainties, which are found in variations of load and variations of hydraulic parameters such as modulus of elasticity. Uncertainties related to the nonlinearities are favored by: external disturbances, leakage flow, unmatched friction, etc. For these reasons, most often for nonlinear systems best control structure is a nonlinear one.

In this paper is developed an algorithm for the analysis of nonlinear systems, which integrates Direct Adaptive Robust Control and Indirect Adaptive Robust Control (D. / I.A.R.C.).

## 2. Nonlinear control algorithm

Functional diagram of the system that is intended to be controlled by two nonlinear methods is presented in Figure 1.

Below is presented a schematic diagram for automatic control system proposed for implementation in a research program [1]. The system contains the following components: 1 - variable displacement pump with axial pistons; 2 - linear hydraulic motor needed to change the angular position of the piston block holder, so modify the flow of the pump; 3 – proportional directional valve, that control the position of the linear motor, 4 - pressure sensors; 5 – diaphragm, needed to measure the flow rate of the pump; 6 – electronic circuits with the following attributes: calculate the pressure drop on the diaphragm, then determine the flow, and then with the signal from a pressure sensor and the signal that represents the flow is obtained the hydraulic power generated by the pump; 7 - electronic comparator, designed to find the error between programmed and actual value of the adjusted parameter (pressure, flow, power); 8 – electronic controller, used to compensate the errors and gives the command signal for the proportional valve; 9 - switches whose state determines the control structure; 10 – fixed displacement pump, provides the necessary flow for positioning hydraulic motor; this flow can be taken from the adjustable pump’s flow, in this case the auxiliary pump is no longer required; 11 – relief valve, protects the system to not exceed the permissible pressure in hydraulic components. Thus without change in pump construction, this can be integrated into any control circuit for adjustable hydraulic machines, by simply actuation of an electrical switch.

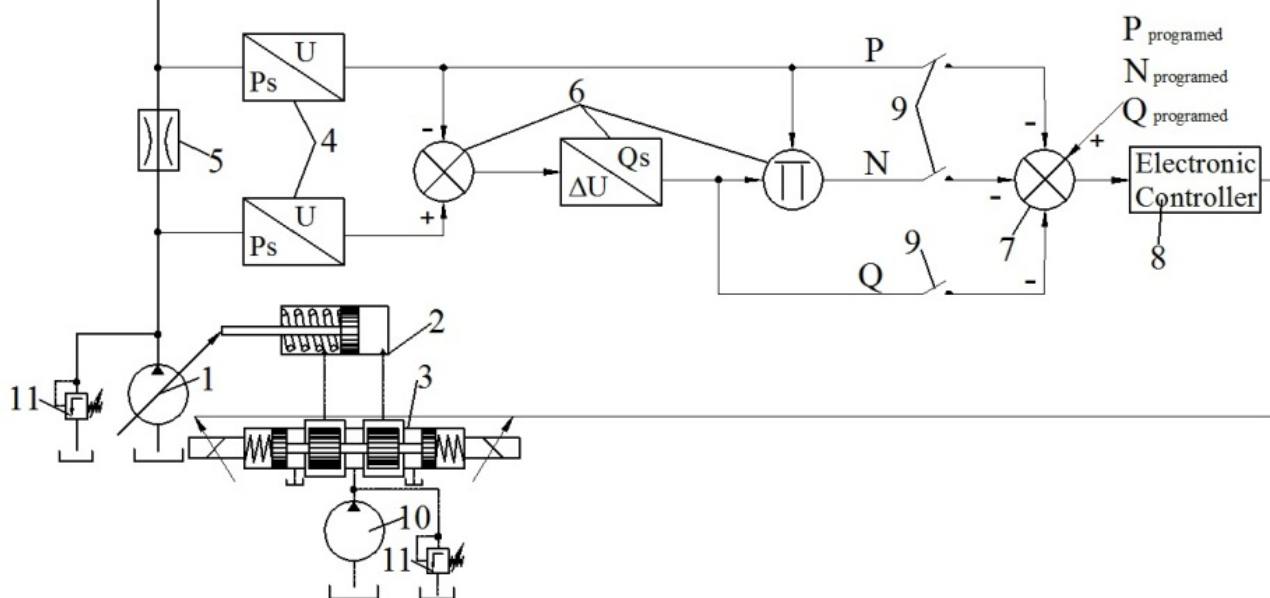


Figure 1 Functional diagram of the investigated system

Mechanical equilibrium equation of the linear motor is:

$$m_p \cdot \ddot{x}_m = A \cdot p_A - \alpha_a \cdot A \cdot p_B + F_{am} + k_m \cdot x_m - (C_3 + C_4) \dot{x}_m + \frac{m \cdot \omega^2 \cdot R^2}{a^2} \cdot x_m - \frac{\pi \cdot d^2 \cdot R}{4a} \cdot p_s + \tilde{f}(t, x_m, \dot{x}_m), \quad (1)$$

where  $\tilde{f}(t, x_m, \dot{x}_m)$  is the error that incorporates external disturbances and friction forces non modelled.

the notations have the following meanings:

$\dot{p}_A$  – the temporal derivative of the pressure function,  $p_A$ , in the large chamber of the linear hydraulic motor;  $\dot{p}_B$  – the temporal derivative of the pressure function,  $p_B$ , in the small chamber of the linear hydraulic motor;  $p_A$  – the pressure in the large chamber of the linear hydraulic motor;  $p_B$  – the pressure in the small chamber of the linear hydraulic motor;  $p_T$  – tank pressure, ( $p_T=0$ );  $p_c$  – the pressure between the pump and control valve;  $p_s$  – load pressure;  $E_U$  – elasticity modulus;  $V_A$  – the volume of oil under pressure  $p_A$ ;  $V_B$  – the volume of oil under pressure  $p_B$ ;  $V_T$  – (dead) volume in the supply circuit of the linear motor, (connecting pipes volume);  $Q_A$  – flow rate that enters or is discharged from the large chamber of the positioning hydraulic motor;  $Q_B$  – the flow rate that enters or is discharged from the small chamber of the positioning hydraulic motor;  $A$  – the piston area (rodless);  $x_m$  – linear position of the hydraulic motor;  $c_{LG}$  – leakage flow coefficient dependent of speed;  $c_{LP}$  – leakage flow coefficient dependent of pressure;  $\alpha$  – piston surface ratio;  $\alpha_Q$  – flow rate coefficient;  $d_v$  – proportional valve diameter;  $x_v$  – linear position of the proportional valve;  $\rho$  – oil density;  $T_v$  – time constant for control valve;  $K_v$  – the gain of control valve;  $m_p$  – linear motor piston mass;  $F_{am}$  – preload force of the spring in the linear motor;  $k_m$  – spring stiffness;  $c_3, c_4$  – viscous damping coefficient;  $m$  – piston and rod of the variable pump's, reduced mass;  $\omega$  – angular velocity of the pump's piston holder;  $R$  – pump's pistons placement radius;  $a$  – tilting radius of the variable pump;  $d$  – pump's pistons diameter.

Continuity equations are:

$$Q_A = \begin{cases} k_v \cdot x_v \cdot \sqrt{p_C - p_A}, x_v \geq 0 \\ k_v \cdot x_v \cdot \sqrt{p_A}, x_v < 0 \end{cases} = g_3; \quad (2)$$

$$Q_B = \begin{cases} k_v \cdot x_v \cdot \sqrt{p_B}, x_v \geq 0 \\ k_v \cdot x_v \cdot \sqrt{p_C - p_B}, x_v < 0 \end{cases} = g_4. \quad (3)$$

The equations that that show the forming of the pressure in the linear motor are:

$$\dot{p}_A = \frac{E_u}{V_T + A \cdot x_m} \left( Q_A - A \cdot \dot{x}_m + \tilde{Q}_A(p_A, p_B, p_S, p_T) \right); \quad (4)$$

$$\dot{p}_B = \frac{E_u}{V_T + \alpha_a \cdot A(L - x_m)} \left( -Q_B + \alpha_a \cdot A \cdot \dot{x}_m - \tilde{Q}_B(p_A, p_B, p_S, p_T) \right), \quad (5)$$

where  $\tilde{Q}_A, \tilde{Q}_B$  is the modelling error for flow equations.

Are defined the following variables in state space:

$$x = [x_1, x_2, x_3, x_4]^T = [x_m, \dot{x}_m, p_A, p_B]^T. \quad (6)$$

Considering  $x_d(t)$  the desired position of the hydraulic differential motor, it seeks to obtain an input  $u$ , so that the size of output  $y=x_1$ , to be as close as possible to the ordered value, despite various uncertainties of the model.

The system is subjected to parametric uncertainties due to the variation of the elasticity modulus, friction and damping, and the nominal value of the modelling errors ( $d, d_n$ ).

The equations system with input size  $u = x_v$  in state space becomes:

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m_p} [x_3 \cdot A - x_4 \cdot \alpha_a \cdot A + x_1(F_{em} + k_1) - (C_3 + C_4) \cdot x_2 - k_2 + d] \\ \dot{x}_3 = \frac{E_u}{V_A(x_1)} [-A \cdot x_2 + g_3(x_3, \text{sign}(u) \cdot u) + \tilde{Q}_A] \\ \dot{x}_4 = \frac{E_u}{V_B(x_1)} [\alpha_a \cdot A \cdot x_2 - g_4(x_4, \text{sign}(u) \cdot u) - \tilde{Q}_B] \\ d = \tilde{f}(t, x_1, x_2) \\ V_A(x_1) = V_T + A \cdot x_1 \\ V_B(x_1) = V_T + \alpha_a \cdot A(L - x_1) \end{array} \right. \quad (7)$$

For simplicity, we have considered only modulus  $E_u$ , and the nominal value of the modelling error  $d, d_n$ . With the remaining parameters can do the same, if necessary.

It defines the following set of unknown parameters:

$$\lambda = [\lambda_1, \lambda_2]^T; \quad (8)$$

$$\lambda_1 = d_n, \lambda_2 = E_u. \quad (9)$$

With this system described in state space can be linearized according to  $\lambda$  as follows:

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m_p} [x_3 \cdot A - x_4 \cdot \alpha_a \cdot A + x_1 (F_{em} + k_1) - (C_3 + C_4) \cdot x_2 - k_2 + d] + \lambda_1 + \tilde{d}(t, x_1, x_2) \\ \dot{x}_3 = \frac{\lambda_2}{V_A(x_1)} \cdot (-A \cdot x_2 + Q_A + \tilde{Q}_A) \\ \dot{x}_4 = \frac{\lambda_2}{V_B(x_1)} \cdot (\alpha_a \cdot A \cdot x_2 - Q_B - \tilde{Q}_B) \\ \tilde{d}(t, x_1, x_2) = d - d_n \end{array} \right. \quad (10)$$

Uncertain parameters and uncertain nonlinearities satisfy the following:

$$\lambda \in \Omega_\theta = \{\lambda : \lambda_{\min} < \lambda < \lambda_{\max}\}; \quad (11)$$

$$|\tilde{d}(t, x_1, x_2)| \leq \delta_d \cdot (x_1, x_2, t); \quad (12)$$

$$|\tilde{Q}_i(x_3, x_4, p_C, p_T)| \leq \delta_{Q_i}(x_3, x_4, p_C, p_T); i: A, B, \quad (13)$$

where:  $\delta_d, \delta_{Q_A}, \delta_{Q_B}$  - known and:

$$\lambda_{\min} = [\lambda_{1\min}, \lambda_{2\min}]^T; \quad (14)$$

$$\lambda_{\max} = [\lambda_{1\max}, \lambda_{2\max}]^T, \quad (15)$$

Are made the following notations:

$\tilde{\lambda}$  is the estimated error of  $\lambda$ ;

$\hat{\lambda}$  - estimation of  $\lambda$ .

$$\tilde{\lambda} = \hat{\lambda} - \lambda. \quad (16)$$

Define the following discontinuous projections:

$$proj_{\lambda_i}(\bullet_i) = \begin{cases} 0, \text{daca } \hat{\lambda}_i = \lambda_{i\max} \text{ si } \bullet_i > 0; \\ 0, \text{daca } \hat{\lambda}_i = \lambda_{i\min} \text{ si } \bullet_i < 0; \\ \bullet_i. \end{cases} \quad (17)$$

And the saturation function:

$$sat_{\lambda_M}(\Gamma \tau) = S_0 \Gamma \tau; \quad (18)$$

$$S_o = \begin{cases} 1, & \text{daca } \|\Gamma \tau\| \leq \lambda_M; \\ \frac{\lambda_M}{\|\Gamma \tau\|} & \text{daca } \|\Gamma \tau\| > \lambda_M. \end{cases} \quad (19)$$

It uses an adaptation law given by:

$$\hat{\lambda} = \text{proj}_{\lambda}^{\lambda_M} \left( \text{sat}_{\lambda_M}^{\lambda} (\Gamma \tau) \right), \quad (20)$$

where:  $\Gamma > 0$  is a diagonal matrix;

$\tau$  – adapting function;

$\lambda_M$  – upper limit of the adaptation rate.

It is known that for  $\forall \tau$  the projection used in equation (20) guarantees:

$$(P_1) \hat{\lambda} \in \bar{\Omega}_{\lambda} = \left\{ \hat{\lambda} : \lambda_{\min} \leq \hat{\lambda} \leq \lambda_{\max} \right\}; \quad (21)$$

$$(P_2) \tilde{\lambda}^T \left( \Gamma^{-1} \text{proj}_{\lambda}^{\lambda_M} \left( \left( \text{sat}_{\lambda_M}^{\lambda} (\Gamma \tau) \right) - \Gamma^{-1} \text{sat}_{\lambda_M}^{\lambda} (\Gamma \tau) \right) \right) \leq 0; \quad (22)$$

$$(P_3) \left\| \hat{\lambda} \right\| \leq \lambda_M. \quad (23)$$

Are defined the following functions:

$$z_1 = x_1 - x_d(t); \quad (24)$$

$$z_2 = \dot{z}_1 + k_1 \cdot z_1 = x_2 - x_{2eq}. \quad (25)$$

## Controller design

Nonlinear controller design requires three steps:

### Step 1

Is chosen a virtual control law given by the relations:

$$\alpha_1(x_1, t) = \alpha_{1a} + \alpha_{1S1}; \quad (26)$$

$$\alpha_{1a} = \dot{x}_d; \quad (27)$$

$$\alpha_{1S1} = -k_{1S1}z_1, \quad (28)$$

where  $k_{1S1}$  is gain factor of the positive feedback.

### Step 2

The second equation of the system (10):

$$-\dot{x}_2 = b_2x_3 + \phi_2^T \lambda_C + \Delta_2; \quad (29)$$

$$b_2 = \frac{A}{m_p}; \quad (30)$$

$$\phi_2^T = [\alpha_a \cdot A(F_{am} + k_1)(C_3 + C_4) - k_2]; \quad (31)$$

$$\lambda_C = [\lambda_1 \lambda_2]. \quad (32)$$

Is chosen the adjustment law for  $x_3$ :

$$\alpha_2 \left( \bar{x}_2, \hat{\lambda}_C, t \right) = \alpha_{2a1} + \alpha_{2a2}; \quad (33)$$

$$\alpha_{2a1} = \frac{1}{\hat{b}_2} \left[ k_{1S1} \cdot x_2 + k_{1S1} \cdot x_d + \ddot{x}_d - \phi_2^T \hat{\lambda}_C - z_1 \right]; \quad (34)$$

$$\alpha_{2a2} = -\frac{1}{\hat{b}_2} \cdot \lambda_2. \quad (35)$$

### Step 3

At this step is defined the desired position  $x_v$  of the valve, that is considered the input of the system.

$$\dot{x}_3 = b_3u + \phi_3^T \lambda_C; \quad (36)$$

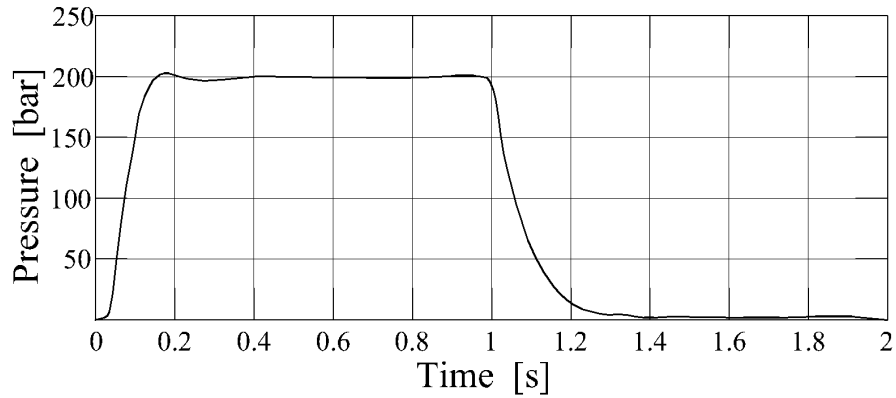
$$u \left( \bar{x}_3, \hat{\theta}_C, t \right) = u_a + u_s; \quad (37)$$

$$F_p = \frac{d_A \cdot A}{V_A} + \frac{Q_B \cdot \alpha_a \cdot A}{V_B} = u; \quad (38)$$

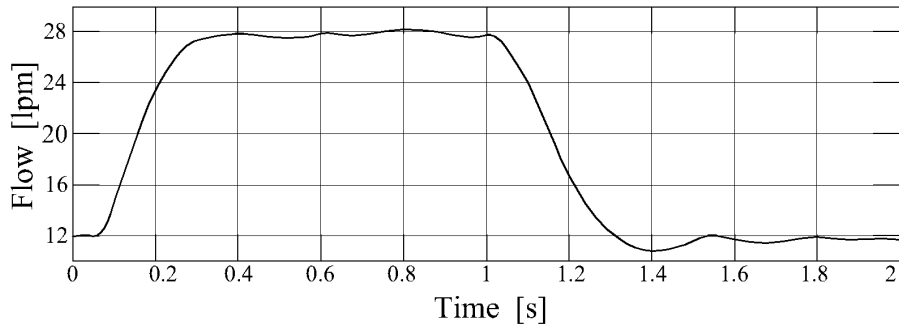
$$x_v = \frac{F_p}{\frac{g_3 \cdot A \cdot k_v}{V_A} + \frac{g_4 \cdot \alpha_a \cdot A \cdot k_v}{V_B}}. \quad (39)$$

### 3. Results and Conclusions

Was analyzed the behavior of the system like response to step command for pressure, flow and power. When adjusting the pressure the control step represents the input signal corresponds to a variation in load pressure from 0 to 200 bar, (Fig. 2a). When setting the flow, the control step representing the input signal corresponds to a variation in flow from 0 to 30 l/min, (Fig. 2b). In control of the power the version the step control that represents the input signal corresponds to a change in power in 0-5 kW, (Fig. 2c).



a)



b)

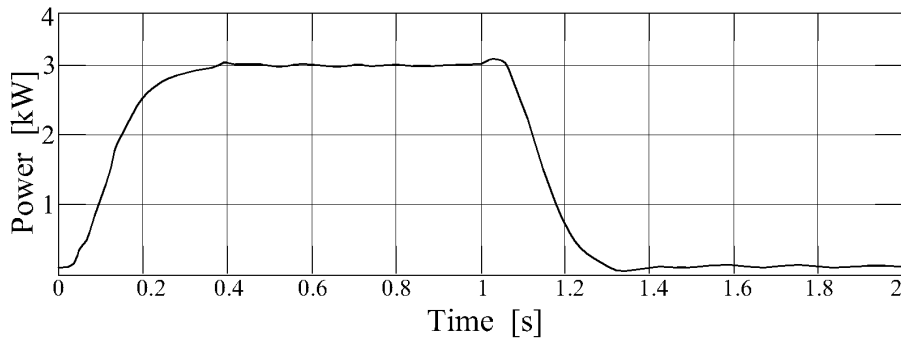


Figure 2 Dynamic behaviour of the system (Nonlinear Control)



The controller presented in this paper have a good potential to use it in electro-hydraulic control systems with variable displacement pumps, it can get both good dynamic behaviour and a more precise estimate of the system parameters, the performance of which are difficult to reach for a nonlinear systems with regulators and classical control strategies.

Axial piston machines with variable displacement and electro-hydraulic control system allow a complete automation, compatible with the operating ciclograms of the complex equipments by interfacing with a PLC or process computer.

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