ANALYTICAL MODEL OF THE CONNECTION PIPES OF THE ALTERNATING FLOW DRIVEN HYDRAULIC SYSTEMS

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Abstract: *The paper presents the specific equations that are defining the transfer functions of the connection pipes of the alternating flow driven hydraulic systems, determined based on each component specific characteristics like fluid and pipe elasticity, inertia, friction and leaks. Also there are presented the relations which are defining the amplitude and the phase alteration.*

Keywords*: analytical model, alternating flow, hydraulic system*

1. General aspects

Alternating flow driven systems involves a new approach of the driving systems using pressurized liquids, because we have here, in the entire system, along the pipes, an energy transmissions without volumetric flow transportation between the energy converters, hydraulic generator and hydraulic motor. [5], [7], [8]

Generally, an alternating flow driven hydraulic transmission consists in a alternating flows and pressures generator (G) and a motor (M), the connection between them being realized with a number of pipes equal with the number of phases (Phase 1, Phase 2 and Phase 3), the pipes being filled with fluid at a certain pressure (pre-established with an hydraulic accumulator Ac), figure 1. During the functioning of the system the pressure and the flow within each pipe varies in a sinusoidal way, around an average value.

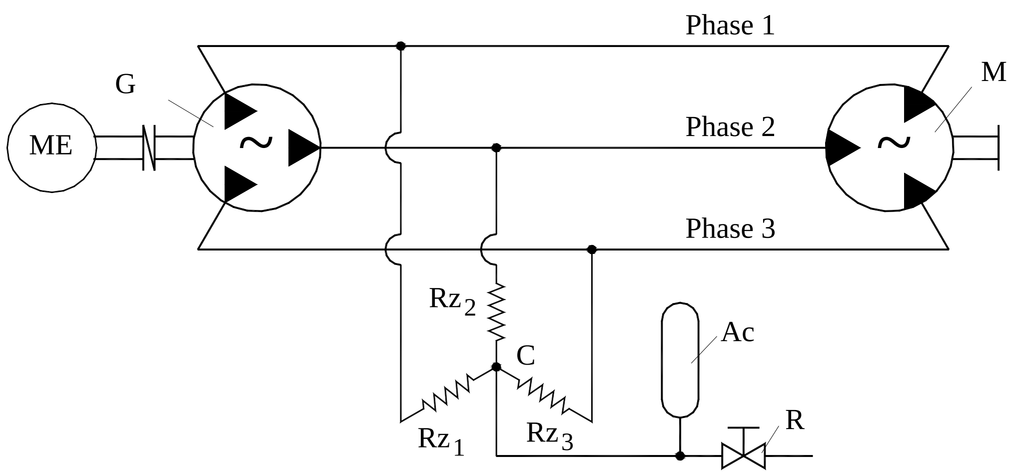

Fig. 1. Principle schema of an alternating flows and pressures drive hydraulic system.

In order to have a proper functioning it is compulsory that this average pressure from each pipe to have the same value and to have a constant value in time. Therefore, to obtain the correct functionality we create from the beginning either a pressure in each phase, higher than the amplitude maximum value, or this pressure is modifying itself during the functioning.

2. Analytical model of the alternating flow driven hydraulic systems

The precision of the analyzed aspects, the design and the physical realization of an automatically system depends on the complete modeling possibilities, using characteristic equations for each component. It is also very important the correct determination of each constant value from the equations structure which involve the establishment of the frequency functions.

The transfer functions of the elements (components) represent the rate between the Laplace transformations of the output respectively input signal, for null initial conditions.

The general effect of using the Laplace transformation is the reducing of difficulty order of the problems. The transfer function algebra contains some rules which allow combining the transfer functions of many components and finally to obtain the transfer function of the entire assembly of individual elements. [1], [4]

The transfer function of the entire system was determined considering that the input signal is a harmonic one (sinusoidal), signal provided by alternative movement of the hydraulic generator piston, and the output signal is also harmonic, but having an amplitude and phase angle alteration. [8]

The dynamic system schema is represented in figure 2.

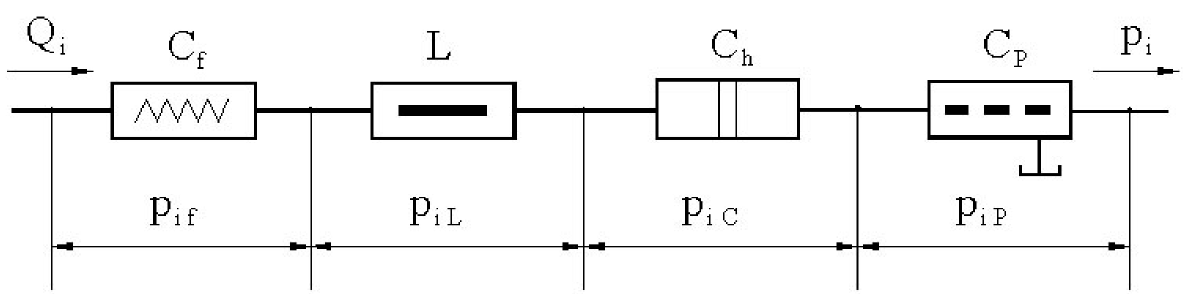

Fig. 2. Representation of the hydraulic pressure losses in the connection pipe.

We assume that the governing equation for the instantaneous flow are: [5], [7]

$$Q_i = Q_{a\max} \cdot \sin(\omega t + \varphi_0) \tag{1}$$

In which:

$$Q_{a\max} = \frac{\omega \cdot h_g \cdot S_g}{2} \tag{2}$$

The instantaneous flow Q_i, equation (1), provided by the generator, is producing a variation due to the combined effect of the frictions, capacity, inertia and leaks, figure 2. The dependences between these instantaneous pressures and instantaneous flow are: [7]

$$p_{if} = C_f \cdot Q_i \tag{3}$$

$$p_{iL} = L \cdot \frac{dQ_i}{dt} \tag{4}$$

$$p_{iC} = \frac{1}{C_h} \cdot \int Q_i \cdot dt \tag{5}$$

$$p_{iP} = \frac{Q_i}{C_P} \tag{6}$$

The friction coeficient is:

$$C_f = \frac{\varepsilon \cdot \gamma \cdot l_c \cdot v_{ef}}{2g \cdot S_c \cdot d_c} \quad \left[\frac{\mathrm{N \cdot s}}{\mathrm{m^5}}\right] \tag{7}$$

in which:

ε - no dimensional coefficient;
γ - the specific weight of the oil [kg/m^3];
l_c - the length of the pipe [m];
V_{ef} - the oil speed [m/s];
g - the gravitational acceleration [m/s^2];
S_c - the pipe area [m^2];
d_c - the inside diameter of the pipe [m].

Using experimental researches the coefficient ε is estimated by the equation: [7]

$$\varepsilon = 0{,}02 + \frac{0{,}18}{\sqrt{V_{ef} \cdot d_c}} \tag{8}$$

The inertia or the hydraulic inductance is defined by the equation:

$$L = \frac{\gamma \cdot l_c}{g \cdot S_c} \quad \left[\frac{\mathrm{N \cdot s^2}}{\mathrm{m^5}}\right] \tag{9}$$

The hydraulic capacity, considering the oil and pipe compressibility is:

$$C_h = V_0 \cdot \left(\frac{1}{E_{ulei}} + \frac{1}{E_{conducta}} \cdot \frac{2{,}5 \cdot \left(\frac{d_{c_{ext}}}{d_{c_{\mathrm{int}}}}\right)^2 + 1{,}5}{\left(\frac{d_{c_{ext}}}{d_{c_{\mathrm{int}}}}\right)^2 - 1} \right) \quad \left[\frac{\mathrm{m^5}}{\mathrm{N}}\right] \tag{10}$$

in which:

V_0 - the initial oil volume from the pipe [m^3];
E_{ulei} - elasticity modulus of the oil [N/m^2];
$E_{conducta}$ - elasticity modulus of the pipe [N/m^2];
$d_{c_{ext}}$ - the inner diameter of the pipe [m];
$d_{c_{int}}$ - the outer diameter of the pipe [m].

The coefficient which is defining the hydraulic leaks can be evaluated using the formula: [2]

$$C_p = \frac{\pi \cdot d \cdot h^3}{12\eta} \tag{11}$$

The combined effect of the friction, inertia, capacity and leaks, figure 2, is expressed by summing the specific pressures, defined by the relations (3), (4), (5) and (6), obtaining in this way:

$$p_i = p_{if} + p_{iL} + p_{iC} + p_{iP} \tag{12}$$

or:

$$p_i(t) = C_f \cdot Q_i(t) + L \cdot \frac{dQ_i(t)}{dt} + \frac{1}{C_h} \cdot \int Q_i(t) \cdot dt + \frac{Q_i(t)}{C_P} \tag{13}$$

Using the Laplace transformation we obtain:

$$p_i(s) = C_f \cdot Q_i(s) + L \cdot s \cdot Q_i(s) + \frac{1}{C_h \cdot s} \cdot Q_i(s) + \frac{1}{C_P} \cdot Q_i(s) \tag{14}$$

or:

$$p_i(s) = \left(C_f + \frac{1}{C_P} + L \cdot s + \frac{1}{C_h \cdot s} \right) \cdot Q_i(s) \tag{15}$$

In this way:

$$p_i(s) = F_c^*(s) \cdot Q_i(s) \tag{16}$$

in which:

$$F_c^*(s) = \frac{p_i(s)}{Q_i(s)} = R_f + \frac{1}{C_P} + L \cdot s + \frac{1}{C_h \cdot s} \tag{17}$$

is representing the transfer function of the pipe.

The equation (17) can be reconsidered like:

$$F_c^*(s) = \frac{L \cdot C_h \cdot s^2 + \left(C_f + \frac{1}{C_P} \right) \cdot C_h \cdot s + 1}{C_h \cdot s} \tag{18}$$

The Laplace operator is $s = j \cdot \omega$ [6], and in this way the equation (18) is:

$$F_c^*(j\omega) = \frac{L \cdot C_h \cdot (j\omega)^2 + \left(C_f + \frac{1}{C_P} \right) \cdot C_h \cdot (j\omega) + 1}{C_h \cdot (j\omega)}$$

or:

$$F_c^*(j\omega) = \frac{1 - \omega^2 \cdot L \cdot C_h + j \cdot \omega \cdot C_h \cdot \left(C_f + \frac{1}{C_P} \right)}{j \cdot \omega \cdot C_h} \tag{19}$$

considering: $j^2 = -1$.

The transfer function of the pipe can by expressed like:

$$F_c^*(j\omega) = \left(C_f + \frac{1}{C_P} \right) + j \cdot \left(\omega \cdot L - \frac{1}{\omega \cdot C_h} \right) \tag{20}$$

The real and the imaginary components are:

$$\mathrm{Re} = C_f + \frac{1}{C_P} \tag{21}$$

and:

$$\mathrm{Im} = \omega \cdot L - \frac{1}{\omega \cdot C_h} \tag{22}$$

In this way, the amplitude and the phase alteration of the transfer function of the pipe is defined by the equations:

$$\left| F_c^*(j\omega) \right| = \sqrt{\left(C_f + \frac{1}{C_P} \right)^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C_h} \right)^2} \tag{23}$$

and:

$$< F_c^*(j\omega) = arctg \frac{\omega \cdot L - \frac{1}{\omega \cdot C_h}}{C_f + \frac{1}{C_P}} \tag{24}$$

Knowing that the input signal, the instantaneous flow is harmonic equation (1), then the output signal will be also harmonic, but having a different amplitude and phase alteration. Then, the output signal which is the instantaneous pressure is defined by:

$$p_i = Q_{a\max} \cdot \sqrt{\left(C_f + \frac{1}{C_P}\right)^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C_h}\right)^2} \cdot \sin\left(\omega t + \varphi_0 + arctg \frac{\omega \cdot L - \frac{1}{\omega \cdot C_h}}{C_f + \frac{1}{C_P}}\right) \tag{25}$$

From equation (25), using the equations (23) and (24), we obtain the amplitude of the output pressure, like:

$$p_{a\max} = Q_{a\max} \cdot \sqrt{\left(C_f + \frac{1}{C_P}\right)^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C_h}\right)^2} \tag{26}$$

and the phase alteration:

$$\psi_c = arctg \frac{\omega \cdot L - \frac{1}{\omega \cdot C_h}}{C_f + \frac{1}{C_P}} \tag{27}$$

If we not taking into account the hydraulic leaks coefficient C_p , then the transfer function defined by the equation (18) is:

$$F_c^*(s) = \frac{1}{s} \cdot \left(L \cdot s^2 + C_f \cdot s + \frac{1}{C_h}\right) \tag{28}$$

Another form of the transfer function of the pipe is:

$$F_c^*(s) = \frac{1}{s \cdot C_h} \cdot \left(\frac{1}{\omega_{nc}^2} \cdot s^2 + \frac{2\delta_c}{\omega_{nc}} \cdot s + 1\right) \tag{29}$$

in which the natural frequency is defined by:

$$\omega_{nc} = \sqrt{\frac{1}{L \cdot C_h}} \tag{30}$$

and the damping ratio of the pipe is:

$$\delta_c = \frac{C_f}{2} \cdot \sqrt{\frac{C_h}{L}} \tag{31}$$

Considering the Laplace operator $s = j \cdot \omega$ we obtain a new expression of the transfer function:

$$F_c^*(j\omega) = \frac{\frac{2\delta_c}{\omega_{nc}} + j \cdot \left(\frac{\omega^2}{\omega_{nc}^2} - 1\right)}{\omega \cdot C_h} \tag{32}$$

The natural frequency becomes:

$$\omega_{nc} = \sqrt{\frac{1}{\frac{\gamma \cdot l_c^2}{g} \cdot \left(\frac{1}{E_{ulei}} + \frac{1}{E_{conducta}} \cdot \frac{2{,}5 \cdot \left(\frac{d_{c_{ext}}}{d_{c_{\text{int}}}}\right)^2 + 1{,}5}{\left(\frac{d_{c_{ext}}}{d_{c_{\text{int}}}}\right)^2 - 1}\right)}} \tag{33}$$

Also, the damping ratio of the pipe will be:

$$\delta_c = \frac{\varepsilon \cdot \gamma \cdot l_c \cdot v_{ef}}{4 \cdot g \cdot S_c \cdot d_c} \cdot \sqrt{\frac{g \cdot S_c^2}{\gamma} \cdot \left(\frac{1}{E_{ulei}} + \frac{1}{E_{conducta}} \cdot \frac{2{,}5 \cdot \left(\frac{d_{c_{ext}}}{d_{c_{\text{int}}}} \right)^2 + 1{,}5}{\left(\frac{d_{c_{ext}}}{d_{c_{\text{int}}}} \right)^2 - 1} \right)} \tag{34}$$

In this way, the new equation defining the amplitude and phase alteration are:

$$\left| F_c^*(j\omega) \right| = \frac{1}{\omega \cdot C_h} \cdot \sqrt{\frac{4\delta_c^2}{\omega_{nc}^2} + \left(\frac{\omega^2}{\omega_{nc}^2} - 1 \right)^2} \tag{35}$$

$$< F_c^*(j\omega) = arctg\left[\frac{\omega_{nc}}{2\delta_c} \cdot \left(\frac{\omega^2}{\omega_{nc}^2} - 1 \right) \right] \tag{36}$$

3. Conclusions

The objective of this study was a new approach of the hydraulic drives, in which the pressure and flow is not continuously transmitted between the energy converters (pumps and motors).

The analytical model concern the connection pipes, taking into account some specific characteristics like fluid and pipe elasticity, the fluid inertia, the fluid friction and also the leaks. Simulating this model we can observe the possibility to adjust, during the functioning, the input parameters, in order to obtain the anticipated output values of some parameters.

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