NUMERICAL STUDY ON THE FLUID MOTION INDUCED BY A ROTATING DISK INSIDE A VESSEL

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Abstract: *The paper presents a numerical study concerning the evolution of viscous friction coefficient and friction moment coefficient at low Reynolds numbers for the case of a rotating disk inside a vessel. The study is useful in the field of turbomachinery, magneto-hydrodynamic flows, mixing processes, hydraulic drives etc. The flow and velocity patterns at low Reynolds numbers provides conclusions concerning the disk radius where maximum shear stress arises and the influence of the vessel walls on overall vortex motion.*

Keywords*: (shear stress, viscous friction coefficient, friction moment coefficient, rotating disk)*

1. Introduction

The flow induced by rotating disks is a classical problem due to large number of applications associated to the phenomenon. Regarding turbomachinery field, flow induced by rotating disks in Tesla turbines was studied [1] in order to obtain torque and power output. Also, multiple-disk Tesla type fan was studied [2] for assessing the performance of such turbomachinery working with low viscosity fluid. Effects of viscosity of fluids on centrifugal pump performance [3] were experimentally driven in order to assess the rapid increase in the disc friction losses. For MHD applications, recent research was developed in order to obtain exact solutions for the flow of a viscous hydromagnetic fluid due to the rotation of an infinite disk [4] or for assessing the heat transfer in a steady MHD laminar flow for the same disk motion [5]. Also various applications were proposed as mixing two-layer stratified fluid by a rotating disk [6], disk-driven vortical flow in a cubical container [7] etc. Some relevant studies are related to calculation of fluid friction for the case of a rotational rough disk in a rough vessel [8], study on the flow and heat transfer over a rotating disk with surface roughness [9], numeric solution for Navier-Stokes equations for unsteady viscous flow over a rotating stretchable disk with deceleration [10].

The present paper is proposing a numerical solution for the motion of a viscous fluid due to a rotating disk inside a vessel. The results are similar with the experimental ones obtained in the same conditions [11].

2. Mathematical model

According to literature the friction moment acting on a rotating body placed in a viscous environment is depending on several variables, M = f (R_2, ω, ρ, ν) and the consumed power is:

$$P_0 = M\omega = c_f\, \rho R_2^5 \omega^3\ (W). \tag{1}$$

where: M is the friction momentum of the disk and c_f is the friction coefficient depending on Reynolds number $Re_\omega = R_2^2\omega/\nu$, relative width ($s/R_2$) between the vassel base *s* and disk radius R_2 and relative roughness Δ/R_2 (Δ is absolute roughness). If one consider the disk is rotating in infinit fluid volume relative width is neglected. The disk rotation is inducing centripetal fluid motion in the axial region and centrifugal motion at the periphery of the disk; near the surface of the disk the boudary layer has the thickness δ. From the theory of laminar boudary layer one can obtain the friction coefficient formula for Re ≤ 30 (Müller):

$$c_{f0} = \frac{64}{3}\frac{1}{Re_\omega} \tag{2}$$

For smooth hydraulic disks (for wich the value of absolute roughness Δ is below the value of the thikness of the boundary layer δ) and for infinit fluid volume condition at $30 \le Re_\omega \le 3\times10^5$ one can use Cochran formula:

$$c_{f0} = \frac{3{,}87}{\sqrt{Re_\omega}} \tag{3}$$

For limits of Reynolds number $3\times10^5 \le Re_\omega \le 10^6$ Kármán formula is available (turbulent):

$$c_{f0} = \frac{0{,}146}{\sqrt[5]{Re_\omega}}, \tag{4}$$

and for $Re_\omega \ge 10^6$:

$$c_{f0} = \frac{0{,}982}{\left(lgRe_\omega\right)^{2{,}58}} \tag{5}$$

If the case of rotating disk in closed volume when secondary current occur relative width (s/R_2) is relevant and the availabe formulas are:

$$c_f = \frac{2\pi}{\frac{s}{R_2}}\frac{1}{Re}\ (Re \le 10^4);\quad c_f = \frac{2{,}67}{\sqrt{Re}}\ (10^4 \le Re \le 3\times10^5);\quad c_f = \frac{0{,}0622}{\sqrt[5]{Re}}\ (Re \ge 3\times10^5) \tag{6}$$

It is demonstrated that rotating moment is smaller when finite volume condition is considered then infinite volume case for $Re \ge 10^4$ due to secondary flows in the vessel and in the boundary layer.

In order to obtain the fluid velocity distribution, Navier-Stokes equations was written in cilindrical coordinates. Using a numerical integration method developed in [12] one can obtain algebric relations for stream lines Ψ_0 and velocity lines w_0 :

$$w_0 = \frac{\sum_1^4 w_i + \frac{K}{2}\cdot\left(w_1 - w_3\right) + \frac{Re}{24\cdot r}\cdot}{4 + K^2 + \frac{Re\cdot K}{12}\cdot\left[8\cdot\left(\Psi_2 - \Psi_4\right) + \Psi_{12} - \Psi_{10}\right]}\times$$
$$\times\left\{\left[8\cdot\left(\Psi_1 - \Psi_3\right) + \Psi_{11} - \Psi_9\right]\cdot\left(w_2 - w_4\right) - \left[8\cdot\left(\Psi_2 - \Psi_4\right) + \Psi_{12} - \Psi_{10}\right]\cdot\left(w_1 - w_3\right)\right\} -$$
$$- w_0\cdot\frac{\frac{Re\cdot K}{12}\cdot\left[8\cdot\left(\Psi_2 - \Psi_4\right) + \Psi_{12} - \Psi_{10}\right]}{4 + K^2}, \tag{7}$$

$$\Psi_0=\frac{1}{20-\frac{15}{2}\cdot K^2+\frac{Re\cdot K}{r}\cdot\frac{25}{24}\cdot\left[8\cdot\left(\Psi_4-\Psi_2\right)+\Psi_{10}-\Psi_{12}\right]}\times$$

$$\times\left\{8\cdot\sum_1^4\Psi_i-2\cdot\sum_5^8\Psi_i-\sum_9^{12}\Psi_i+K\cdot\left[4\cdot\left(\Psi_3-\Psi_4\right)+\Psi_5+\Psi_8+\Psi_9-\Psi_6-\Psi_7-\Psi_{11}\right]+\right.$$

$$+K^2\cdot\left[\frac{1}{4}\cdot\left(\Psi_9+\Psi_{11}\right)-4\cdot\left(\Psi_1+\Psi_3\right)\right]+K^3\cdot\left[2\cdot\left(\Psi_1-\Psi_3\right)+\frac{1}{4}\cdot\left(\Psi_{11}-\Psi_9\right)\right]+$$

$$+\frac{Re}{3\cdot r}\cdot\left\{\left[2\cdot\left(\Psi_2-\Psi_4\right)+\frac{1}{4}\cdot\left(\Psi_{12}-\Psi_{10}\right)\right]\cdot\right.$$

$$\left[2\left(\Psi_3-\Psi_1\right)+\frac{1}{2}\cdot\left(\Psi_5+\Psi_8+\Psi_9-\Psi_6-\Psi_7-\Psi_{11}\right)\right]+$$

$$+\left[2\cdot\left(\Psi_3-\Psi_1\right)+\frac{1}{4}\cdot\left(\Psi_9-\Psi_{11}\right)\right]\cdot\left[2\cdot\left(\Psi_4-\Psi_2\right)+\frac{1}{2}\cdot\left(\Psi_5+\Psi_6+\Psi_{10}-\Psi_7-\Psi_8-\Psi_{12}\right)\right]+ \tag{8}$$

$$+K\cdot\left\{\left[\frac{1}{3}\cdot\left(\Psi_5+\Psi_7-\Psi_6-\Psi_8\right)+\frac{1}{48}\cdot\left(\Psi_{14}+\Psi_{16}-\Psi_{13}-\Psi_{15}\right)\right]\cdot\right.$$

$$\left[2\cdot\left(\Psi_1-\Psi_3\right)+\frac{1}{4}\cdot\left(\Psi_{11}-\Psi_9\right)\right]+\left[2\cdot\left(\Psi_4-\Psi_2\right)+\frac{1}{4}\cdot\left(\Psi_{10}-\Psi_{12}\right)\right]\cdot$$

$$\left.\left.\left[4\cdot\left(\Psi_1+\Psi_3\right)-\frac{1}{4}\cdot\left(\Psi_9+\Psi_{11}\right)+\frac{8}{3}\cdot\left(\Psi_2+\Psi_4\right)-\frac{1}{6}\cdot\left(\Psi_{10}+\Psi_{12}\right)\right]\right\}+$$

$$+K^2\cdot\left[2\cdot\left(\Psi_1-\Psi_3\right)+\frac{1}{4}\cdot\left(\Psi_{11}-\Psi_9\right)\right]\cdot\left[2\cdot\left(\Psi_2-\Psi_4\right)+\frac{1}{4}\cdot\left(\Psi_{12}-\Psi_{10}\right)\right]+$$

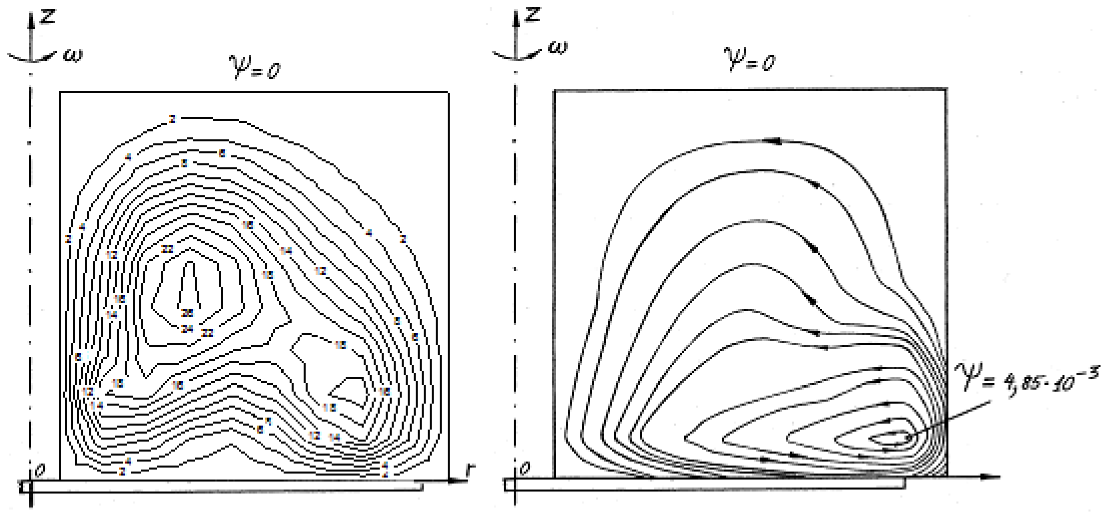
$$\left.\left.+3\cdot r^4\cdot K^2\cdot w_o\cdot\left(w_4-w_2\right)\right\}\right\}-\Psi_0\cdot\frac{\frac{Re\cdot K}{r}\cdot\frac{25}{24}\cdot\left[8\cdot\left(\Psi_4-\Psi_2\right)+\Psi_{10}-\Psi_{12}\right]}{20-\frac{15}{2}\cdot K^2}$$

where: $\vec{V}\left(V_R,V_\theta,V_Z\right)=\vec{V}\left(R,Z\right)$, $V_R=\frac{1}{R}\cdot\frac{\partial\Psi}{\partial Z}$, $V_z=-\frac{1}{R}\cdot\frac{\partial\Psi}{\partial R}$, z=Z/$D_c$, r=R/$D_c$, $v_r=\frac{V_R}{U}=\frac{1}{r}\cdot\frac{\partial\Psi}{\partial z}$, $v_z=\frac{V_z}{U}=\frac{1}{r}\cdot\frac{\partial\Psi}{\partial r}$, $w=\frac{V_\theta}{U}$, $\psi=\frac{\Psi}{U\cdot D_c^{\,2}}$, $Re=\frac{U\cdot D_c}{\nu}$, $\chi=\frac{\delta R}{D_c}=\frac{\delta Z}{D_c}\langle 1$ (relative grid step), $K=\frac{\chi}{r}$.

The boundary condition on the disk surface and shaft are the same – equal velocity – and zero values on the vessel walls. As for the stream line function Ψ_0 the values are zero on the boundaries because of the closed domaine.

3. Results

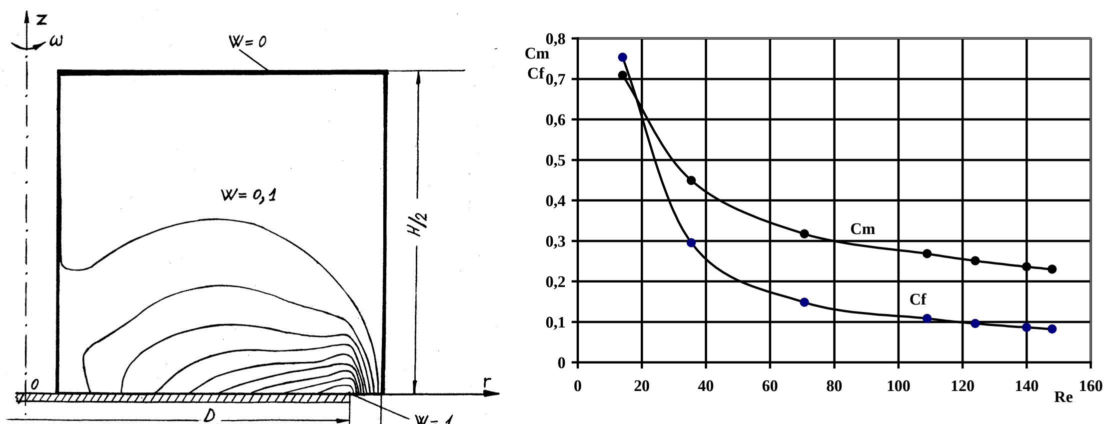
The numerical solutions for low Reynolds numbers (Re_1= 100 and Re_2= 148) was obtained for rotating velocities of the disk of n_1=146 rot/min and n_2=190 rot/min using an oil viscosity of ν= *0,00037[m²/s]*. In figure nr. 1 are presented the stream lines for Re_1= 100 and Re_2= 148



a. **b.**

Fig. 1 Stream lines for a. Re_1= 100 and b. Re_2= 148

In figure nr.1 one can observe that as increasing the Reynolds number the vortex motion (two nucleus for Re_1 and one nucleus for Re_2) is moving for the margins of the disk. The effect is due to increasing the shear stress on the disk surface as an consequence of centrifugal forces in the fluid flow. In figure 2 is presented the velocity lines according to Re_2= 190 and the friction c_f and momentum c_M coefficients for six different Reynolds numbers.



b.
Fig 2 a. Velocity lines; b. Friction and momentum coefficients

The momentum coefficient was obtained by integration the shear stress on the moving surfaces of disk and shaft. The theoretical results were compared with experimental measurements made on a closed installation (D_c =100 mm, H =100 mm, shaft diameter d = 10 mm). One found good concordance between theoretical and experimental data.

Conclusions

A numerical study was performed in order to obtain viscous friction coefficient and friction moment coefficient at low Reynolds numbers for the case of a rotating disk inside a vessel. The results are in a good conformity with experimental data obtained in similar conditions. By simulating the flow for various Reynolds numbers between 50 and 300, it was observed a modification of the number and position on vortices due to the increase of centrifugal forces and reverse flow as a wall effect. Using velocity values in the integration domain it was derived the evolution of friction and moment coefficients by integration the shear stress on the moving surfaces.

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