Dynamics of Hydraulic Cylinders. Classical Mathematical Models and Simulations

Prof. PhD Eng. **Anca BUCUREȘTEANU**[1]

[1] University POLITEHNICA of Bucharest; ancabucuresteanu@gmail.com

Abstract: *This paper presents mathematical models in dynamic mode for the study of hydraulic cylinders. It analyses the influence of the distributor type used for cylinder actuation but also the necessary correlation of pump flow, cylinder features and adjustment of pressure valve. The paper has a practical character because some unwanted dynamic aspects, such as, for example, the jerky start can be avoided since the designing phase of an installation.*

Keywords: *Cylinders dynamics, stabilization of pressure and speed, adjustment of pressure valve*

1. Introduction. Hypotheses for calculation

At first consider the calculation diagram in Figure 1.

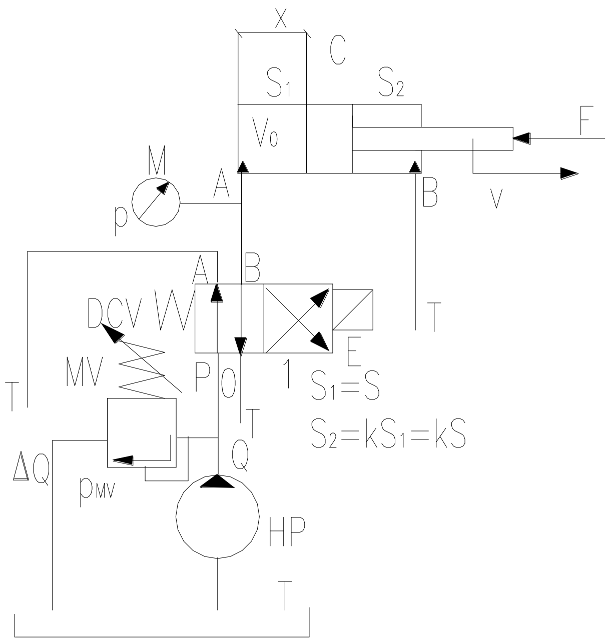


Fig. 1. The calculation diagram

Hydraulic pump HP supplies cylinder C through the electro-hydraulic distributor DCV. Surface chamber S_1 will be supplied by path A while the oil goes freely from surface chamber S_2 to tank T. The cylinder operates against force *F* and its rod moves with current speed *v*. The initial volume of oil from the left chamber of cylinder is V_0. As long as the distributor is not actuated (E-), the pump discharges freely on path P-A-T. When distributor (E+) is actuated, the connection P-B-A is performed (at cylinder). Pressure valve MV ensures the adjustment of the maximum pressure admitted at value p_{MV} [1]. The instantaneous pressure will be read on pressure gauge M. Depending on pressure, it is considered that through pressure valve MV can pass a quantity of oil ΔQ that can start from 0 and can reach value *Q*, where *Q* is pump flow, assumed constant. The instantaneous size *X* can have values included in interval [0, *c*] where *c* is the maximum stroke of cylinder rod.

We shall consider that the pump is initially turned on, then distributor DCV is actuated; this one switches instantaneously. These hypotheses on how the actuation is performed are shown in Figure 2.

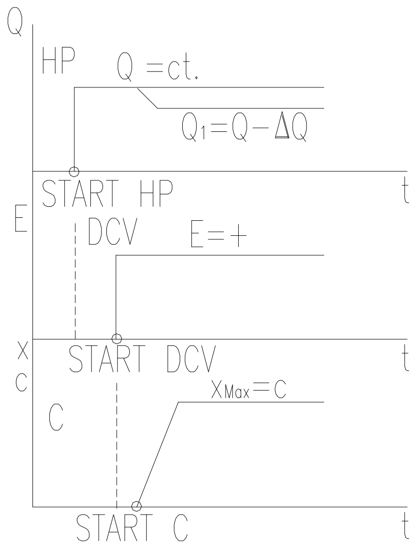


Fig. 2. The hypotheses on how the actuation is performed

If pressure valve does not open, then all the flow of the pump reaches the cylinder. If pressure valve opens, in cylinder C arrives only flow Q_1 [1].

2. Mathematical- theoretical models

For such a system, the specialized literature [1] takes into consideration the mathematical model below:

$$Q = S \cdot v + \frac{V_0}{E_O} \cdot \frac{dp}{dt} \tag{1}$$

$$M \cdot \frac{dv}{dt} + b \cdot v + F = p \cdot S \tag{2}$$

In the relations (1) and (2) there were also noted: *M* - mass of powered mobile assembly; E_0 - modulus of oil elasticity; *p* - instantaneous pressure; *t* – time; *b* - cylinder damping factor.
PHASE 1. In the first phase, the cylinder rod does not move (v = 0), the pump sends fluid until reaching the value corresponding to resistance force *F*, covering the compressibility effects too. In these conditions we can consider:

$$v = 0 \tag{3}$$

$$Q = \frac{V_0}{E_O} \cdot \frac{dp}{dt} \tag{4}$$

$$dp = \frac{E_O \cdot Q}{V_0} \cdot dt \tag{5}$$

$$\int_0^{\frac{F}{S}} dp = \frac{E_0 \cdot Q}{V_0} \cdot \int_0^{t_1} dt \qquad (6)$$

From relation (5) we can determine the time needed to this phase:

$$t_1 = \frac{F}{S} \cdot \frac{V_0}{E_O \cdot Q} \qquad (7)$$

After time t_1, the pressure $p = F/S$ is reached, and speed is $v = 0$.
PHASE 2. From this moment on, the mathematical model becomes:

$$v \neq 0 \qquad (8)$$

$$Q = S \cdot v + \frac{V_0 + x \cdot S}{E_O} \cdot \frac{dp}{dt} \qquad (9)$$

$$M \cdot \frac{dv}{dt} + b \cdot v + F = p \cdot S \qquad (10)$$

The volume of oil in section chamber S_1 is variable over time, depending on position. It is recommended to replace the value $V_0 + xS$ with an average volume V_M:

$$Q = S \cdot v + \frac{V_M}{E_O} \cdot \frac{dp}{dt} \qquad (11)$$

From relations (10) and (11) we obtain the differential equation below:

$$\frac{d^2 v}{dt^2} + \frac{b}{M} \cdot \frac{dv}{dt} + \frac{S^2 \cdot E_O}{M \cdot V_M} \cdot v = \frac{S \cdot E_O}{M \cdot V_M} \cdot Q \qquad (12)$$

In this equation, the following notations will be made:

$$A = \frac{b}{M} \qquad (13)$$

$$B = \frac{S^2 \cdot E_O}{M \cdot V_M} = \omega_0^2 \qquad (14)$$

$$C = \frac{S \cdot E_O}{M \cdot V_M} \cdot Q = \omega_0^2 \cdot \frac{Q}{S} \qquad (15)$$

With these notations, the equation (12) becomes:

$$\frac{d^2 v}{dt^2} + A \cdot \frac{dv}{dt} + B \cdot v = C \qquad (16)$$

In order to solve it, consider the attached equation [2]:

$$k^2 + A \cdot k + B = 0 \qquad (17)$$

Depending on the values of expression A^2 - 4B, the following solutions will be obtained:

1. $A^2 - 4 \cdot B > 0$

$$v = \frac{\frac{C}{B}}{k_1 - k_2} \cdot \left(k_2 \cdot e^{k_1 \cdot t} - k_1 \cdot e^{k_2 \cdot t}\right) + \frac{C}{B} \qquad (18)$$

Where k_1 and k_2 are the solutions of equation (17).

2. $A^2 - 4 \cdot B = 0$

$$v = \frac{-C}{B} \cdot e^{k \cdot t} + \frac{C}{B} \cdot k \cdot e^{k \cdot t} \cdot t + \frac{C}{B} \qquad (19)$$

In the relation (19), k is the double solution of equation (17).

3. $A^2 - 4 \cdot B < 0$

$$v = e^{\alpha \cdot t} \cdot \left(\frac{-C}{B} \cdot \cos \beta \cdot t + \frac{\alpha}{\beta} \cdot \frac{C}{B} \cdot \sin \beta \cdot t \right) + \frac{C}{B} \qquad (20)$$

In the relation above, it is considered that the complex conjugate roots ($i^2 = -1$) of equation (17) are:

$$k_{1,2} = \alpha \pm i \cdot \beta \qquad (21)$$

Out of these three cases, the case 3 [3] has practical applicability. Therefore we shall continue to refer only to this one. Analyzing the equation (17) it can be noticed that $\alpha = -A/2 < 0$ always. If $\alpha = 0$, namely no account is taken of cylinder damping, then $\beta = \omega_0$.
From the relations above we can determine the evolution of pressure in this second phase:

$$p = \frac{F}{S} + \frac{M \cdot Q}{S^2} \cdot e^{\alpha \cdot t} \cdot \frac{\alpha^2 + \beta^2}{\beta} \cdot \sin \beta \cdot t \qquad (22)$$

We can consider that the maximum value of pressure p from the relation above is:

$$p_{MAX} = \frac{F}{S} + \frac{M \cdot Q}{S^2} \cdot \frac{\alpha^2 + \beta^2}{\beta} \qquad (23)$$

For determining the duration of this second phase (t_2) there are two distinct cases:
a. Maximum pressure resulted from relation (22) is higher than the pressure adjusted at pressure valve MV. This one will open and will limit the pressure in the system. In this situation there is the relation:

$$p_{MAX} > p_{MV} \qquad (24)$$

In this case the duration of the phase in which the speed increases and the pressure is limited is:

$$t_2 = \frac{1}{\beta} \cdot Arc\sin \frac{\left(p_{MV} - \frac{F}{S}\right) \cdot S^2}{M \cdot Q \cdot e^{\alpha \cdot t_{MAX}}} \cdot \frac{1}{\beta} \approx \frac{1}{\omega_0} \cdot Arc\sin \frac{\left(p_{MV} - \frac{F}{S}\right)}{M \cdot Q \cdot \omega_0} \qquad (25)$$

Figure 3 shows pressure evolution in this case.

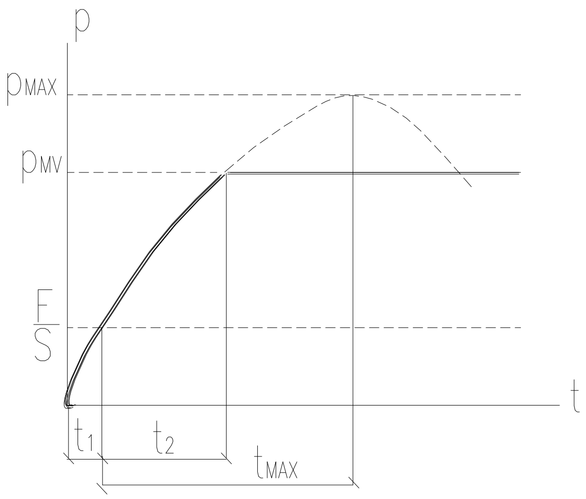


Fig. 3. Pressure evolution in case "a"

In Figure 3 it was noted with t_{MAX} the time for reaching maximum pressure for the first time, if pressure valve would not open. Valve opening has a damping effect.
b. If the valve does not open and the pressure develops according to relation (22), the pressure characteristic in time is shown in Figure 4.

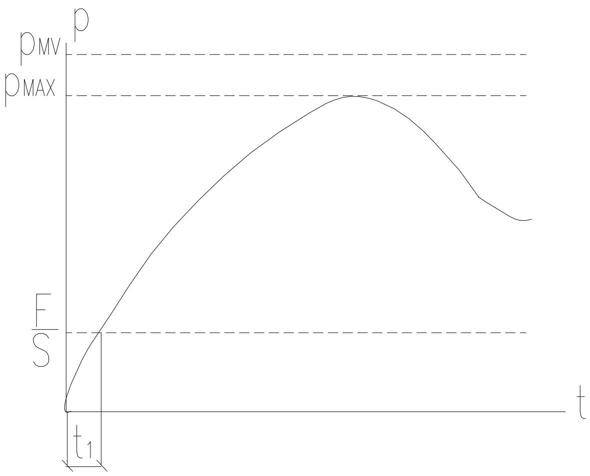


Fig. 4. Pressure evolution in case "b"

This case is undesirable in reality, the pressure oscillations resulting in jerky movements of the cylinder rod.
PHASE 3. Movement continues until reaching maximum speed. This time, a part of pump flow can be discharged through the pressure valve. The system pressure is constant and equal to the value set to pressure valve MV.
The mathematical model is:

$$M \cdot \frac{dv}{dt} + b \cdot v + F = p_{MV} \cdot S \tag{26}$$

After solving the differential equation, we get the expression of speed in this phase:
If *b* differs from 0:

$$v = C_1 \cdot e^{-\frac{b}{M} \cdot t} + \frac{p_{VM} \cdot S - F}{b} \tag{27}$$

Constant C_1 is determined by stipulating the condition that speed at t = 0 is equal to speed v_2 from the end of the previous phase.
If b = 0:

$$v = v_2 + \frac{p_{MV} \cdot S - F}{M} \cdot t \tag{28}$$

At the end of this phase, the speed reaches the values resulted from using the entire flow, namely:

$$v_3 = \frac{Q}{S} \tag{29}$$

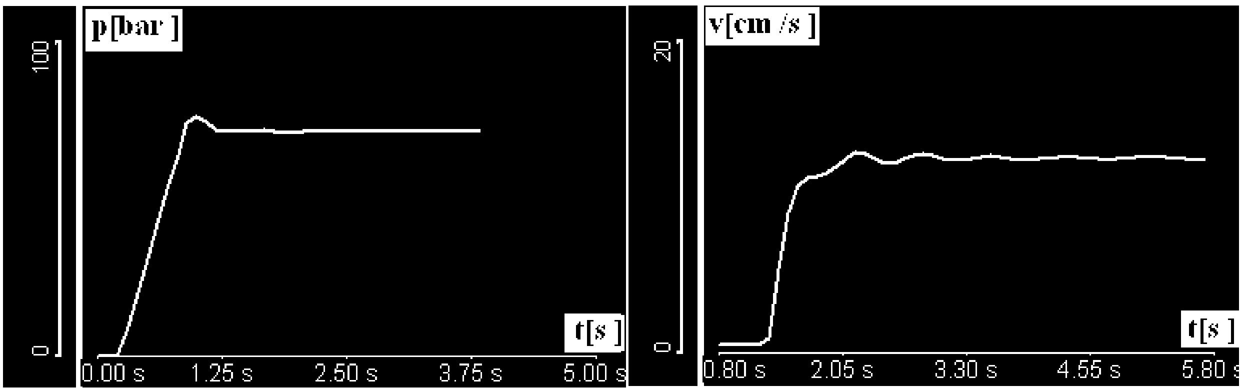
The duration of this phase is determined with the relation:

$$t_3 = \frac{\frac{Q}{S} - v_2}{p_{MV} \cdot S - F} \cdot M \tag{30}$$

PHASE 4. Starting from this moment the movement is performed with speed v_3, at pressure $p = F/S$ until reaching travel end.
The relations above help to determine speeds, pressures and strokes performed in each one of the four phases [3].

3. Simulation of hydraulic cylinders operation [4, 5]

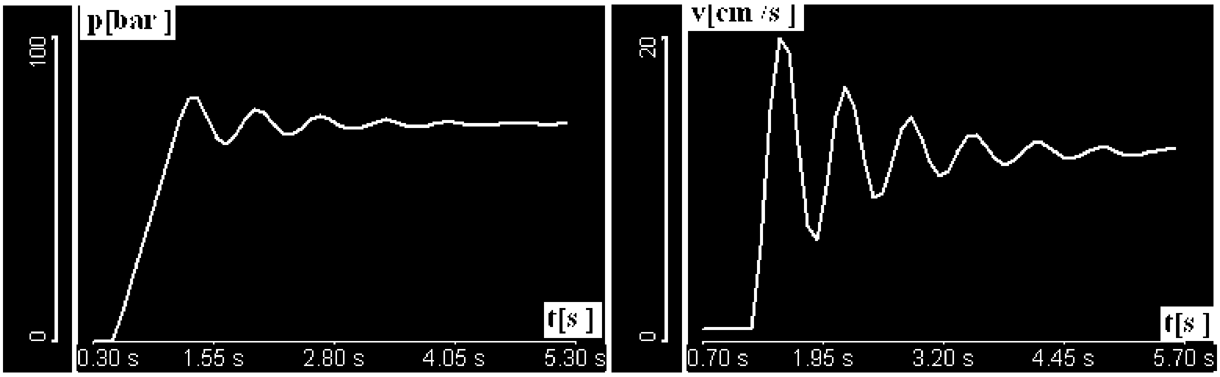
Currently, the specialized programs of simulation of the hydraulic systems [6] in dynamic mode are a real help in design activity. It is recommended to take into account the instructions above in the case of some real units.
For example, consider the case of the cylinder actuated as in Figure 1. Value $p = F/S$ is 71.3 bar, and the second term of the sum in the relation (23) is 33 bar. In these conditions it is required a correct adjustment at pressure valve at 72 bar and a second one at a higher pressure of 120 bar.
For the first case, we obtain the pressure and speed characteristics in Figure 5.



a. b.

Fig. 5. The pressure (a) and speed (b) characteristics for the pressure valve MV adjusted at 72 bar

If pressure valve MV is adjusted at 120 bar, we obtain the characteristics in Figure 6.



a. b.

Fig. 6. The pressure (a) and speed (b) characteristics for the pressure valve MV adjusted at 120 bar

From the comparison of these two cases, it can be noticed that a pressure well adjusted at pressure valve MV influences rod speed and stabilizes its movement obviously.
The influence of damping can be increased by throttling the oil output from section chamber S_2 [6].

4. Conclusions

Smooth operation of a hydraulic cylinder is also conditioned by the correct correlation of source flow with the maximum pressure adjusted at this one. It is recommended that calculations of verification or sizing of the hydraulic cylinders are made in the preliminary phase of designing, not forgetting to determine the own beats ω_0. After determining them, even approximately, you can avoid faulty pressure settings. An exaggerated increase of the pressure adjusted at pressure valve MV does not lead to the elimination of jerking motion, and it may even amplify it. A pressure valve can reduce the oscillations of the entire system.

References

[1] D. Prodan, "Machine-Tools Hydraulics" („Hidraulica Maşinilor Unelte"), Printech Publishing House, Bucharest, 2004;
[2] B. Demidovitch, "Collection of Exercises and Problems of Mathematical Analysis" ("Recueil d’Exercices et de Problemes d’Analyse Mathematique"), MIR Publishing House, Moscow, 1974;
[3] F. Esnault, P. Beneteau, "Hydrostatics 1. Power Transmission" ("Hydrostatique 1. Transmission de Puissance"), Ellipses Publishing House, Paris, 1997;
[4] *** Software package: Automation Studio by Famic Technologies Inc.;
[5] *** Catalogues and leaflets ATOS, BOSCH REXROTH, PARKER;
[6] D. Prodan, "Machine-Tools. Modelling and Simulation of Hydrostatic Elements and Systems" („Maşini-unelte. Modelarea şi simularea elementelor şi sistemelor hidrostatice"), Printech Publishing House, Bucharest, 2006.