Sensitivity Analysis of Sharp-Crested Weirs as a Function of Shape Opening, for Small Discharges

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Abstract: *The paper investigates the degree of measurement accuracy of several sharp – crested weirs of different shapes, based on the experimental results obtained in the laboratory. A sensitivity analysis is performed to assess the influence the weir shape has on the coefficients of discharge and to identify which one provides the best accuracy for measuring small discharges.*

Keywords: *Sharp-crested weir, coefficient of discharge, weir sensitivity, rectangular, triangular, trapezoidal, proportional weir.*

1. Introduction

A weir consists of a vertical wall that blocks the flow, usually placed at the outlet of a tank, flume, channel or basin, being used for discharge measurement. Its basic operation principle is very simple: the liquid depth gradually increases till reaching the barrier height and then flows over its crest. The liquid level over the crest is a measure of the flow rate [1]. Weirs can be classified in accordance with several criteria (shape of the opening, shape of the crest, effect of sides on the nappe) [2], but one of the most common division is that of grouping them in sharp – crested and broad – crested weirs. The fundamental distinction between sharp and broad-crested weirs relates to the location of the critical depth that in facts controls the discharge [3].
Due to their accuracy [4], [5] sharp crested weirs have many practical applications, being widely used devices for low flow measurements in hydraulic laboratories as well as in situ for smaller rivers and canals [6]. Their main drawback in the field is that they frequently become clogged with debris or floating objects and must be periodically cleaned.
Aydin et al. [5] showed that the most critical parameter defining the discharge accuracy measurement is the weir's sensitivity, which is exclusively controlled by the head reading. The weir's sensitivity is defined as the change of head per unit change of discharge, i.e., the $\frac{dh}{dQ}$ ratio or, for a range discharge range ΔQ as $\frac{\Delta h}{\Delta Q}$. The higher this ratio, the weir is more sensitive, and the accuracy of the flow estimate is higher.
The notion of sensitivity is coupled with that of accuracy. The accuracy of the flow measurements provided by a weir depends not only on the accuracy of the calibration curve or of its corresponding formula, but also on the accuracy of the vernier height gauge scale and its readings. The reading error of this instrument, like any measurement operation, is subject to a certain absolute error, hence the occurrence of a flow measurement error. Usually, the quantity of concern is the relative discharge $\frac{\mathrm{d}Q}{Q}$ error resulting from a dh nappe height measurement error. As well, other possible sources of errors are possible, such as those resulting from an inaccurate design or an inappropriate calibration of the weir. The resulting error $\frac{\mathrm{d}Q}{Q}$ is a measure of the degree of uncertainty associated with flow measurement [4], [7]. Stability of a weir implies having a constant discharge coefficient over the whole range of flow rates.
The present paper investigates the sensitivity of 4 sharp – crested weir shapes (rectangular, triangular, trapezoidal and proportional), to assess which one provides the best accuracy for measuring small discharges.

2. Theoretical background

The head-discharge formula for a simple opening weir has the general form of a power law

$$Q = Kh^n = C_d Q_t \tag{1}$$

where Q is the actual flowrate, K is a dimensional coefficient, C_d- a non-dimensional discharge coefficient, Q_t the theoretical discharge and n - the exponent of the weir head. The exponent has different values pending on the weir's shape (Fig.1). To take into consideration the influence of the liquid viscosity and surface tension a correction term h_k is added to provide an effective head, $h_e = h + h_k$ [4]. As well, to avoid drawdown errors, head must be measured at an upstream distance equal with 3÷5 times its value [8].

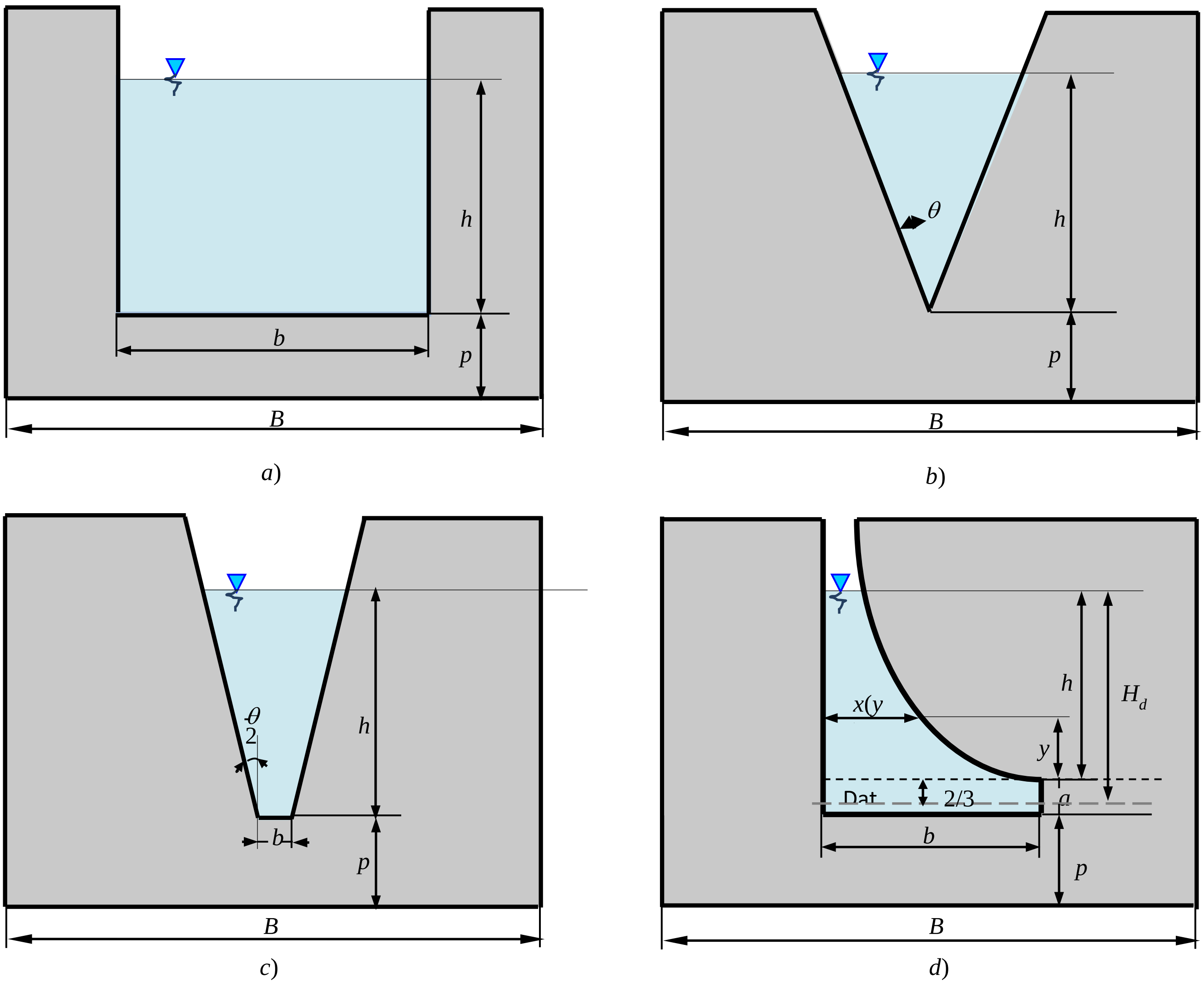

Fig. 1. Sharp-crested weir opening shapes used in the study: a) rectangular; b) V-notch; c) trapezoidal; d) proportional (Sutro)

The non-dimensional coefficient of discharge, C_d is a complex parameter, being triggered by a set of physical phenomena such as friction, surface tension, pressure distribution due to streamline curvature, nappe lateral contraction and vertical drawdown, velocity profile in the approach section and dependent on the weir geometry. For small heads, surface tension induces the clinging of the nappe to the weir and therefore a minimum head is required, to allow an accurate measurement. Depending on the weir geometry, equation (1) takes different forms.

2.1 Rectangular weir

Rectangular weirs (Fig.1.a) may be suppressed or contracted and are usually used in open channels, wastewater and sewage systems, being thus suited for relatively larger flowrates. To provide an accurate measurement it is recommended the head to be greater than 3 cm [8]. The relationship giving the flowrate of a rectangular weir with both side contractions is

$$Q_r = C_{dr}\frac{2}{3}\sqrt{2g}(b-0.2h)h^{3/2} \quad (2)$$

where: C_{dr} is the non-dimensional discharge coefficient of the rectangular weir, *b* - the notch width and *h* - the head above the weir horizontal crest [9], [10] and *g* - the acceleration due to gravity.
A variety of rectangular weir is the slit weir, having a narrow width, so that the $\frac{b}{B}$ ratio of the weir width over the channel width should not be greater than 0.17. Such weirs were used for measuring discharges smaller than 5 l/s in the laboratory [5].

2.2 Triangular weir

The triangular weir or V-notch sharp-crested weir (Fig.1.b) is a very useful tool for measuring small discharges in engineering fields such as hydraulics, environmental, chemical and irrigation [11]. Flow and head are related by the formula

$$Q_{tr} = C_{dtr}\frac{8}{15}\sqrt{2g}tg\left(\frac{\theta}{2}\right)h^{5/2} \quad (3)$$

where C_{dtr} is the non-dimensional discharge coefficient of the V-notch weir and *h* the head over the weir vertex.

A V-notch weir can measure relatively low flow rates and [4] reports its good precision and stability, i.e., errors smaller than 2%, provided both design conditions and recomended flow range for the weir are met, and water level measurements errors are kept under 1%.

2.3 Trapezoidal (Cipolletti) weir

A trapezoidal weir integrates a rectangular weir with a triangular one (Fig.1.c) and therefore the discharge flowing over it is given by the addition of the relations (2) and (3):

$$Q_{trap.} = C_{dtrap}\left(\frac{2}{3}\sqrt{2g}bh^{\frac{3}{2}} + \frac{8}{15}\sqrt{2g}tg\frac{\theta}{2}h^{5/2}\right) \quad (4)$$

where C_{dtrap} is the non-dimensional discharge coefficient of the trapezoidal weir, *θ*/2 – angle between the weir side and vertical, *h* the head over the weir crest and *b* – the crest width.
The trapezoidal weir is used for larger flowrates and for a given crest length, the discharge it provides is greater than the discharge provided by a rectangular weir having its width equal to the crest length. Cipoletti showed the weir with the angle between the side and the vertical of 14.28° has a minimum lateral contraction of the nappe, and therefore the shape became standardised [4]. Even though this type of weir is used for large flow rates, for the present study an unusual Cipoletti weir was designed with a very small bottom width to allow measurement of very low discharges.

2.4 Proportional weir (Sutro)

The proportional sharp-crested weir is a type of weir providing a flow rate in direct proportion to the head, being used in a variety of engineering fields such as: sanitation, agriculture, environmental, chemical engineering, hydraulic [12-15]. Literature in the field presents several shapes of proportional weirs [14], [4], one of them being the Sutro weir (Fig. 1.d).
The flowrate of the Sutro weir is given by the formula

$$Q_S = K_S\left(h + \frac{2}{3}a\right) = K_S H_d \quad (5)$$

where *h* is the head over a rectangular base of height a above the weir crest, K_S - a proportionality constant, $K_S = C_{dS}b\sqrt{2ga}$, H_d - the head over a datum, chosen at *a*/3 above the crest and C_{dS} the nondimensional coefficient of discharge. The shape of the weir curvilinear profile determined by Sutro has the form provided by the equation:

$$\frac{x}{a} = 1 - \frac{2}{\pi} arctg \sqrt{\frac{y}{a}} \quad (6)$$

A considerable advantage of the proportional weir when used at the downstream end of the grit chambers is that it maintains an almost constant velocity in these, which is very important in the process of gritt sedimentation for the varying sewage flows incoming at the wastewater treatment plants.

3. Experimental setup

The experimental setup consists in a hydraulic bench, equipped with a rectangular plexiglas flume (Fig.2.), four weirs of different shapes (rectangular, V-notch, trapezoidal and proportional-Sutro) were designed for a specified discharge range, built and used for measurements. Table 1 summarises the main geometrical and hydraulic features of the experimental setup. The rectangular and V-notch weir are made up of 3mm thick stainless steel plates with baveled downstream edge whereas the other two weirs were laser cut from 2 mm plexiglass plates using a numerically controlled machine. Water is recirculated through the hydraulic bench by means of a centrifugal pump and pipes. The discharge is measured by means of a calibrated diaphragm.
The invert of the flume was maintained horizontal along the its length with the help of a bubble levelling indicator. The weir head was measured using a vernier height gauge level indicator having an accuracy of 0.2 mm, placed in a control section at a distance of 3-5 times the maximum head.

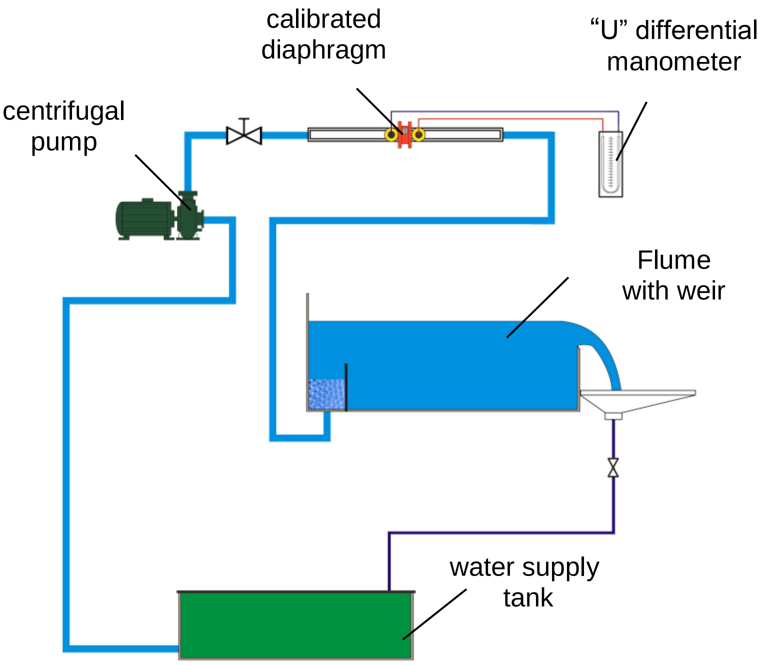

Fig. 2. Sketch of experimental setup

4. Method

To obtain reliable data, care was taken to allow flow to reach steady state conditions before each set of measurements, i.e. enough time for the flow to stabilise. Measurements were performed for increasing and decreasing flow rates between the maximum value delivered by the pump and minimum value for avoiding clinging nappe.

Table 1: Main geometric and hydraulic features of the experimental setup

Geometric and hydraulic feature	Rectangular	V-notch	Trapezoidal	Proportional - Sutro
b (m)	0.03	-	0.01	0.04
$b_{min.}$ (m)	-	-	-	0.0072
a (m)	0.005	-	-	-
p (m)	0.065	0.065	0.065	0.065
θ (°)	-	30	30	-
Q_{min} (l/s)	0.03			
Q_{max} (l/s)	0.25			
B (m)	0.135			
L_{flume} (m)	0.8			
$H_{max.\ flume}$ (m)	0.25			

5. Results

In Fig. 3. are shown the head-discharge relationships for the studied weir shapes. One may observe that the Sutro weir dependence is close to a linear function, as expected for a proportional weir.

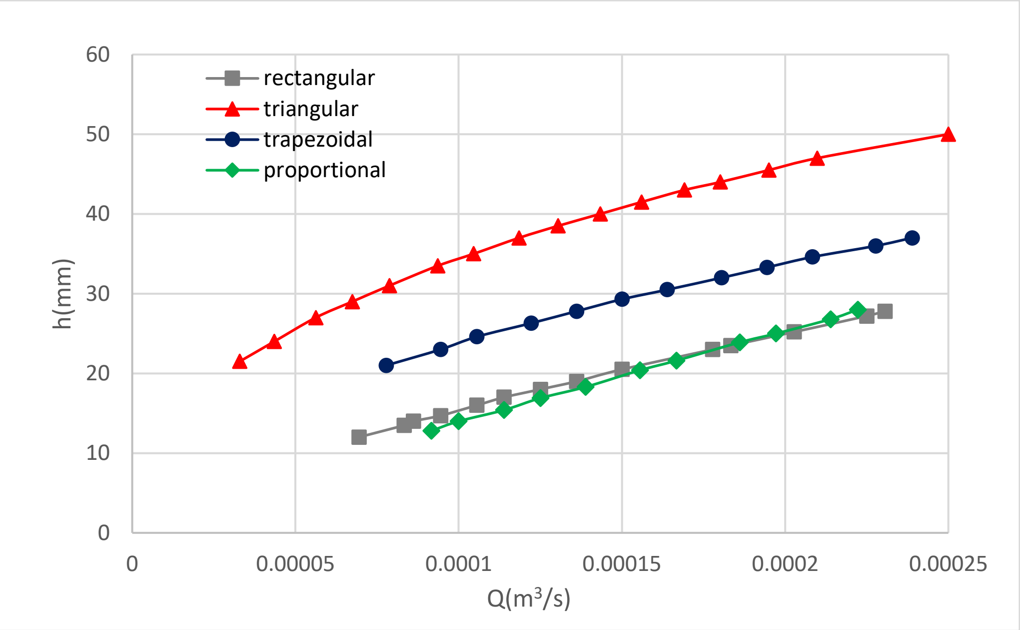

Fig. 3. Head-discharge relationships for the studied weir shapes

From Fig. 4. one may see that discharge coefficients show an increase with increasing flowrates for the case of trapezoidal weir, whereas for V-notch and rectangular weirs they decrease as flowrate increases. Discharge coefficients of the proportional weir practically remain constant. Higher and smaller than usual discharge coefficients are obtained for the cases of triangular and rectangular weirs respectively, possibly due to the fact that the prescribed minimum head requirement of 5 cm for the former and 2 cm for the latter [4] are not complied with for such small discharges measured in the laboratory.

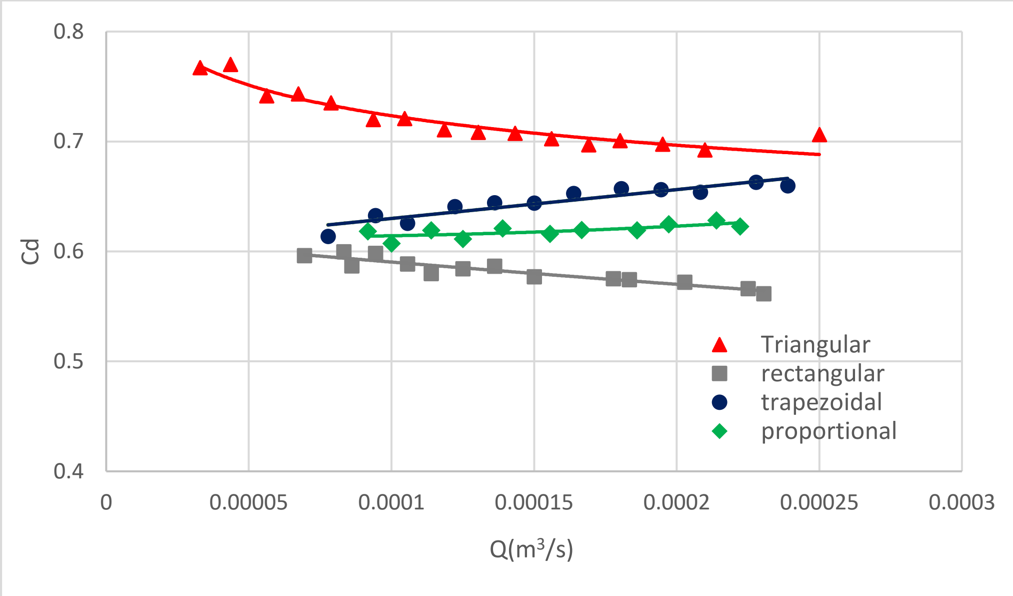

Fig. 4. Discharge coefficients for the studied weir shapes

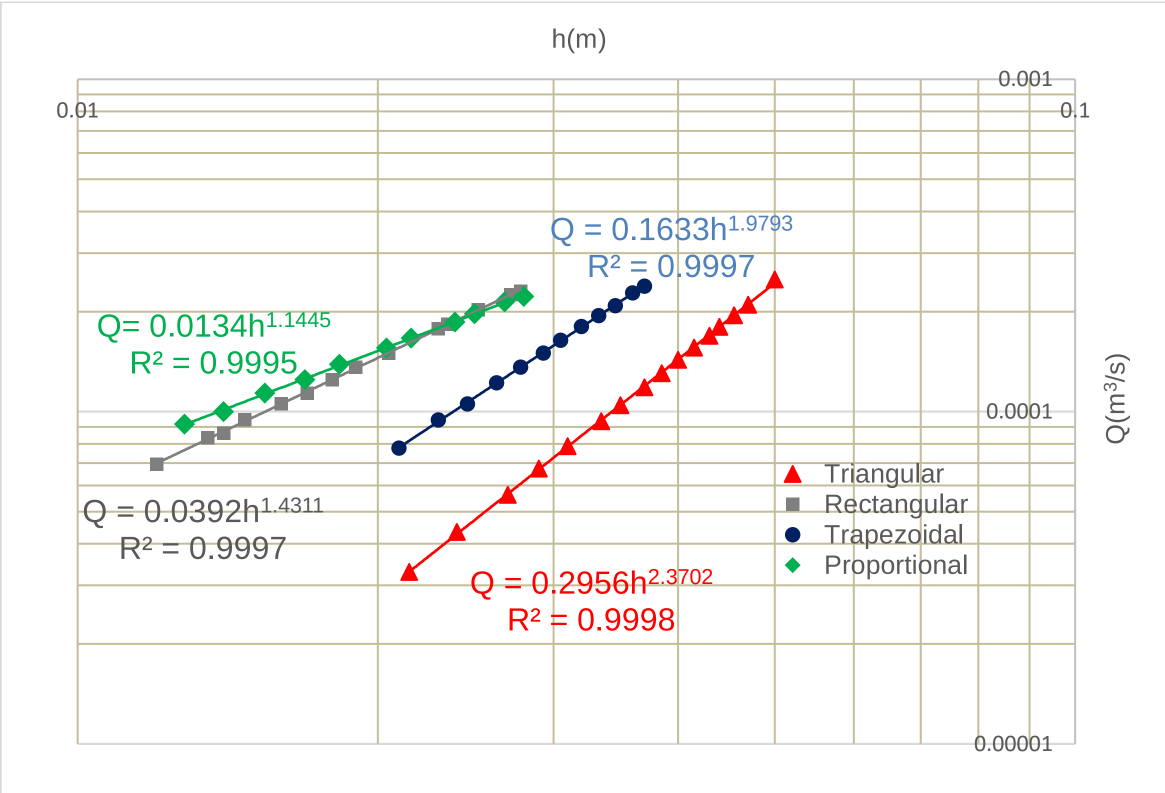

Fig. 5. Approximated head-discharge relationships for the studied weir shapes

The experimental head-discharge curves were fitted (with very good correlation coefficients) with power functions (Fig. 5.) to identify the mean two parameters in equation (1): coefficient *K* and exponent *n*. For all studied weirs the resulted exponents are close to the corresponding theoretical values such as: 1.4311 (instead of 1.5), 2.3702 (instead of 2.5), 1.1445 (instead of 1) and 1.9793 (in the 1.5 and 2.5 range).

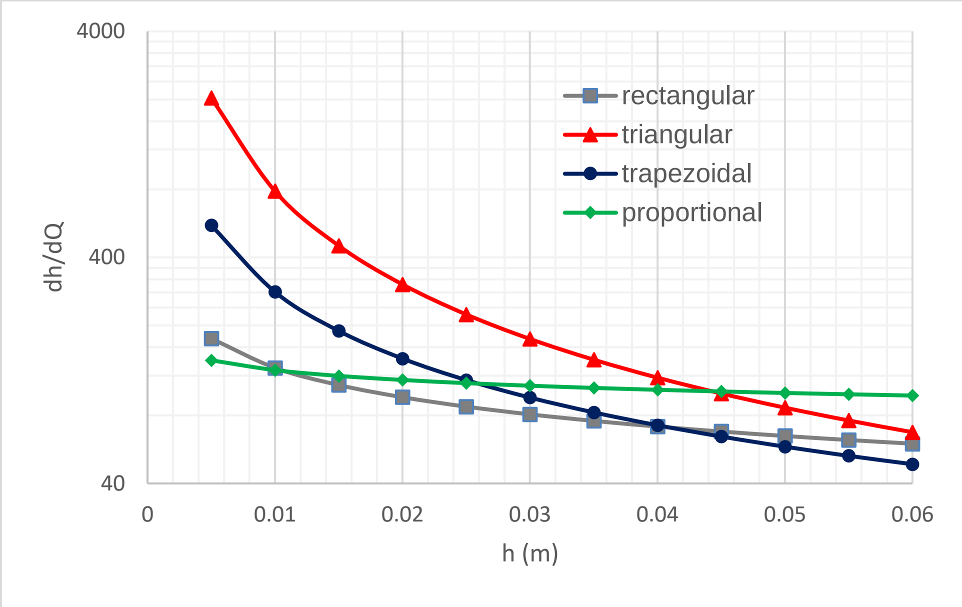

Fig. 6. Sensitivity of the studied weirs as a function of head

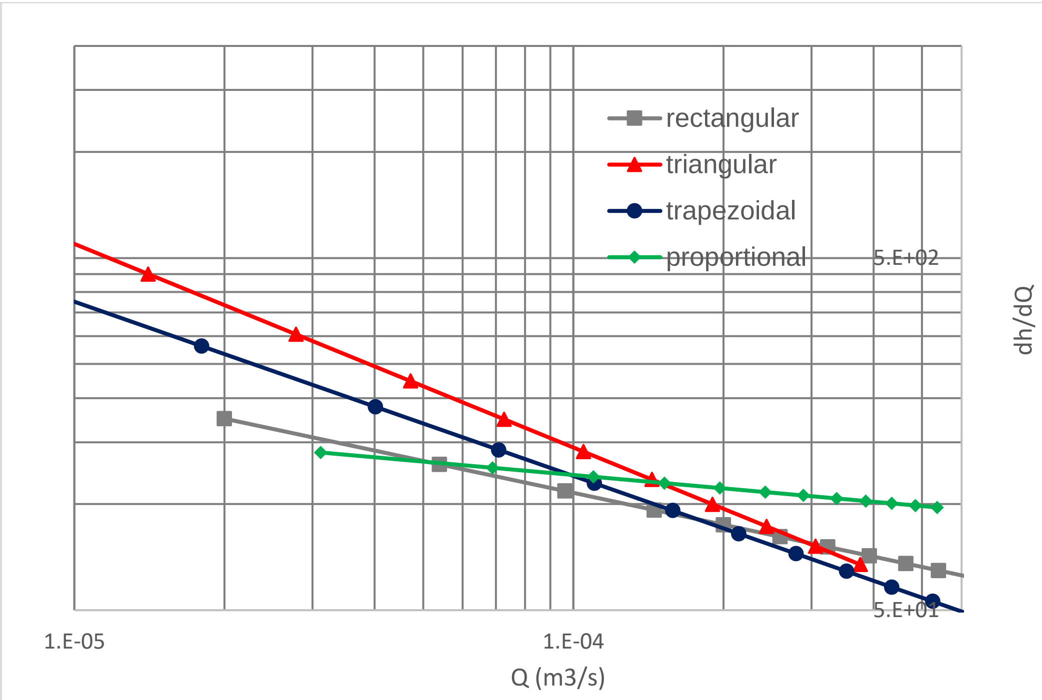

Fig. 7. Sensitivity of the studied weirs as a function of discharge

Fig. 6 and Fig.7 present the sensitivity of the studied weirs as a function of head and discharge, respectively. For heads smaller than 4.5 cm the best sensitivity belongs to the V notch, but the proportional weir becomes the most sensitive for heads greater than this value. As well, the triangular weir has the highest sensitivity for discharges smaller than about 0.152 l/s, whereas for greater values of flowrates the proportional weir takes the lead. For heads lower than 2.5 cm and flowrates smaller than 0.11 l/s, the second-best sensitivity belongs to the trapezoidal weir. Results also show a surprisingly better sensitivity of the rectangular weir, as compared with the proportional weir, for heads smaller than 1 cm and discharge values under 0.068 l/s. This could be attributed to the fact that accurate measurements for the rectangular weir can be made only for heads greater than 2 cm [4], However, when computing the relative discharge $\frac{dQ}{Q}$ error resulting from a $dh = 0.2$ mm nappe height measurement error, one can see that the highest degree of uncertainty associated with flow measurement lies with the V notch, whereas the proportional weir has the best accuracy for all head ranges (relative discharge errors under 2%).

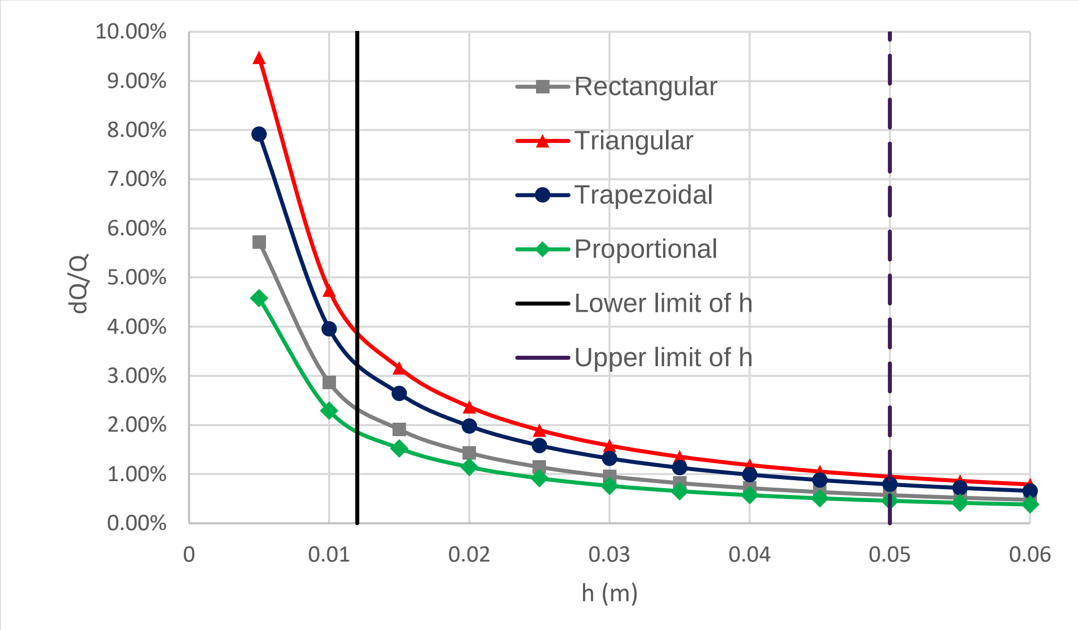

Fig. 8. Relative error of the measured discharge for 0.5 mm error of measured head as a function of head

6. Conclusions

The superiority of the Sutro weir over the non-linear weirs such as the rectangular, triangular and trapezoidal weirs is obvious in terms of multiplication of the head reding errors into the discharge ones. The former induces practically an equal error in discharge, whereas the V notch, rectangular and trapezoidal weirs induce errors of 1.42, 2.37 and 1.97 times into the discharge values respectively.
For flowrates smaller than 0.152 l/s the best sensitivity among the studied weirs belongs to the V-notch. Above this value, in the studied discharge range, the proportional weir has the best sensitivity. Therefore, these two opening-shapes of sharp-crested weirs proved to be the best for applications of measuring very low discharges.

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