Applicability of Video-Based Water Velocity Measurements on Ice-Drifting River Sections

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Abstract: Video-based velocity analysis is a novel, experimental procedure onto the definition of the surface water velocity of water flows. The method may be effective to ground data collections, when any other measurement procedure inapplicable, i.e. flash-flood events, analysis of longer water flow sections, monitoring places which can be difficult to approach.

In this paper I take an overview of the theoretical bases of the video-based velocity analysis (LSPIV), then, according to the application of two numerical methods I take an estimate onto the cross-sectional distribution of the water velocity and the discharge using the measured LSPIV velocities. At last I make proposals concerning the practical adaptability of the method.

Keywords: Large Scale Particle Image Velocimetry, Ice-drifting, Water discharge measurement

1. Introduction

The LSPIV-method is an indirect way to analyse the surface velocity conditions of a river or a freesurface flow. Opposed to the common hydrometric practice, this process doesn't require using any mechanical or acoustic device that physically contacts the flowing water. The usability of conventional water velocity and flow measuring instruments are strongly influenced by measurement conditions. Due to safety issues these instruments have limited usability in case of flash floods or ice-covered water surfaces. As a result, in most cases the hydrological-hydraulic parameters of such hydrological events can only be determined by estimates or approximate calculations, with no measured parameters. The LSPIV procedure can be an approximate method onto these hardly measurable situations, because this method requires only a video, which records the flowing water surface.

2. Large Scale Particle Image Velocimetry (LSPIV) – Theoretical foundations

The method is based on a captured a video of a particular section of the watercourse. In order to analyse the flow a series of pictures have to be done from the video, which has a determined Δt gap in time between the particular pictures. By analysing this sequence of images, a processing software determines an instantaneous velocity field by tracking the trace of a tracer floating on the surface.[1] It is obligate that the video has to be taken from that kind of section of a watercourse where sufficient tracer (leaf, foam made by turbulence, tiny cane debris, other sediment, ice) floats on the surface, as their absence impairs the mapping of the velocity field. As the video is usually made from an external viewpoint, the images that captured from the video must be transformed into a 2D orthogonal coordinate system. This method is called orthorectification. The method base is to find well-defined points that can be seen on the images, and can be identify in real conditions as well. It is also crucial to know the geographic coordinates (or the physical distance components in a North-East coordinate system) of these reference points (called GRPs - Ground Reference Points). Using the field and image points, the coordinates between the two reference systems can be determined using the following conventional photogrammetric relationship: [2]

$$x = \frac{A_1 X + A_2 Y + A_3 Z + A_4}{C_1 X + C_2 Y + C_3 Z + 1} \tag{1}$$

$$y = \frac{B_1 X + B_2 Y + B_3 Z + B_4}{C_1 X + C_2 Y + C_3 Z + 1}$$
(2)

where

x, y – the coordinates of GRPs in the coordinate system of the image,

X, Y, Z – the coordinates of the GRPs in the real space

A1 ... C3 – transformation coefficients using field reference points

There are many options to define the coordinates of ground reference points, e.g. GNSS-based measurements or to identify the points from a well-detailed aerial image (e.g. Google Earth). The LSPIV procedure determines the surface water velocity of a stream by analyzing the similarity of patterns on each image pairs, determining a so-called similarity index. The comparison made in a pixel-defined query area defined in the first image of the sequence of images (This is called interrogation area - [hereinafter IA]) in the same area of the next image of the sequence in relation to the search area ([hereinafter SA]) also defined in pixels. The maximum value of the similarity index represents the most likely movement of a tracer between two consecutive frames. With orthorectification the extent of displacement of a tracer is determined, and knowing the Δt sampling frequency between the two frames, the surface water velocity can be calculated. [1]

The method used in my analysis determines the similarity index by the following cross-correlation algorithm:

$$R_{ab} = \frac{\sum_{x=1}^{MX} \sum_{y=1}^{MY} (a_{xy} - \overline{a_{xy}})(b_{xy} - \overline{b_{xy}})}{\sqrt{\sum_{x=1}^{MX} \sum_{y=1}^{MY} (a_{xy} - \overline{a_{xy}})^2 \sum_{x=1}^{MX} \sum_{y=1}^{MY} (b_{xy} - \overline{b_{xy}})^2}}$$
(3)

Cross-correlation is determined between the areas of the first frame IA and the IAs within the SA of the subsequent frame.

In the equation:

MX, MY – IA size in pixels,

 a_{xy} , b_{xy} – Change of 8-bit grayscale (0 to 255) in IA; the values marked with the upper sign are the average values for IA

The advantage of this method is to able to estimate the water speed conditions from a low-resolution video.[1]

As a result of this procedure, the instantaneous surface velocity field of the test area can be obtained. The LSPIV vector field hereinafter allows further analysis, e.g. it is suitable for determining the average surface velocity and also to estimate the direction and turbulence of the flow; or with the geodetic measurement of the riverbed (even after the particular event) onto the calculation of the water discharge.[1]

Videos (i.e., sequences of images with a defined sampling time) can be analyzed by using FUDAA-LSPIV, a free software specifically designed to support LSPIV-based velocity vector field definition. The program is freeware and its graphical user interface runs under Java Runtime Environment.

The program analyzes 8-bit grayscale images (pgm extension) as input format, and performs orthorectification and velocity field computation on a defined sequence of images with the mentioned cross-correlation analysis. A pgm image (actually an ASCII text file) can be created from a video file of any length and resolution using free converter programs. A sample cube of the input image sequence is shown in Figure 1.

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Fig. 1. FUDAA-LSPIV input file format

To perform 2D orthorectification, at least 6 interface points per image sequence have to be selected. The orthorectification can be done in one step in case of static reference points for the whole sequence, or even frame by frame in case of moving camera.

2.1 Define the instantaneous velocity field and filter the results

To calculate the instantaneous velocity field on an image sequence the parameters needs to be defined in FUDAA-LSPIV are as follows: [3], [4]

- discretization of computational grid
- defining correlation parameters
- defining Δt time step

The correct setting of these parameters basically determines the correctness of the final result of the analysis thus their determination requires attention.

The basis of determination of the instantaneous surface velocity field is that the motion of the tracer in the 2D image sequence can be followed by the best possible correlation. The most important step in this procedure is to determine the size of the IA and SA query fields in pixels in the FUDAA-LSPIV program. Defining these parameters requires considerable caution and care. If the IA value is too small, the physical size of the surface tracer, and its change between two frames will impair the identification of the displacement, a too high IA value will dramatically increase the software's runtime, but its result will not be necessarily more reliable.

Before the analysis the detection area (the flowing water surface) also have to discretized, which means a fixed grid in pixels in the orthogonal image sequence. An example of grid points is shown in Figure 2.



Fig. 2. Computational grid, FUDAA-LSPIV

Finding the optimal value for IA and SA parameters is an experimental procedure in almost every case. It's expedient to perform some sensitivity testing onto the determination of these values. Several tests have shown that 1 second sample rate for some rapidly changing tracers is not sufficient to detect their track continuously, since the turbulence of the flow between two consecutive images rearranges the surface tracer to such an extent that the process cannot reproduce the locally experienced velocity conditions.

The result of the sequence analysis is a surface velocity field defined for the grid points. With the knowledge of the velocity field, additional numerical methods can be used to make an approximate calculation of the water discharge, which is the most important measure of hydrological practice.

3. Velocity profile estimation, determining water discharge

The calculated LSPIV velocity field will be useful in practice if we can determine the flow of the river with sufficient accuracy. To do this, it is essential to know the geometry of the wetted cross section, the water level at the time of video recording, and the distribution of the velocity profile along the Z (depth) axis. Such tasks can usually occur during the subsequent reproduction of the hydraulic characteristics of extreme flow events. Subsequent determination of riverbed geometry and water levels (e.g. from water traces) is a routine activity at the state of the art. In the following part I'm going to introduce two numerical methods for determining the assumed shape of a velocity profile in a preferred transverse section of the LSPIV surface velocity field.

3.1 Estimation of velocity profile using turbulent boundary layer theory

A generally used method of vertical velocity profile estimation is to determine a logarithmic velocity distribution based on the turbulent boundary layer theory.

Along a flat boundary (e.g. wall, riverbed) the flow state is considered to be fully fledged when the velocity doesn't change in time or along the boundary (x axis of the flow), only in the direction perpendicular to the boundary (z-axis). For this case, the simplified Reynolds equation has the following form (omitting the zero derivatives) [5]

$$0 = -\frac{1}{\rho} * \frac{\partial P}{\partial x} + \frac{\partial}{\partial z} \left(v \frac{\partial U}{\partial z} - \overline{u' * w'} \right) = \frac{1}{\rho} * \frac{\partial P}{\partial x} + \frac{1}{\rho} * \frac{\partial \tau_v}{\partial z} + \frac{1}{\rho} * \frac{\partial \tau_t}{\partial z}$$
(4)

where we separate the shear stresses due to viscosity and turbulence, and from which

$$0 = -\frac{\partial P}{\partial x} + \frac{\partial \tau_e}{\partial z} \tag{5}$$

we get a simple bivariate differential equation.

Since the pressure gradient in the equation is only a function of x and the resultant shear stress is only the function of z, both of them have to be constant for an existing solution. Consequently, they are linear functions on x or z axes. Along the above-mentioned simplifications, now let's evaluate the case of a frequently used, so-called hydraulically rough boundary. The shear stress resulting from the viscosity is generally negligible relative to the turbulence, particularly in the case of rough boundary.

After the simplification we can consider the only part that does not even require indexing

$$\tau = -\rho * \overline{u' * w'} \tag{6}$$

the turbulent shear stress. Introducing I (concept of mixing length [Prandtl, 1925], which means that distance at which a characteristic change in velocity u' can occur, [5]

$$u = U(z) + u' = U(z + l)$$
 (7)

whereof,

$$u' = U(z+l) - U(z)$$
(8)

The latter equation is solved along the z-axis in Taylor-series and keeping the first two terms

$$U(z+l) = U(z) + \frac{\partial U}{\partial z} * l$$
⁽⁹⁾

that is,

$$u' = U(z) + \frac{\partial U}{\partial z} * l - U(z) = \frac{\partial U}{\partial z}l$$
(10)

Substituting this into the shear stress formula and applying time averaging rules:

$$\tau = -\rho * \frac{\partial \upsilon}{\partial z} * l * w' = -\rho * \overline{l * w'} * \frac{\partial \upsilon}{\partial z}$$
(11)

$$sign(\tau) = sign\left(\frac{\partial U}{\partial z}\right), sign\left(w'\right) = -sign\left(l\right),$$
 (12)

assuming isotropy, o(u')=o(w') that is $u' = \frac{\partial u}{\partial z} * l$ and u' = -w' relations

$$w' = -\frac{\partial U}{\partial z} * l \tag{13}$$

Substituting this into the latest formula for shear stress and dividing it by density

$$\frac{\tau}{\rho} = -\overline{u' * w'} = l^2 * \left| \frac{\partial u}{\partial z} \right| * \frac{\partial u}{\partial z}$$
(14)

results this equation, which defines the shear stress as the multiplication of the mixing length and the square of time-averaged velocity gradient.

Let's introduce the concept of sliding speed, as follows:

$$u_* = \sqrt{\left|\frac{\tau}{\rho}\right|} = \sqrt{\left|\overline{u' * w'}\right|} = l * \left|\frac{\partial u}{\partial z}\right|$$
(15)

whereof

$$\tau = \rho * u_*^2 \text{ és } \left| \frac{\partial U}{\partial z} \right| = \frac{u_*}{l} \tag{16}$$

We also introduce the approach that mixing length, as a kind of free path length, increases linearly with the distance from the boundary, so that

$$l = \kappa * z \tag{17}$$

where κ is the so-called Kármán-constant, it's value with good accuracy is 0.4. As we use it for velocity gradient:

$$\frac{\partial U}{\partial z} = \frac{u_*}{\kappa_* z} \tag{18}$$

we get a first order differential equation which is rearrangeable (by separating the variables)

$$\partial U = \frac{u_*}{\kappa} * \frac{\partial z}{z} \tag{19}$$

we get an equation which solution is

$$U(z) = \frac{u_*}{\kappa} * (lnz + C)$$
(20)

We can introduce z0, called roughness-height, that is, the distance from the rough boundary where the U velocity is still considered to be zero, and using it as a boundary condition

$$0 = \frac{u_*}{\kappa} * (lnz_0 + C)$$
(21)

the analytical solution of the velocity profile of the turbulent boundary layer is as follows.

$$U(z) = \frac{u_*}{\kappa} * \ln \frac{z}{z_0} \tag{22}$$

By using this formula, the velocity profile of a section in the LSPIV velocity field can be determined. The u* bottom sliding speed can be estimated with the following formula [5]

$$u^* = \sqrt{g * R * I} \tag{23}$$

where

g – gravitational acceleration $(9,81 \text{ m/s}^2)$

R – hydraulic radius

I – riverbed slope

Using boundary layer theory and LSPIV surface velocity field the vertical velocity profile of a chosen cross section can be derived using the analytical solution of the velocity profile of the turbulent boundary layer. The sliding velocity (u^*) is derivable by estimating the roughness height (z0) in the knowledge of riverbed geometry with the following formula [5]

$$\frac{u_{LSPIV^{\kappa}\kappa}}{\ln\frac{H}{x_0}} = u_* \tag{24}$$

Based on this, the depth-averaged velocities of each vertical profile can be calculated with the following formula: [5]

 $U_{\acute{a}tl} = \frac{u_*}{\kappa} * \left[-\ln\left(\frac{z_0}{H}\right) - 1 \right]$ (25)

3.2 Derivation of the velocity profile from LSPIV velocity

This procedure is used for the calibration of built-in Doppler acoustic water velocity probes (H-ADCP). [6],[7]

With some modification the method can be adaptable to estimate the vertical velocity profile based on LSPIV velocity data.

Based on this numerical method, the velocity of each vertical profile in a certain cross section can be estimated as follows:

$$v_{i,j} = \alpha_i * [\gamma * (h_{max} - h_{i,j})]^{\beta}$$
(26)

where

 β - coefficient for bed roughness and flow profile shape (1/6 ... 1/12)

$$\alpha_i = \frac{v_i^{LSPIV}}{(h_i^{max})^{\beta}} \tag{27}$$

$$\gamma = \frac{v_{max}^{becs}}{v_i^{LSPIV}} \tag{28}$$

The meaning of these factors is shown graphically in Figure 3.



Fig. 3. Estimation of velocity distribution in a vertical profile

Applying this numerical method to a particular cross section of the LSPIV velocity field, water discharge can be calculated within the knowledge of the wetted area of the cross section.

$$Q = \sum_{i=1}^{n} \sum_{j=1}^{n} \Delta s * \Delta h * v_{i,j}$$
⁽²⁹⁾

By setting parameters β and γ , you can calibrate the shape of the vertical velocity profile to the shape of the velocity profile of a control ADCP measurement. Table 1 shows three typical speed profiles with the parameter values.

I. linear velocity distribution

$$V^{\text{LSPIV}} = V_{i}$$

II. estimation with a power function $v^{LSPIV} = v_{max}$,

III. estimation that may best fits to a natural free surface flow, whereby the highest velocity zone occurs in the upper third of the velocity profile

Table 1: Vertical velocity profile estimation

	β	γ	velocity profile
I	0	1	v
11	1/6	1	v
111	1/12	1.3	v

These procedures have been tested several times under controlled conditions (control flow measurements with ADCP, accurate GRPs on the video measured via GNSS RTK, appropriate tracer, prismatic riverbed) [8], [9] and all tests showed good agreement with the control results using either of the two numerical methods. However, the question may arise that if we do not explicitly accomplish LSPIV's specific information needs (correct GRPs, appropriate camera position, proper video-resolution, accurate riverbed geometry, control water discharge measurements, etc.) are we still capable of using this method with an adequate result? To prove this, I made an evaluation of a video what is captured from the Danube in an ice drifting period.

4. Evaluation of ice drifting on the Danube

The Lower Danube Valley Water Directorate (Baja, Hungary) operates a webcam at Baja, Hungary at the bank of the Danube to examine the ice conditions of the lower Danube section in Hungary. [10], [11] The directorate has provided me a full day (11 February 2012) of series of 30-second shots, which was recorded during an intense Danube ice drifting.

They also provided me a previously measured cross section of this section of the river. My goal was to determine the daily mean water discharge using the sequence of images and the cross section.

The first difficulty, of course, was the orthorectification of the sequence of images, since measured points were not found in the images. The transformation was done with the help of Google Earth using the pillars of the Danube bridge shown on the pictures, as well as two points that can be identified along the shores. The points used in the images and their alignment under Google Earth are shown in Figures 4 and 5.



Fig. 4. Danube-Baja February 11, 2012; points used for transformation



Fig. 5. The captured area with the defined GRPs (Google Earth)

Google Earth determines the points in WGS84 coordinate system with latitude-longitude values. In order to make a correct orthorectification the points have to be transformed into a local planimetric coordinate system, called EOV. It's available with a free converter program, called EHT2. The surface velocity field was determined by the parameters IA = 36 px and SA = 20 px, discretized on a 25x25 m grid on 100 images of the available series, which means an average of about 8 hours taking into account the 30 s sampling time of the images. The results were filtered above the 0,6 value of R_{ab} similarity index.

The calculated surface velocity field and the orthorectified image are shown on Figure 6.





Fig. 6. Calculated LSPIV surface velocity field (Baja, Hungary, February 11, 2012)

The transverse surface velocity distribution of the orange-marked cross section is shown on figure 7.



Fig. 7. Danube - Baja, february 11, 2012, Calculated surface velocity distribution

The water levels of the Danube are available in the Hungarian Hydrologic Database (MAHAB), thus the water level at the day of video-capture is known.



Fig. 8. Danube-Baja water level, februar 2012

Within the knowledge of the water level and a previously measured cross section of this river section, an approximate cross section can be estimated for the evaluation:



Fig. 9. Cross-section estimation for LSPIV evaluation, Danube - Baja

In view of all this the water discharge can be computable with the help of the previously mentioned LSPIV III velocity profile estimation method. The water discharge for the whole profile resulted 1994 m3/s. The result plotted on Danube - Baja profile's rating curve is the following:



Fig. 10. Rating curve of Danube – Baja profile with the LSPIV discharge

The estimated LSPIV discharge is approximately 15% higher than the rating curve, which is above the usually agreed standard in the hydrographic practice, but according to the uncertainty of the used information and data in the calculations, it means a satisfactory match.

Conclusions

One of the most important conclusions of the analysis of ice-drifting video is that orthorectification can be performed on the images even in the absence of properly measured reference points, if the images containing clearly identifiable terrain points. A similarly important conclusion is that the quality of the surface tracer materially determines the feasibility of the analysis. In case of previously processed measurements, it was an important experience that the observed natural foaming on water surface was a relatively fast changing pattern. Real velocity field hasn't been realized even beside 1second image sampling rate, only with its 3-times densified frequency (3 frames per second) was enough to reproduce the real velocity conditions, however this method necessarily increasing the runtime of the analysis. On the other hand, the ice-floats on the Danube represent a much more constant pattern on the river surface, so in this case the 30 second image sampling rate and the relatively small image resolution of 704x576 pixels did not make it impossible to determine the velocity field. Of course, the determination of the wetted cross-section is essential for the calculation of the water discharge.

Hydrometric devices of the present day - especially acoustic velocity and flowmeters - are hightech, robustly designed tools for everyday use, which multiplies the measurement efficiency of the previous - mostly mechanical - instruments. They can be used over a very wide range of water courses; they are suitable for high definition determination of water flow and many other flow characteristics.

One might well ask, with such technical knowledge and high-tech instruments, where can be the benefit of the LSPIV method, which is a fundamentally technical estimation procedure?

In the recent years in context with the climate changing there's a significant growth in the number of flash-floods, because of the increasing number of the heavy torrential rainfalls mainly in the summer period. These events mean a great challenge to the hydrologic experts, because these floods' behaviour not really known in their practice. The main problem with these events is that the hydrological measurement groups of the water directorates usually cannot "reach" the peak of the floods, partly due to lack of capacity and mainly due to the short duration of the flash floods. So, in most cases the whole flood wave drains immeasurably, and in small river basins usually no hydrographic measurement station operates to reconstruct the peak discharge, or any hydraulic parameters of these floods. Thus only a few, low-confidence hydrological estimation methods are available to determine the hydrological characteristics of the floods. Video-based procedures can be an effective tool for such cases. As some 10-second video recording from a suitable location is sufficient to estimate the surface velocity magnitude, the method can be done by anyone even via social media channels. There are some great examples to the strength of the social media e.g. Waze - a social media-based navigation application, or idokep.hu - a meteorological site, also based on social media.

The method can also be integrated into the work of the water directorates. These events usually generate an extraordinary need of information, which hardly can be performed with the current workforce. In recent years on several occasions such events occurred that have exceeded the resources available for this purpose (flood discharge measurement). The method can be successfully applied in all cases where any mechanical or acoustic measurement method fails to work. In the case of a well-organized measurement and data collection campaign, where several members of the defense organization are involved in the data collection beyond hydrographic measurement groups, significantly more numerical hydrographic information may be available for hydrological assessment of a major meteorological event. Of course, after the floods all cross sections have to be measured geodetically, including the signs of the peak water levels. In the knowledge of all these factors the peak discharges can be assessed retrospectively.

The advantage of this method is that a measurement is actually means a video recording, so it can be done by anyone.

Summarizing the results of the measurements and data processing, the LSPIV-based water velocity measurement and discharge calculation can be an appropriate complement to traditional water discharge measurement methods, even with subsequent determination of the wetted cross section.

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