Determining the Optimal Curves for Stator or Rotor Profiles of Multi-Action Hydraulic Pumps and Motors

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Abstract: *This paper presents a method of profiling the stator or rotor of the multi-action hydraulic motors and pumps in order to reduce the flow pulses rate (speed). On the basis of the presented theoretical elements, it is possible to determine correctly the profile curves, their number and also the number of pistons for the hydraulic machines of this type. The curves thus determined can be used for the creation of the programs necessary for the machining operations on numerical control machine-tools.*

Keywords: *Rotary hydraulic machines, diminution of pulses, profiling curves*

1. Introduction

The rotary hydraulic machines, pumps or motors are characterized by a specific size named capacity (stroke capacity). In the case of pumps, the flow that they supply (Q_P) is calculated using the relation [1]:

$$Q_P = q_P \cdot n \quad (1)$$

The rotation speed of a motor, neglecting the losses, can be done by means of the relation:

$$n_M = \frac{Q}{q_M} \quad (2)$$

The notations used in the relations above are the following: q_P, q_M – capacities of the pumps and motors, respectively; Q - flow; n – rotation speed (rpm).
Actually, these sizes noted by q_P, q_M are not constant, they have a pulsating and periodic character [1, 2, 3]. These pulses can introduce unwanted effects in the operation of the units. The size of their amplitude define the pulsation characteristic of the machine in the form [2]:

$$\delta = \frac{q_{MAX} - q_{min}}{q_{Med}} [\%] \quad (3)$$

It is desired that this size be as small as possible, even zero [2,4].
The frequency of pulses depends on the number of profiles, the number of pistons and, in the case of pumps, on the rotation speed of the driving electric motor.
These machines can have simple action or multiple action The machine is a simple action one if the pistons make a suction and a discharge during a full rotation. If the pistons make two or several suctions and discharges during a single rotation, the machine is a multi-action one [1, 5, 6].

2. Determining the profile curves

The case of the machines with multi-action radial pistons [1, 2, 6] is studied hereby.
It is considered that on the stator 1 operate the pistons 3 located in the rotor 2, as shown in Figure 1. This one rotates with the angular velocity ω, considered a constant.
Figure 1 has also the following notations: d – diameter of the pistons; α, β – profile defining angles; φ - current angle; $\rho(\varphi)$ - instantaneous radius expressed in polar coordinates; R_0 - minimum radius; h – difference between the maximum radius and the minimum radius (travel on profiles); ω – angular velocity (supposed to be constant); A, B, C – points defining the conditions for curves C_1

and C_2; $\rho \in [R_0; R_0 + h]$; $\alpha + \beta = \frac{\pi}{z_p}$, where z_p - number of profiles. During a complete rotation, the total stroke is executed as follows: $c_T = z_p \cdot h$.

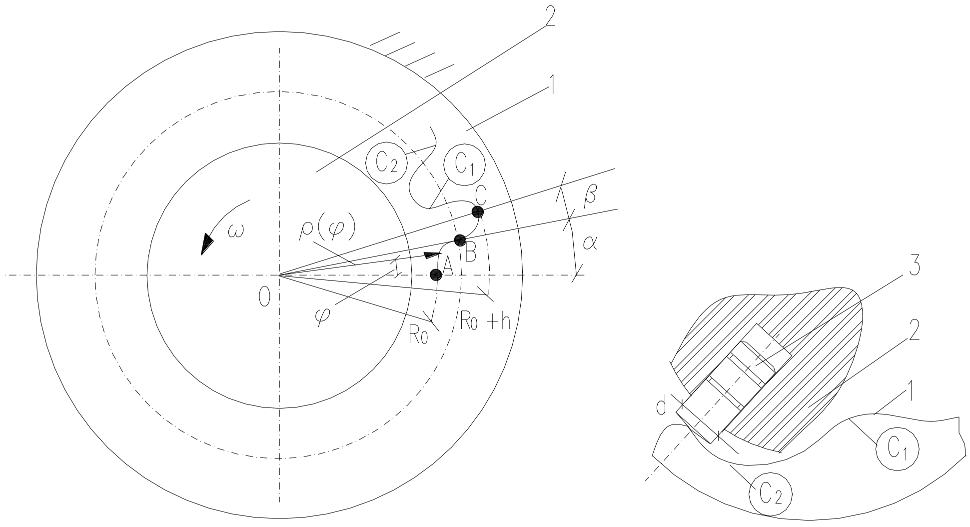

Fig. 1. Operation of the machines with piston and multiple action

The capacity of the machine is:

$$q = \frac{\pi d^2}{4} \cdot x_p \cdot z_p \cdot h \tag{4}$$

where x_p is the number of pistons.
The capacity of the machine (pump or motor) is theoretically constant, but in reality is pulsating [1, 2, 4].
The technical requirements are listed below:

- Capacity as big as possible in dimensions as small as possible;
- The curves C_1 and C_2 are determined so the capacity is constant for a certain number of pistons x_p and a certain number of profiles z_p;
- The pistons execute movements of relative translation with constant acceleration.

Figure 2 shows the exploded view on the angle (α + β).

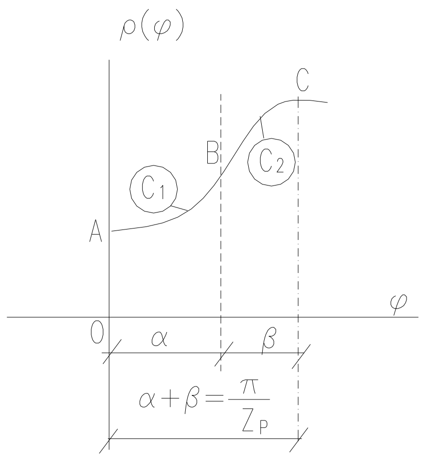

Fig. 2. Curves that define the profile

If the instantaneous radius for a piston is $\rho(\varphi)$, then the speed and acceleration have the expressions:

$$v = \frac{d\rho}{d\varphi}\frac{d\varphi}{dt} = \omega\frac{d\rho}{d\varphi} \tag{5}$$

$$a = \omega^2\frac{d^2\rho}{d\varphi^2} \tag{6}$$

If we consider that the driving speed ω is constant during the displacement on the angle $0 \rightarrow \alpha$ (curve C_1), one must write the necessary condition that the acceleration be constant too:

$$\frac{d^2\rho}{d\varphi^2} = c_1 \tag{7}$$

On the angle $\alpha \rightarrow \alpha + \beta$ (curve C_2) the constant acceleration means:

$$\frac{d^2\rho}{d\varphi^2} = c_2 \tag{8}$$

If the relations (7) and (8) are integrated twice, it follows:
- curve C_1:

$$\rho_1(\varphi) = a_1\varphi^2 + b_1\varphi + c_1 \tag{9}$$

- curve C_2:

$$\rho_2(\varphi) = a_2\varphi^2 + b_2\varphi + c_2 \tag{10}$$

According to the relations (9) and (10), the curves C_1 and C_2 are parabolas and the following conditions should be verified [7]:

$$\rho_1(0) = R_0 \tag{11}$$

$$\rho_2(\alpha+\beta) = R_0 + h \tag{12}$$

$$\left.\frac{d\rho_1}{d\varphi}\right|_{\varphi=0} = 0 \quad \text{(minimum)} \tag{13}$$

$$\left.\frac{d\rho_2}{d\varphi}\right|_{\varphi=\alpha+\beta} = 0 \quad \text{(maximum)} \tag{14}$$

$$\rho_1(\alpha) = \rho_2(\alpha) \quad \text{(continuity)} \tag{15}$$

$$\left.\frac{d\rho_1}{d\varphi}\right|_{\varphi=\alpha} = \left.\frac{d\rho_2}{d\varphi}\right|_{\varphi=\alpha} \text{(continuous derivative - common tangent)} \tag{16}$$

The relations (11) to (16) represent a system of 6 equations with 6 unknowns: a_1, a_2, b_1, b_2, c_1, c_2. After solving the system, you get:
- curve C_1:

$a_1 = \frac{h}{\alpha(\alpha+\beta)}$; $b_1 = 0$; $c_1 = R_0$.

$$\rho_1(\varphi) = \frac{h}{\alpha(\alpha+\beta)}\varphi^2 + R_0 \tag{17}$$

$$\rho_1'(\varphi) = \frac{2h}{\alpha(\alpha+\beta)}\varphi \tag{18}$$

- curve C_2:

$a_2 = -\frac{h}{\beta(\alpha+\beta)}$; $b_2 = \frac{2h}{\beta}$; $c_1 = -\frac{\alpha}{\beta}h + R_0$.

$$\rho_2(\varphi) = -\frac{h}{\beta(\alpha+\beta)}\varphi^2 + \frac{2h}{\beta}\varphi - \frac{\alpha}{\beta}h + R_0 \quad (19)$$

$$\rho_2^{'}(\varphi) = -\frac{2h}{\beta(\alpha+\beta)}\varphi + \frac{2h}{\beta} \quad (20)$$

The relations (17) ÷ (20) verify the conditions (11) ÷ (16).
In Figure 3 is shown the speed graph consistent with the relations (17) and (19).

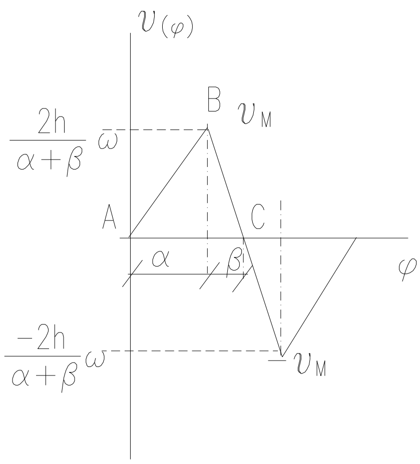

Fig. 3. Dependence of the pistons speed on their position

The derivative is maximum in point B:

$$\left(\frac{d\rho}{d\varphi}\right)_{Max} = \frac{2h}{\alpha+\beta} \quad (21)$$

Thus, the maximum speed of the pistons is:

$$v_M = \frac{2h}{\alpha+\beta}\omega \quad (22)$$

It is recommended that:

$$\frac{v_M \cdot \frac{\pi d^2}{4}}{\frac{\pi d_A^2}{4}} < v_{AD} \quad (23)$$

where d_A is the minimum diameter of the supply ports and v_{AD} – maximum speed of flow from laminar flow conditions (< 3 m/s).

3. Determining the optimum number of profiles and pistons

Further it is considered that the aspiration (filling of the piston chambers) is performed for $\frac{d\rho}{d\varphi} \geq 0$ and the discharge (emptying of the piston chambers) is performed for $\frac{d\rho}{d\varphi} < 0$. It is also considered that ω is constant.
In these conditions it is desired that, at any moment, the sum of the speeds be constant for the pistons located in the discharge area; as well as for those located in the suction area. For this reason, the number of pistons x_p will be even.

The technical conditions (spindle unloading) lead to the same work hypothesis.
If the number of pistons is 2, then surely the speed of a piston will not be constant. Therefore, the minimum number of pistons must be 4. So, two pistons are located in the discharge area and two in the suction area and it is desired that the sum of the piston speeds in the same state be constant.
We consider that there are 4 pistons and 3 profiles.
Figure 4 shows the initial position of the pistons.

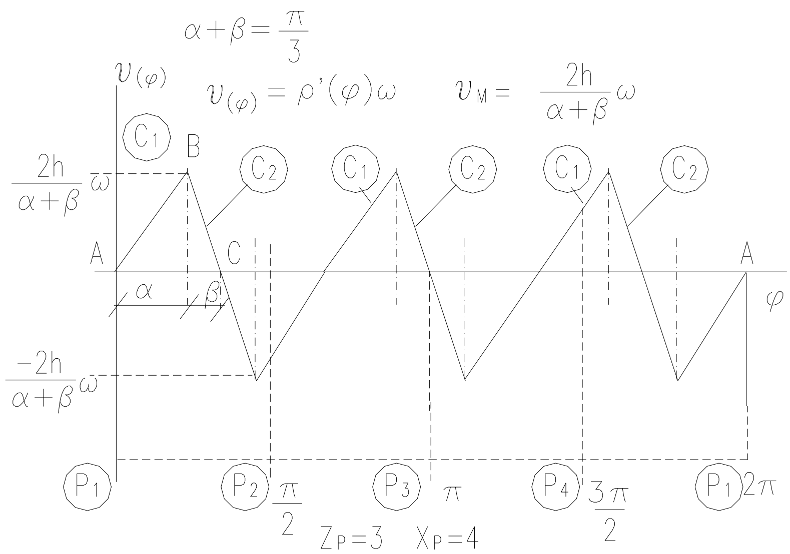

Fig. 4. Defining the initial position of the pistons for different α and β

At this moment, the speeds of the pistons P_1 and P_3 are summed in the discharge stage. We notice that the sum of these speeds cannot be constant because the speeds will increase during the movement to the right as both pistons are placed on the curve C_1. So, it is necessary for the pistons in the same state to be on different curves. Hence: α = β:

$$12\alpha = 2\pi \Rightarrow \alpha = \frac{\pi}{6} \tag{24}$$

In the new conditions, the initial position of the pistons is shown in Figure 5.

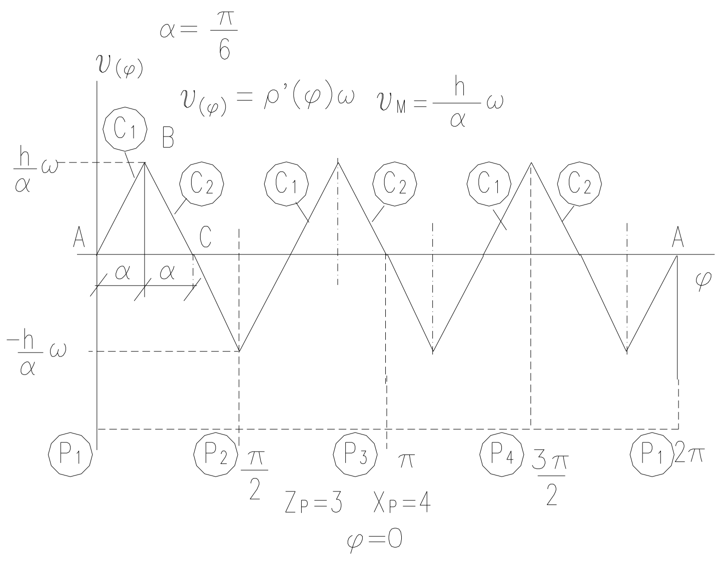

Fig. 5. Defining the initial position of the pistons for α = β

If we consider a rotation with the angle φ ($\varphi < \alpha$), the new position of the pistons is the one in Figure 6.

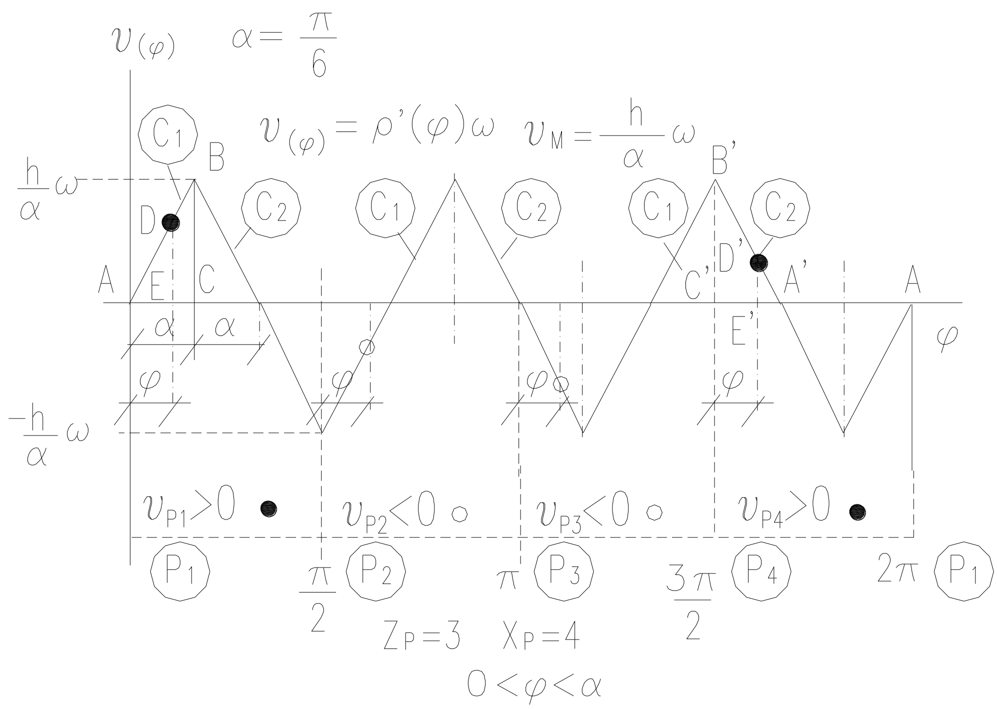

Fig. 6. Defining the current position of the pistons for $\alpha = \beta$

From the similarities of the triangles ABC and A'B'C' it follows:

$$\frac{v_{P1}}{\frac{h\omega}{\alpha}} = \frac{\varphi}{\alpha} \tag{25}$$

From the similarities of the triangles A'B'C' and A'D'E' it follows:

$$\frac{v_{P2}}{\frac{h\omega}{\alpha}} = \frac{\alpha-\varphi}{\alpha} \tag{26}$$

According to the relations (25) and (26) it follows:

$$v_{P1} + v_{p2} = \frac{h}{\alpha}\omega = ct. \tag{27}$$

If $\alpha < \varphi < 2\alpha$, the pistons P_1 and P_2 become active (discharge).
If $2\alpha < \varphi < 3\alpha$, the pistons P_2 and P_3 become active.
If $3\alpha < \varphi < 4\alpha$, the pistons P_3 and P_4 become active. Then the phenomenon is periodic.
Depending on the size (q) desired, other variants can be also achieved.
Numerical example:
h - imposed
d - imposed
$z_P = 3$ ($\alpha = 30^0 = \pi/6$; $\alpha = \frac{2\pi}{hz_p}$)
$x_p = 4$ (number of pistons)
The capacity is:

$$q = \frac{\pi d^2}{4} \cdot x_p \cdot z_p \cdot h = \frac{\pi d^2}{4} \cdot h \cdot 3 \cdot 4 \qquad \text{(according to relation (4))}$$

The flow is:

$$Q = q \cdot \frac{\omega}{2\pi} = \frac{\pi d^2}{4} 12h \cdot \frac{\omega}{2\pi}$$

We calculate the flow using also the relation (27):

$$Q = \frac{\pi d^2}{4} \cdot (v_{P1} + v_{P2}) = \frac{\pi d^2}{4} \frac{h}{\alpha} \omega = \frac{\pi d^2}{4} 12h \cdot \frac{\omega}{2\pi}$$

So, the theoretical flow is equal to the instantaneous flow and it is constant.

4. Experimental achievements

Based on the theoretical elements presented, a rotary hydraulic moment and a multi-acting pump were designed and manufactured. They are shown in Figure 7.

a. b.

Fig. 7. Hydraulic motor (a) and pump (b) with profiled elements with parabola arches

The basic characteristics of these hydraulic machines are shown in the Tables 1 and Tables 2.

Table 1: Characteristics of the rotary hydraulic motor

	q_M [cm³]	x_P	z_P	h [mm]	p_{Max} [bar]	T_{Max} [Nm]
Motor	45	8	6	3	220	150

Table 2: Characteristics of the pump

	q_P [cm³]	x_P	z_P	h [mm]	p_{Max} [bar]	Q at 1500 RPM [l/min]
Pump	4	4	3	3	220	6

5. Conclusions

For the multi-action machines with pistons, the profiles of the actuators can be made according to parabolas. In this case, the accelerations are constant and the relative speeds of the pistons relative to the bores depend linearly on the current angle.
By correlating the number of pistons with the number of profiles, constant capacities can be obtained. Due to size considerations, the number of these profiles, as of the pistons too, is limited.
Based on the presented curves, it is possible to create programs that allow the machining of the stators and rotors on numerical control machine tools.

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