Comparison of Rainfall Interpolation Methods for Obtaining Mean Annual Maximum Precipitation Isohyets in a High Elevation Zone in Mexico

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Abstract: *Average rainfall calculation by isohyets method is considered in hydrology as one of the most accurate ones because traditionally this procedure takes into account site topography to draw lines of equal precipitation; also having isohyets in a region allows estimating mean precipitation at a point that does not have measurements. The spatial interpolation process consists of estimating values that a variable Z reaches in a set of points defined by a pair of coordinates (X, Y), known Z values of a set of points located in a study area. Almost all interpolation procedures are based, roughly, on the use of statistical techniques. To quality verification of an interpolated map, a validation set consisting of a series must be used of sampling points (of which the real value is therefore known) at which an estimate is to be made of said real value (without using of course the value measured in them). In this study, Inverse Distance Weighed (IDW) method and ordinary Kriging method (OK) with Gaussian variogram were applied in addition to Cokriging method (OCK) that uses a geographic information system in order to compare them from the density and shape of the isohyets obtained with these methods. For this, weather stations from CLICOM database belonging of Mexico City and the Valley of Mexico were considered. Additionally, design isohyets were obtained for 10 years return period to make comparisons with traditional isohyets existing in databases of an official agency. It was found that IDW method allows to draw closed isohyets of low precipitation compared to those obtained with Kriging, even though Kriging method in design events reports isohyets more parallel to traditional method ones; on the other hand and this method gave the best determination coefficient and lower mean square error, Cokriging method, which considers elevation as an additional variable, gave parallel shapes similar to those drawn historically. Therefore, it is considered that Kriging and Cokriging methods are valid for estimating design isohyets in the high elevated zone of Mexico analyzed.*

Keywords: *Isohyets, average maximum annual rainfall, Mexico City, Kriging, IDW, Cokriging*

1. Introduction

Average rainfall occurring in a basin is a fundamental concept of hydrology that is used in many practical problems, especially for rainfall transformation into runoff and subsequent obtainment of hydrographs that subsequently have to be simulated for a basin or river. The methods traditionally used for its estimation are the arithmetic method, Thiessen polygons and the Isohyets method [1,2],latter being most recommended because it considers study site topography and gives a better indication precipitation contribution at each site of the basin, considering watershed, although there are authors who have reported that depending on the study site, one method may be better than another [3].
Geographic information systems, which use has become widespread since 1980s, use various interpolation tools to plot surfaces and lines of equal value of a variable, this are used as aids for plotting contour lines and climatic variables such as isotherms and isohyets. Among these methods, it stands out the Kriging methods and the inverse of the weighted distance method (IDW) of which persists the discussion of whether the Kriging methods have advantage over the IDW method due to the fact that the first considers, not only space influence, but also of time in analyzed variable, while IDW method only highlights spatial aspect or data separation according to

site analysis [4], also authors report that Kriging method gives better values of equal-valued curves depending on climatic variable [5,6]. [6] found that precipitation interpolation at a site improves when taking into account variables addition of precipitation and distance, if elevation is contemplated as an additional variable; from their comparisons between interpolation methods, they reported as the best the ordinary Cokriging method applied on monthly precipitation data in a basin of Australia. In areas of the Gulf of Mexico, elevation variable does not show a high correlation with precipitation, according to the study by [7] and Cokriging method reports better results than Idw technique, but does not improve on thin plate smoothing spline (TPSS) techniques.
The previously cited studies indicate that one rainfall interpolation method may be better than another depending on the physiography of the analyzed site; for that reason, this study investigates which of these interpolation methods reports the best results in precipitation estimation with isohyets in a region of high elevations and subject to convective rainfall. For this purpose, a comparison of isohyet maps calculated with three ArcGis© tool is made: one of Ordinary Kriging methods (in this case one that uses a Gaussian semivariogram), IDW method, and also using Cokriging method that takes into account precipitation, distance and elevation of measurement points. Mean square error and determination coefficient are used as basis of comparison Additionally, a comparison of design isohyets obtained with these methods was made with respect to isohyets drawn "by hand" reported by databases of official agencies in Mexico [8], i.e., with traditional method for design isohyets for a return period Tr=10 years. The study area used was Mexico City, which has elevations up to 2250 meters above sea level, and some stations for Mexico State, in Mexican Republic, were also considered.

2. Methodology

2.1 Isohyets Method

P Isohyets are defined as curves of equal precipitation. This method makes it possible to obtain

Weighted average precipitation can be acquired in the following way [9] (Figure 1):
a) Isohyets are drawn from stations, considering high topography points.
b) Each isohyet is assigned an area equal to sum of half of areas between two contiguous isohyets, or each area between two isohyets is assigned an average value of precipitation Pi of the isohyets.
c) Area Ai of the surfaces is estimated, the total area A_t will be sum of these areas.
d) Mean value is calculated using the following expression (eq. 1):

$$P = \frac{1}{A_t}\sum_{i=1}^{n} P_i A_i \quad (1)$$

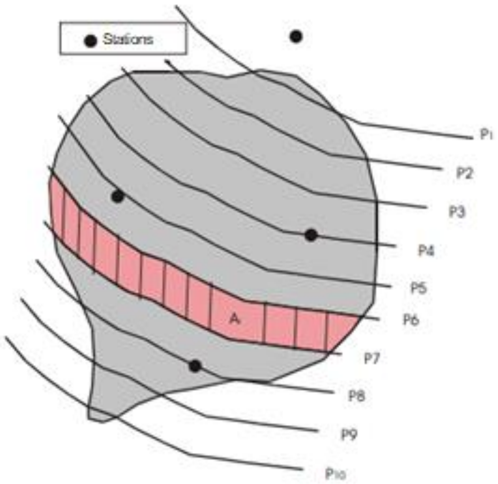


Fig. 1. Isohyets method. (Source: [9])

2.2 Inverse Distance Weighted Method (IDW)

The Inverse Distance Weighted Method (IDW) is used and described in ArcGis© or Arcmap of Esri©, it was presented in 80's by [10]. [11] performs interpolation of the value of a cell with the help of a weighted linear combination of sample points taken at a certain distance; by default, power of distance of the points to one of interest is set to 2 and the greater the distance of sample points to one of interest less participation it will have in estimation of its value. It is considered a deterministic method because it uses a mathematical equation and considers values measured in surroundings. [12]. Equation used by IDW method, to estimate precipitation P ̃(S_0) at a point S_0, known precipitation information P(Si) at n points S_i , with i=1,2,...,n , is as follows [6] (eq. 2):

$$\hat{P}(S_0) = \sum_{i=1}^{n} \omega_i P(S_i) \tag{2}$$

Where: $\omega_i = \frac{d_{i0}{}^{-k}}{\sum_{i=1}^{n} d_{i0}{}^{-k}}$, ω_i are the weight factors, d_{i0} Euclidean distance between points i with known information and point 0, k is a power given as a control parameter.

This method interpolates a raster surface from points using inverse distance weighting (IDW) technique. Main parameters considered by this method are described in [13, 14].

2.3 Variogram

Let Z be a variable that takes values at points i and j; if a new variable y is defined as difference (eq. 3):

$$y = z_i - z_j \tag{3}$$

Variogram of Z is variance of the prior y variable and is denoted (eq. 4):

$$\gamma_{ij} = E((z_i - z_j)^2) \tag{4}$$

Variogram depends only on separation of the two points i and j, but not on the location of these points [15].

2.4 Kriging Methods

Kriging methods [16, 17] are geostatistical interpolation methods and take into account the spatial correlation between the data, i.e. not only the distance of the sample data to the data to be estimated, but also the way in which the sample points are spatially positioned. two steps that Kriging methods follow are: obtaining statistical dependence rules with the help of creating variograms and covariance functions to calculate the statistical dependence values (called spatial autocorrelation) that depend on autocorrelation model (fitting a model) and in a second step performs unknown values prediction. [11].

There are two methods for Kriging: ordinary and universal [13].

Ordinary Kriging can use the following semivariogram models:

1) Spherical: spherical semivariogram model. This is the default.
2) Circular: circular semivariogram model.
3) Exponential: exponential semivariogram model.
4) Gaussian: normal or Gaussian distribution semivariogram model.
5) Linear: linear semivariogram model with a sill.

Universal Kriging can use the following semivariogram models:

1) Linear with linear derivative: universal Kriging with linear derivative.
2) Linear with quadratic derivative: Universal Kriging with quadratic derivative.

There are options available through the Advanced Parameters dialog box. These parameters are:

1) Lag size: default value is the cell size of output raster.
2) Major range: represents a distance beyond which there is little or no correlation.
3) Partial threshold: the difference between the nugget and the threshold.
4) Nugget: represents error and variation at spatial scales too fine to detect. The Nugget effect is seen as a discontinuity at the origin.

In this analysis, the lowest variances were obtained with the ordinary Kriging method and Gaussian semivariogram.

Estimating precipitation equation for ordinary Kriging method is, eq 5:

$$\widehat{P(s_0)} = \sum_{i=1}^{n} \omega^{ok}{}_i \, P(S_i) \tag{5}$$

Where: $\sum_{i=1}^{n} \omega^{ok}{}_i = 1$, $\omega^{ok}{}_i$ are the weight factors of the ordinary Kriging method that are estimated by solving a system of n+1 linear equations (eqs. 6 and 7):

$$\sum_{i=1}^{n} \gamma(s_i - s_j)\omega_i^{ok} + \mu_1^{ok} = \gamma(s_i - s_0) \tag{6}$$

$$\sum_{i=1}^{n} \omega^{ok}{}_i = 1 \tag{7}$$

Where γ (s_i − s_j) are the variogram values between sampling locations, s_i and sj, γ (s_j − s_0) are variogram values between sampling location sj and target location, and s_0, is Lagrange multiplier parameter. Experimental variogram using Ordinary Kriging method can be found in [6]; in hydrology it is common to use defined positive exponential variogram, Gaussian and spherical type to model experimental variogram (Table 1).

Table 1: Examples of positive definite variograms used in Ordinary Kriging method (Source: [6])

Variograms model	Equation
Exponential	$\gamma(d) = C_0 + C_1[1 - e^{(-\frac{2d}{a})}]$
Gaussian	$\gamma(d) = C_0 + C_1[1 - e^{\left(-\frac{2d^2}{a^2}\right)}]$
Spherical	$\gamma(d) = \begin{cases} C_0 + C_1\left[\frac{2}{3}\left(\frac{d}{a}\right) - \frac{1}{2}\left(\frac{d^3}{a^3}\right)\right], d < a \\ C_0 + C_1, d \geq a \end{cases}$

C_0 = Nugget coefficient, C_0 + C_1 = S_{ill}, a = variogram model range. d = separation distance between two locations

2.5 Cokriging Method

Cokriging method proposes to improve Ordinary Kriging method by considering a second variable to improve estimation of search variable; for case of precipitation estimating at a point, as a precipitation function at other sites, the distance between them and elevation as a second variable, equations of method are by next form (eq. 8):

$$\widehat{P(s_0)} = \sum_{i1=1}^{n} \omega^{ock}{}_{i1} \, P(S_{i1}) + \sum_{i2=1}^{n} \omega^{ock}{}_{i2} \, E(S_{i2}) \tag{8}$$

Where: $\sum_{i1=1}^{n} \omega^{ok}{}_{i1} = 1, \sum_{i2=1}^{n} \omega^{ok}{}_{i2} = 0$, $\omega^{ok}{}_{i1}$ are the weight factors associated with first variable P (precipitation in this study) and $\omega^{ok}{}_{i2}$ are weight factors associated with second variable E (elevation in this analysis). Such factors are obtained by solving a system of n+2 simultaneous equations [6] (eqs. 9):

$$\sum_{i1}^{n} \gamma_{PP}(S_{i1} - S_{j1})\omega_{i1}^{ock} + \sum_{i1}^{n} \gamma_{PE}(S_{i2} - S_{j1})\omega_{i2}^{ock} + \mu_1^{ock} = \gamma_{PP}(S_{i1} - S_0)\ for\ j1 = 1, \dots, n$$
$$\sum_{i1}^{n} \gamma_{EP}(S_{i1} - S_{j2})\omega_{i1}^{ock} + \sum_{i2}^{m} \gamma_{EE}(S_{i2} - S_{j2})\omega_{i2}^{ock} + \mu_2^{ock} = \gamma_{EP}(S_{i2} - S_0)\ for\ j2 = 1, \dots, m$$
$$\sum_{i1=1}^{n} {\omega^{ock}}_{i1} = 1 \sum_{i2=1}^{m} {\omega^{ock}}_{i2} = 0 \tag{9}$$

Where: $\gamma_{PE}(S_{i2} - S_{j1})$ and $\gamma_{EP}(S_{i1} - S_{j2})$ are the values of variograms crossed between the values of variables P and y E; μ_1^{ock} and μ_2^{ock} are Lagrange multipliers that take into account two unbiased conditions. The key in this method is to establish a model that establishes continuity and cross-dependence. Further details of this theory are explained by [6].

2.6 Characteristics of errors in interpolation

Error estimation in a point is difference between t-measured value and estimated one. Likewise assigning and error to each point.
Errors set must have the following characteristics:
1. Errors means and errors square must be near zero
2. Errors values must be independent of their location on space and not be autocorrelated.
3. Errors distribution function must be approximated to a normal distribution.

2.7 Study site

A portion of Mexico comprised by Mexico City was considered for analysis; from CLICOM database [18], climatological stations were selected for mean annual maximum precipitation data from 40 revised stations in Mexico City (with 20 or more years of record, some stations with data from 1921 to 2018), from the regionalization study of [2] . (Figure 2).

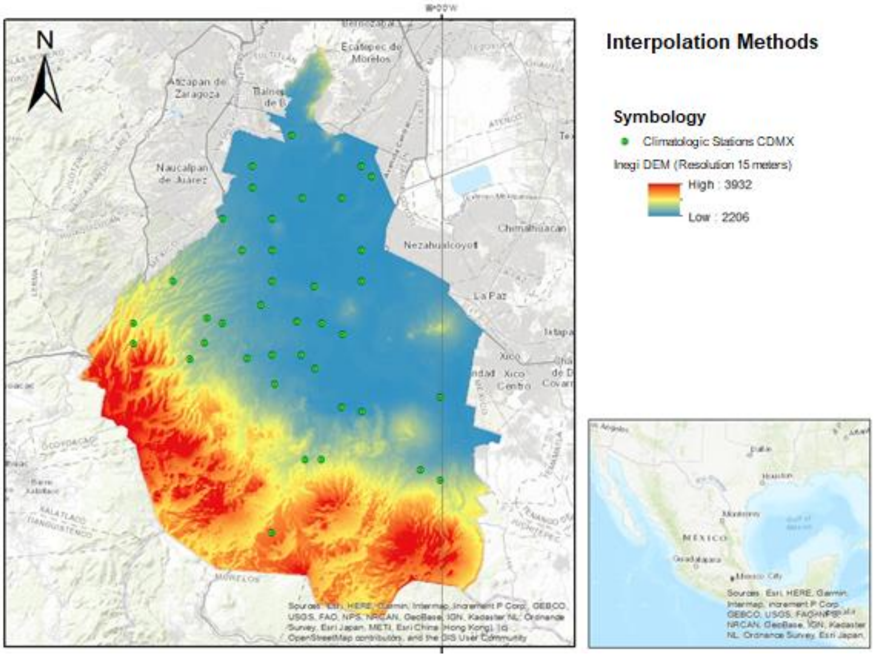


Fig. 2. CLICOM climatological stations in Mexico City. (Source: own design)

The digital elevation model (DEM) provided by INEGI, with a cell size of 15 m, was used. We had .shp files of the points of climatological stations, DEM and CDMX polygon.

3. Results and discussion

Spatial Analysis tool of ArcGis was used to apply different interpolation methods that this application has: a) IDW method b) Ordinary Kriging method (OK) and free tool compatible with ArcGis was added for the case of the c) CoKriging method. In each case, default parameters provided by Arcmap were used.
Figure 3 shows the isohyet map obtained with IDW method considering all climatological stations and Figure 4 shows isohyet map drawn by eliminating three stations with locations at high, intermediate and low elevations in the Mexico City basin, to see the sensitivity in the plotting of the isohyets by each method, three stations that were eliminated have elevations of 2229, 2260 and 2850 masl. Figures 5 and 6 show similar maps obtained with Ordinary Kriging method and Gaussian semivariogram with same considerations of using all stations and removing three stations mentioned above.

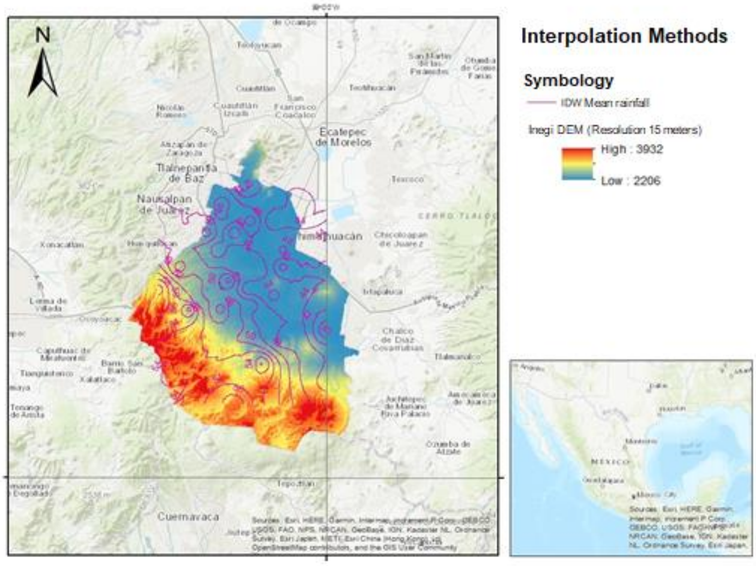


Fig. 3. Historical maximum annual mean rainfall isohyets map of Mexico City. IDW method using the stations. (Source: own design)

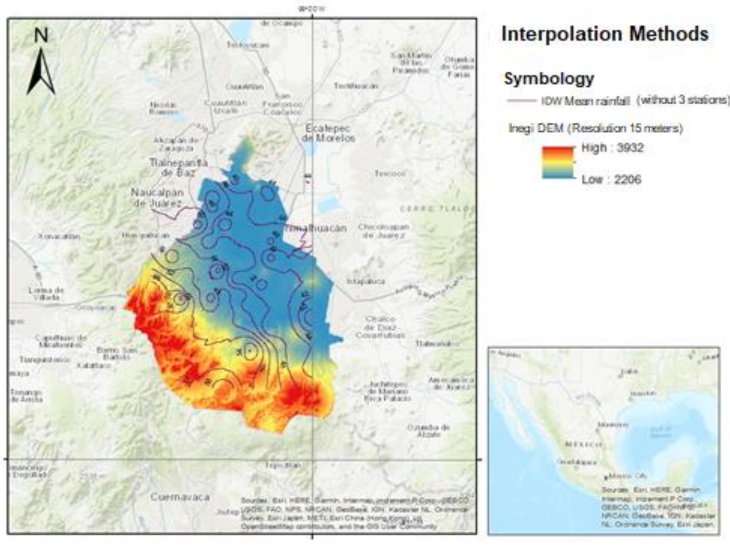


Fig. 4. Maximum annual mean rainfall isohyets map for Mexico City. IDW method and removing 3 stations. (Source: own design)

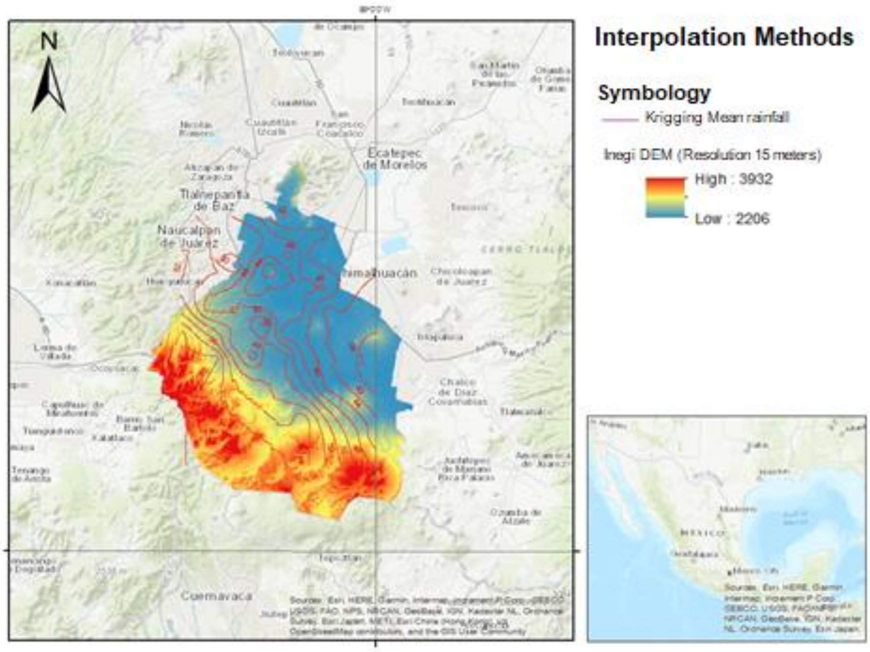


Fig. 5. Maximum annual mean rainfall isohyets map for Mexico City. Kriging Gaussian method and all stations. (Source: own design)

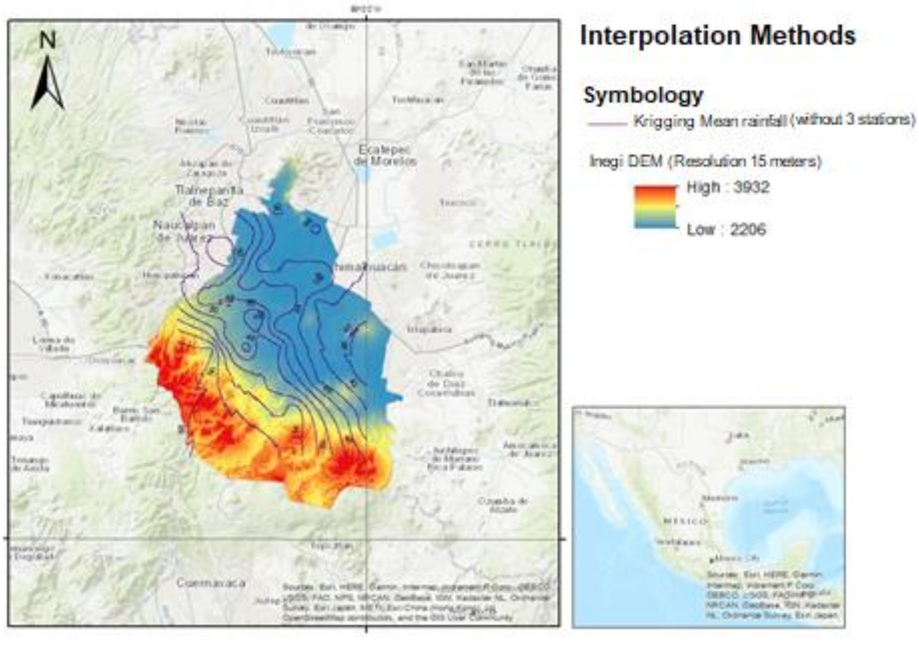


Fig. 6. Historical maximum annual mean rainfall isohyets map of Mexico City. Gaussian Kriging method and removing 3 stations. (Source: own design)

From figures 4 to 6 it can be noticed that Gaussian Kriging method has greater difficulty in obtaining closed isohyets, while IDW method does manage to represent closed precipitation isohyets of more than 70 mm. On the other hand, traces of the SCT curves, particularly those that do not manage to close, are a little more similar to those of the Kriging method.

Using design factors, considering a General Extreme Values function and regionalization methodology by [2] (Table 2) and with mean of maximum annual historical precipitation, the design isohyets were calculated for a return period of 10 years, using IDW method and Gaussian Kriging Method. Figures 7 and 8.

Table 2: Mexico City regional factors. General Extreme Value Function

Tr years	Factor
2	0.94326
5	1.2053
10	1.3868
20	1.567
50	1.8095
100	1.9984
500	2.4597
1000	2.6691
5000	3.1824
10000	3.4158

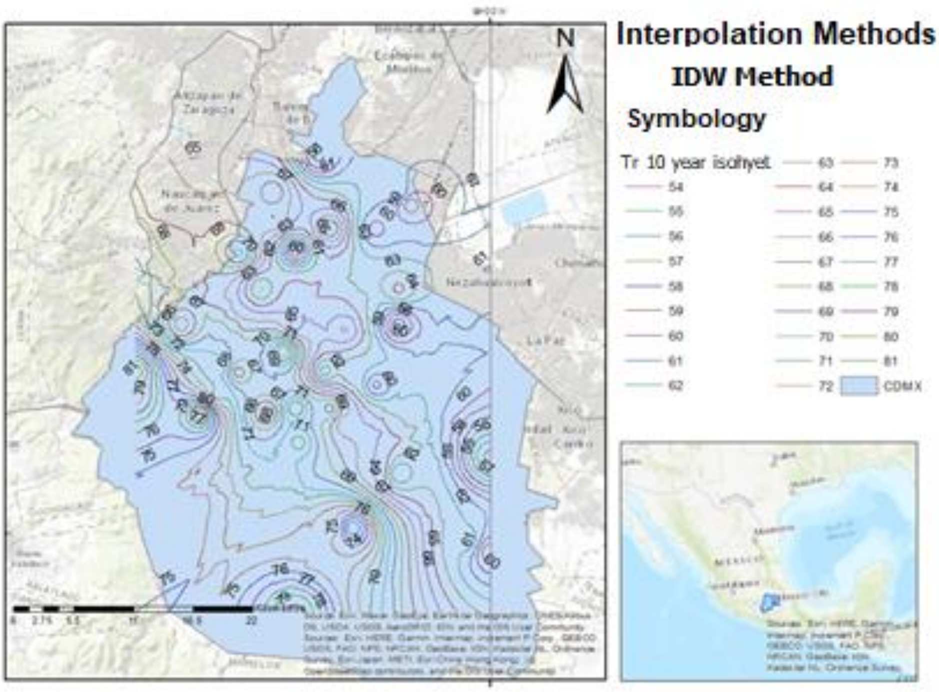


Fig. 7. Maximum annual mean rainfall isohyets map for Tr=10 years for Mexico City. IDW method. (Source: Own design)

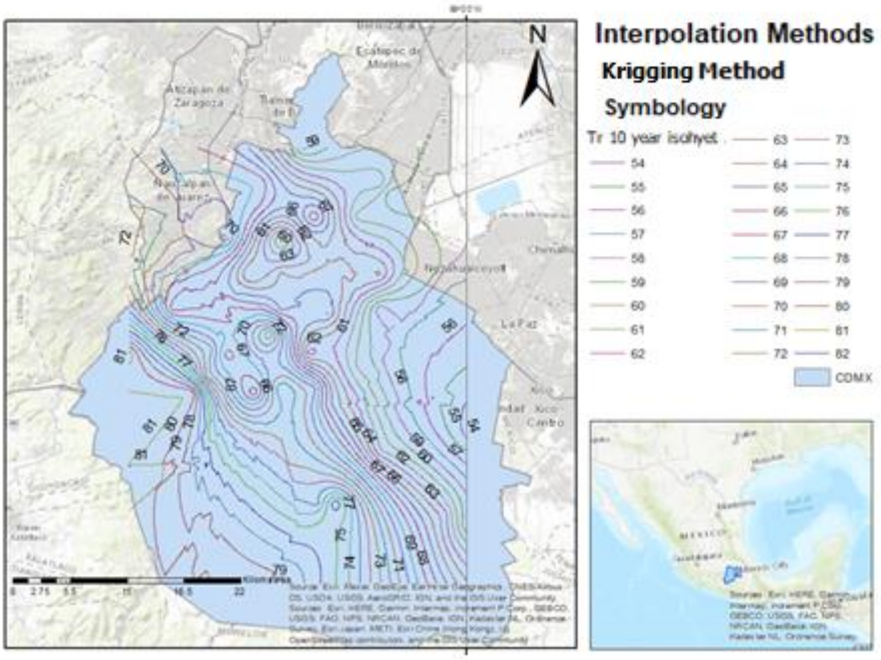
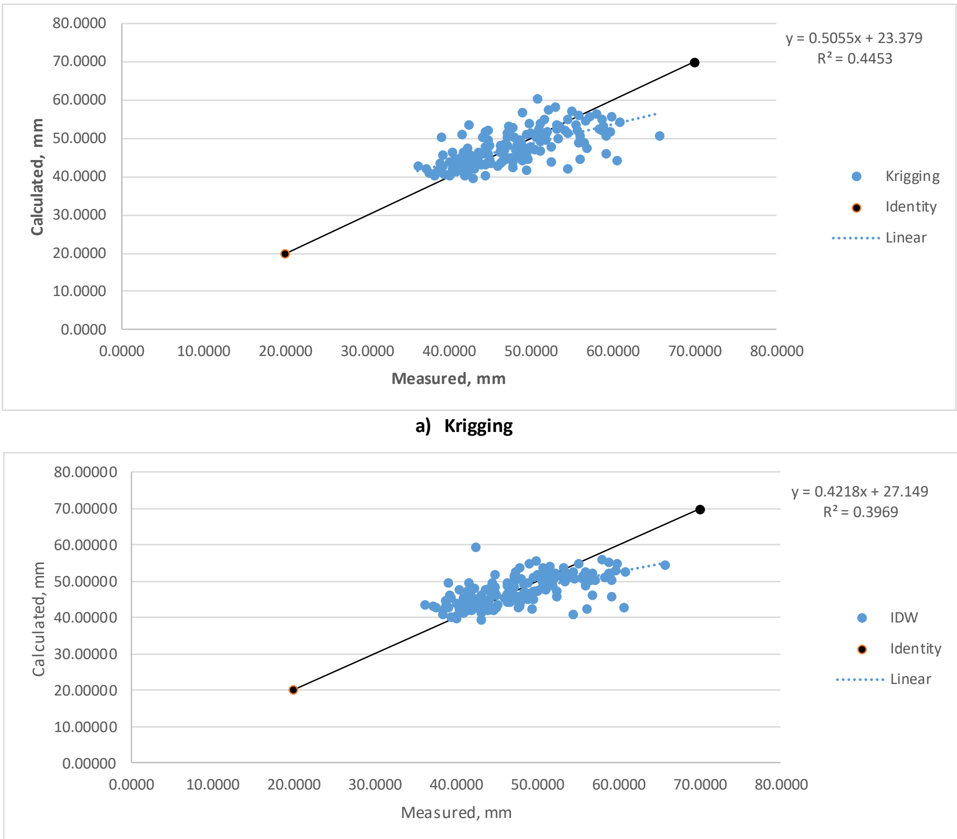


Fig. 8. Maximum annual mean rainfall isohyets map for a Tr=10 years for Mexico City. Gaussian Kriging method (Source: Own design)

Helped with the ArcGIS software, measured and calculated values with the Kriging and IDW methods were obtained. Measured data graphs against those calculated with these methods are shown in Figure 13 a and b.



b) IDW

Fig. 13. Comparison of the measured and calculated values with the methods a) Kriging and b) IDW

The mean square error obtained with the Kriging method was 21.19 while with the IDW method it was 22.83, that is, Kriging gave a lower error in the estimation of the isohyets. From Figures 13 a and b, a determination coefficient R2 of 0.4453 and 0.3969 was estimated with Kriging and IDW, which implies a correlation coefficient of 0.67 and 0.63, so that the Kriging method resulted in a better fit.
Additionally, the descriptive statistics of the errors obtained with the Kriging and IDW methods as well as their histograms were performed (Table 3 and Figures 14 and 15).

Table 3: Descriptive statistics results. Kriging and IDW methods

Concept	Kriging	IDW
Mean	0.0593	0.2567
Typical error	0.3673	0.3807
Median	-0.4365	-0.1418
Mode	none	none
Standard deviation	4.6174	4.7858
Variance	21.3208	22.9043
Curtosis	1.4628	1.8356
Skewness	0.6680	0.4633
Range	27.6204	34.6957
Minimum	-11.3389	-16.8333
Maximum	16.2815	17.8624
Sum	9.3631	40.5624
Data number	158.0000	158.0000
Confidence level (95.0%)	0.7256	0.7520

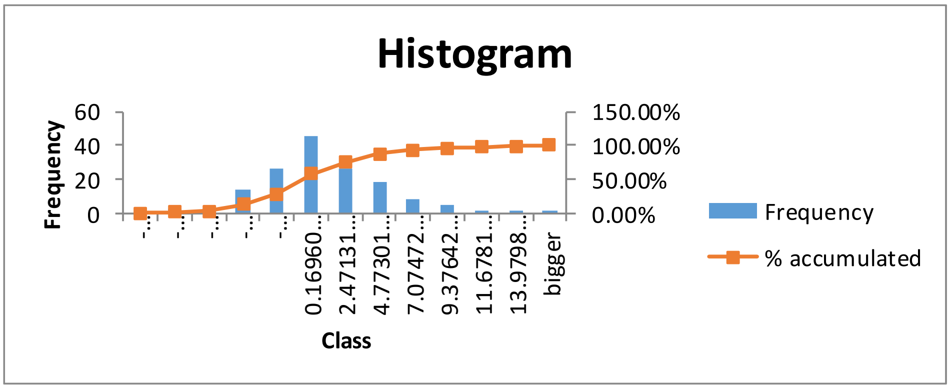


Fig. 14. Histogram of errors. Kriging method

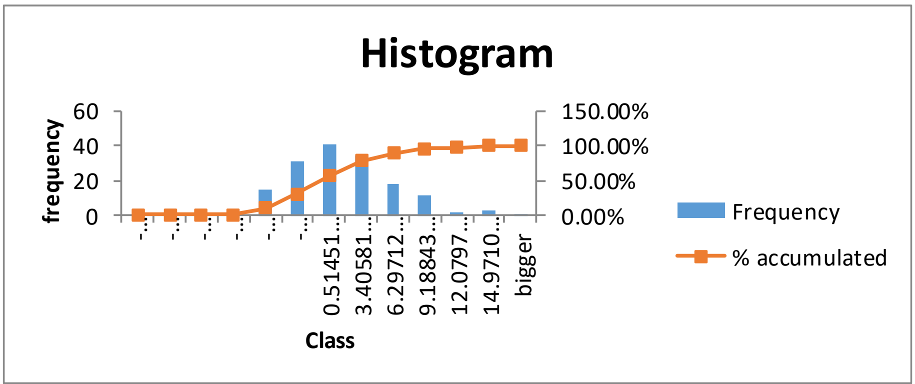


Fig. 15. Histogram of errors. IDW method

Regarding error statistics and their histograms, from figures 14 and 15 is observed that the errors have a grouping similar to those of a normal distribution, in addition from Table 3 the mean of the errors in the case of the Kriging method is close to zero and the standard deviation of the Kriging method error is less than the IDW method error.

To deepen into what happens with isohyets in case of design storms, we selected an isohyets map prepared "hand made" from the [8], corresponding to design isohyets for a Tr=10 years for maximum 24-hour rainfall in mm, which has closed isohyets and data similar to those of historical rainfall, for Mexico City case (Figure 16) and was superimposed with a transparency with respect to what is reported by IDW method and Gaussian Ordinary Kriging method in Figures 17 and 18.

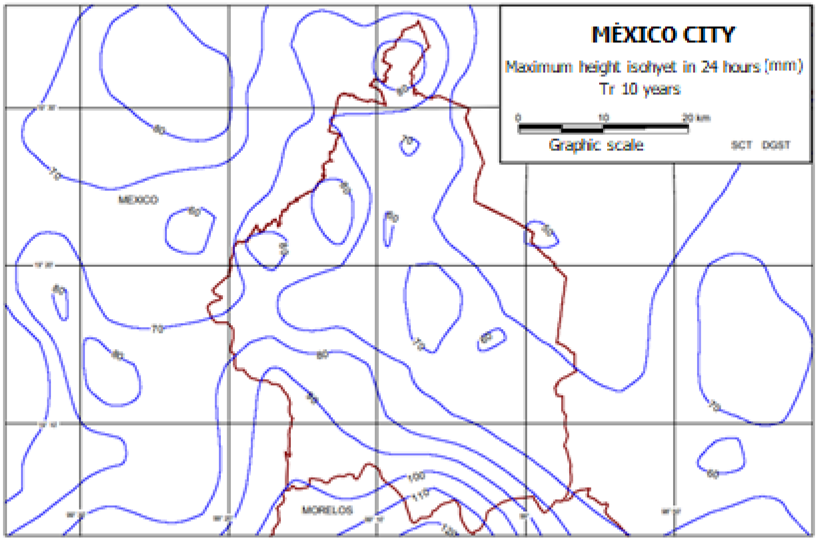


Fig. 16. Maximum annual mean rainfall isohyets map for a Tr=10 years for Mexico City. (Source:[8])

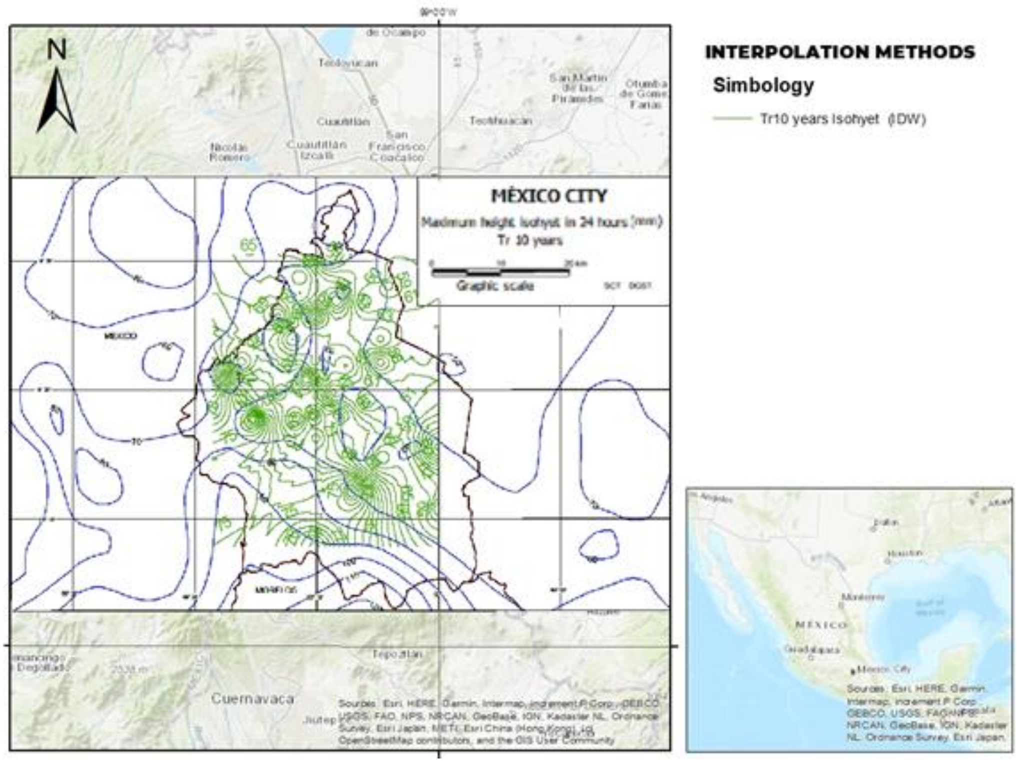


Fig. 17. Maximum mean annual rainfall isohyets map for Tr=10 years. Mexico City. IDW method. (Source: Own design)

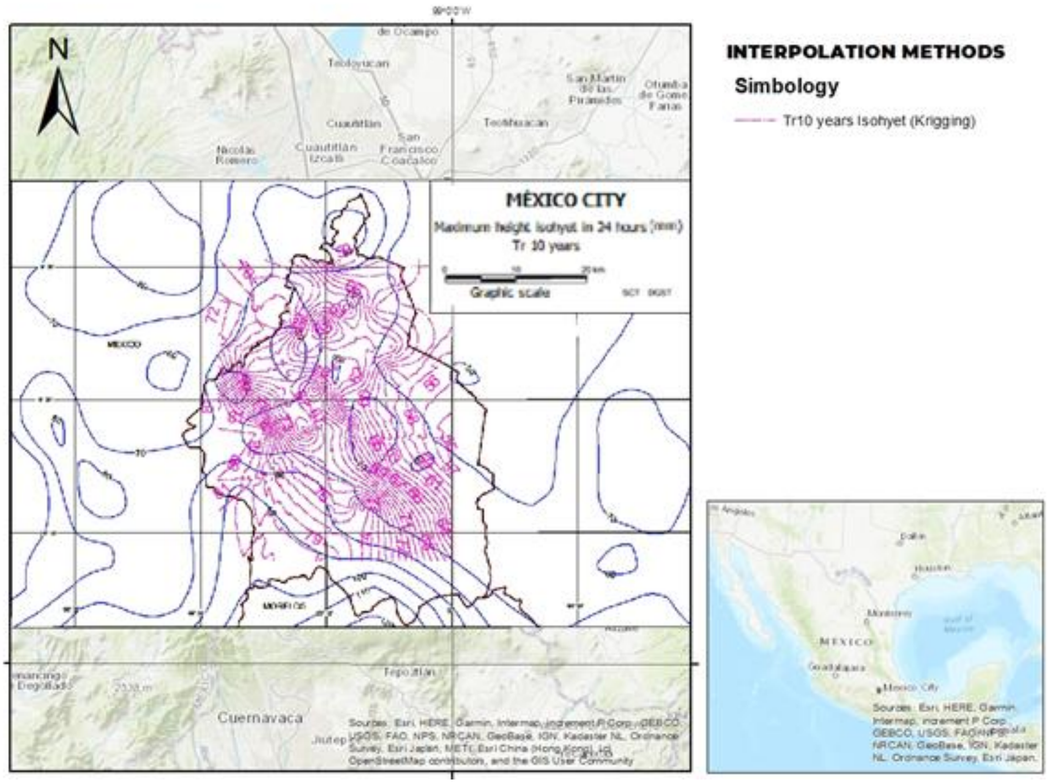


Fig. 18. Maximum mean annual rainfall isohyets map for Tr=10 years. Mexico City. Gaussian Kriging method. (Source: Own design)

Figures 17 and 18 show that Gaussian Kriging method has greater difficulty in obtaining closed contour lines; while IDW method does manage to represent closed precipitation isohyets of more than 60 mm; on the other hand, the traces of the SCT curves, particularly those that fail to close, are parallel to those of Kriging method in some of the 80 and 90 mm isohyets of the SCT.
In order to see effect of adding elevation variable in design isohyets estimation, Cokriging method was used and a new comparison was made with respect to the SCT isohyets for return period Tr=10 years (Figure 19).

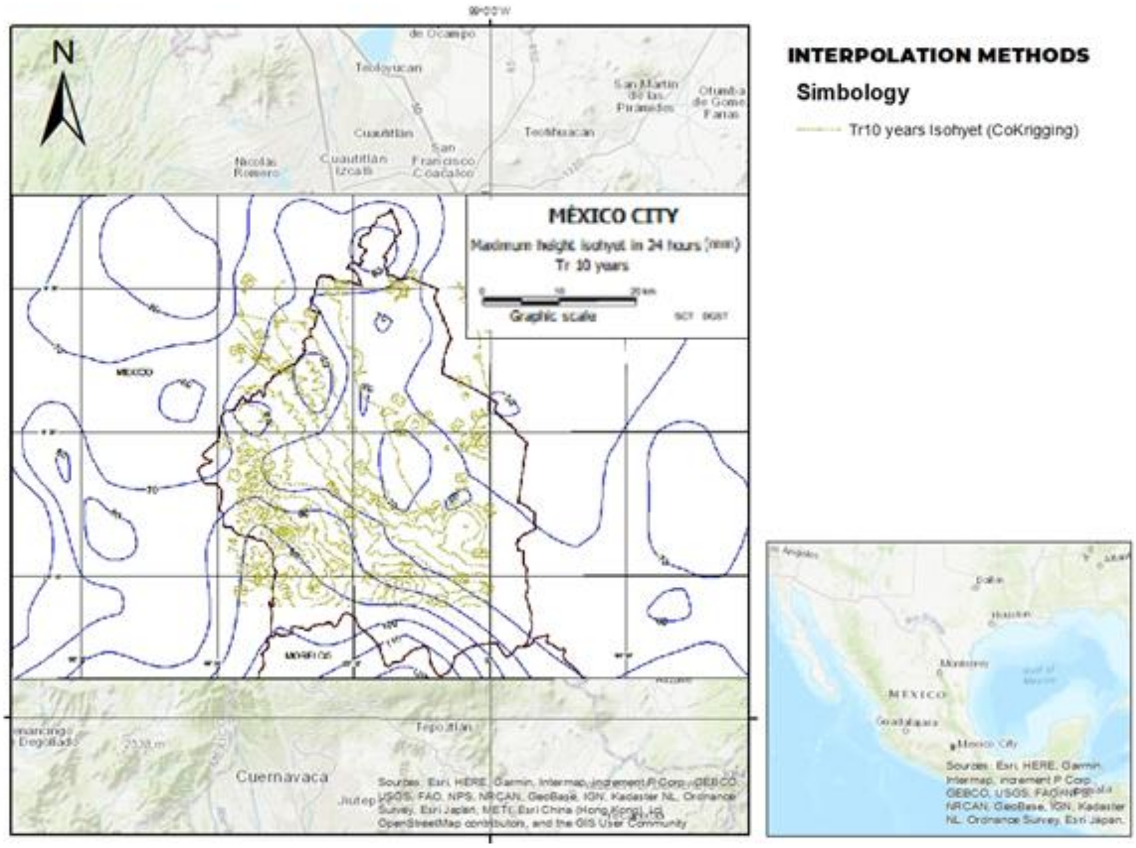


Fig. 19. Maximum mean annual rainfall isohyets map for Tr=10 years. Mexico City. Cokriging method. (Source: Own design)

Figure 19 shows that isohyets density decreases when using Cokriging method, with respect to Ordinary Kriging and IDW, closed isohyets are obtained and those that are open are more parallel to the SCT isohyets.

4. Conclusions

Applying different interpolation methods as a tool for drawing isohyets in a region with high elevations and their comparison with the traditional isohyet method in the case of historical and design events leads to the conclusion that the IDW method achieves a plotting of isohyets with regular geometric shapes and closures in them, but it has the disadvantage of only taking into consideration the precipitation variable and not the geography of the site and it provided a higher mean square error compared to the Kriging method.
Cokriging method, which takes into account both the precipitation variable and elevation as a secondary variable, gave isohyets that were parallel to those of traditional method provided by the official SCT, but the density of these isohyets was low in comparison to the results provided by the Kriging method. Additionally, the Kriging method obtained a higher correlation coefficient between the historical and calculated data, in addition to the fact that the mean square error was also lower with respect to the IDW method. The standard deviation of the error series was also lower when using the Kriging method; it is a simpler model than Cokriging and reports acceptable results that makes it a valid method to use for the estimation of for both historical and design isohyets in Mexico City, which is a site located at high elevations and characterized by convective rains, which in some cases are of high intensity, but short duration.

Acknowledgments

Authors would like to thank the student Juan Emmanuel Baños for reviewing texts of this document.

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