Determining the Relation between the Size of the Air Bubble Immersed in Water and the Dissolved Oxygen Concentration

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Abstract: *Starting from the equation of oxygen transfer rate to water, a mathematical connection is established between the concentration of dissolved oxygen in the water and the diameter of the air bubble introduced in water.*
For diameters of the oxygen introduction orifice in water between 0.05 ÷ 0.1 mm, the diameter of the air bubble entering the water is calculated and the variation of the dissolved oxygen concentration in the water as a function of time is exposed.

Keywords: *Water aeration, dissolved oxygen, fine bubbles, oxygen transfer.*

1. Introduction

Aeration is necessary to improve water quality, to avoid the occurrence of oxygen deficiency in systems where there is a biochemical consumption of oxygen above the self-aeration capacity of water, to eliminate toxic gases that can be found in water and in the water treatment process [1].
The main purpose of water aeration, regardless of the industry and the reason for its use, is to increase or maintain an optimal level of dissolved oxygen in a body of water.
The oxygen needed for the aeration process is taken from the atmospheric air and introduced into the water. For this aeration to be effective, a uniform dispersion of air must be ensured throughout the body of water in a tank or basin; the air must be evenly distributed to provide the required oxygen. Dissolved oxygen content is the most important indicator of water quality. Fish, for example, need up to 5mg / dm^3 of dissolved oxygen to survive. The amount of oxygen in the water is consumed by various biological or chemical processes. The amount of water left in these processes depends on the rate of deoxygenation and the rate of oxygenation (aeration), which can occur naturally or artificially. By aeration of water is meant the transfer of oxygen from atmospheric air to water, which is, in fact, a phenomenon of the transfer of a gas into a liquid. The most common method of removing organic impurities under the action of an aerobic bacterial biomass is to introduce gaseous oxygen into the wastewater. By introducing air into the water there is an oxygenation of the water, a process called aeration.
The term oxygenation is mainly used when introduced into water [1]:
- A mixture of air + pure oxygen from a cylinder.
- Pure oxygen from cylinders.
- Air with low nitrogen content (95% O_2 and 5% N_2) delivered by oxygen concentrators.
- Air + ozone given by ozone generators.

The notion of dissolved oxygen in water becomes clear from the analysis of Figure 1.

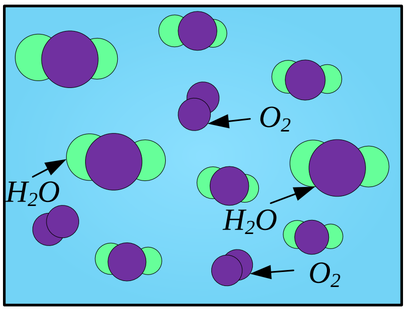

Fig. 1. View of molecular structure: dissolved oxygen

Figure 1. shows that each molecule of water consists of one molecule of oxygen connected to two molecules of hydrogen (the green sphere coupled to two purple spheres).
Dissolved oxygen molecules (purple spheres) can be found among water molecules. The maximum amount of oxygen that can be dissolved in water depends on several physical and chemical parameters, such as: atmospheric pressure, water temperature, water salinity, the degree of water turbulence [2] [3].
Water temperature is an important factor, so the warmer the water, the lower the dissolved oxygen concentration. Therefore:
- at t = 10ºC, in clean, fresh water, an amount of 11.3 mgO_2 / dm^3 can be absorbed;
- at t = 25ºC, in clean water, only 8.3 mgO_2 / dm^3 can be absorbed.

2. The equation of the oxygen transfer rate to water

The rate of transfer of dissolved oxygen in water is given by the relation [3] [4]:

$$\frac{dC}{d\tau} = a \cdot k_L \cdot \left(C_s - C_0\right) \ [kg / m^3 s] \quad (1)$$

where:
dC / dτ - rate of change of dissolved oxygen concentration in water (rate of oxygen transfer to water);
ak_L - volumetric mass transfer coefficient [s^{-1}];
C_s - mass concentration of oxygen at saturation in the liquid phase [kg / m^3];
C - current mass oxygen concentration in the liquid phase [kg / m^3].
The term "ak_L" includes:
k_L- mass transfer coefficient [m / s];
a - specific interphase contact surface:

$$a = \frac{A}{V}\left[\frac{m^2}{m^3}\right] \quad (2)$$

where:
A- gas bubble area [m^2];
V - the volume of the biphasic system (air + water) [m^3];
If boundary conditions C = C_0 are imposed for τ = 0, equation (1) can be integrated [5] [6] [7]:

$$\frac{dC}{C_s - C} = a \cdot k_L \, d\tau \quad (3)$$

In the hypothesis that C <C_s, after integration, it results:

$$-ln\left(C_s - C\right) = a \cdot k_L \cdot \tau + ct \quad (4)$$

The constant is obtained from the limit condition: C = C_0 for τ = 0
and has the value:

$$ct = -ln\left(C_s - C_0\right) \quad (5)$$

Introducing (5) in (3) one can obtain:

$$-ln\left(C_s - C\right) = a \cdot k_L \cdot \tau - ln\left(C_s - C_0\right) \quad (6)$$

$$ln\left(C_s - C\right) = ln\left(C_s - C_0\right) - a \cdot k_L \cdot \tau \quad (7)$$

$$ln\left(C_s - C\right) = ln\left(C_s - C_0\right) + ln\, e^{-a \cdot k_L \cdot \tau} \quad (8)$$

$$ln\left(C_s - C\right) = ln(\left(C_s - C_0\right) \cdot e^{-a \cdot k_L \cdot \tau}$$

$$C_s - C = ln(\left(C_s - C_0\right) \cdot e^{-a \cdot k_L \cdot \tau}$$

$$C = C_s - \left(C_s - C_0\right) \cdot e^{-a \cdot k_L \cdot \tau} \quad (9)$$

τ=0

3. Determining the mathematical relation between the diameter of the air bubble at the orifice exit and the variation of the dissolved oxygen concentration in the water as a function of time

The equation for the rate of oxygen transfer to water is rewritten as follows:

$$C_o = C_s - \frac{C_s - C_0}{e^{a \cdot k_L \cdot \tau}} \tag{10}$$

The value of the interfacial area, considering the air bubble as a small sphere of radius R, is [1] [8]:

$$a = \frac{A}{V} = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{4\pi (d_b/2)^2}{\frac{4}{3}\pi (d_b/2)^3} = \frac{6}{d_b} \tag{11}$$

where d_b = the bubble diameter immediately at the orifice exit.
The equation of the rate of oxygen transfer to water is rewritten as follows:

$$C_o = \frac{C_s - \left(C_s - C_0\right)}{e^{a \cdot k_L \cdot \tau}} = C_s - \frac{C_s - C_0}{e^{a \cdot k_L \cdot \tau}} \tag{12}$$

Given that C_s, C_0, k_L, are constant, the relation (10) can be written as:

$$C_{o_2} = C_{t,1} - \frac{C_{t,2}}{e^{C_{t,3} \cdot \frac{\tau}{d_b}}} \tag{13}$$

where $C_{t,1}$, $C_{t,2}$, $C_{t,3}$ is constant.
From this relation one can see the following:
- When d_0 decreases from 0.1 mm to 0.05 mm, the value of d_b will decrease and τ will increase from 1 to 120 minutes.
- As a result, the size $e^{C_{t,3} \cdot \frac{\tau}{d_b}}$ will increase during aeration (τ = 1 ,,,,,, 120 min).
- As a result, the fraction in relation (13) will decrease during aeration, so C_{o_2} will increase during aeration if d_b decreases [9].

Thus, it is mathematically demonstrated that when the water inlet decreases (d_0 = 0.1 → 0.05 mm) the value of the dissolved oxygen concentration in the water will increase [10] [11].

4. Determining the mathematical relation between the orifice diameter (d_0) in the perforated plate and the air bubble diameter (d_b) at the water entrance

The air bubble radius at the exit of the orifice (R_b) is given by the relation [8] [9]:

$$R_0 = \left(\frac{3}{2} \cdot \frac{r_0 \cdot \sigma}{\rho_l \cdot g}\right)^{\frac{1}{3}} \tag{14}$$

where: σ = surface tension water coefficient; $\sigma = 73 \cdot 10^{-3}$ N / m
ρ_l = water density: $\rho = 10^3$ kg / m^3
g = gravitational acceleration; g = 9.81 m / s^2
r_0 = orifice radius [m].

$$d_b = 2 \cdot R_0 = 2 \cdot \left(\frac{3}{2} \cdot \frac{r_0 \cdot \sigma}{\rho_l \cdot g}\right)^{\frac{1}{3}} \tag{15}$$

Replacing d_0, the data in table 1 results in:

Table 1

Version no.	1	2	3	4	5	6
$d_o \cdot 10^{-3}$ [m]	0.05	0.06	0.07	0.08	0.09	0.10
$d_b \cdot 10^{-3}$ [m]	1.324	1.407	1.461	1.528	1.589	1.646

Table 1 shows that as the orifice diameter (d_0) in the perforated plate increases by 0.05 → 0.1 mm, the diameter of the air bubble when detached from the plate increases from 1.324 mm to 1.646 mm.

5. The results of theoretical and experimental researches

a) For version I, in which the orifice diameter in the perforated plate is 0.1 mm, the initial data used for the theoretical calculation and for performing the experimental measurements are presented:
V_{H2O} = 0.125 m^3; d_b = 0.1 mm
H = 500 mm H_2O
C_0 = 5.84 mg / dm^3
t_{H2O} = 24 ° C → C_s = 8.4 mg / dm^3
p_{air} = 573 mm H_2O; t_{air} = 24.1 °C
τ = 120 min
b) For version II, the same data are maintained, only d_b = 0.05 mm is modified.
To determine the change over time in the dissolved oxygen concentration in water, a computation program was prepared, the logical diagram of which is shown in Figure 2.

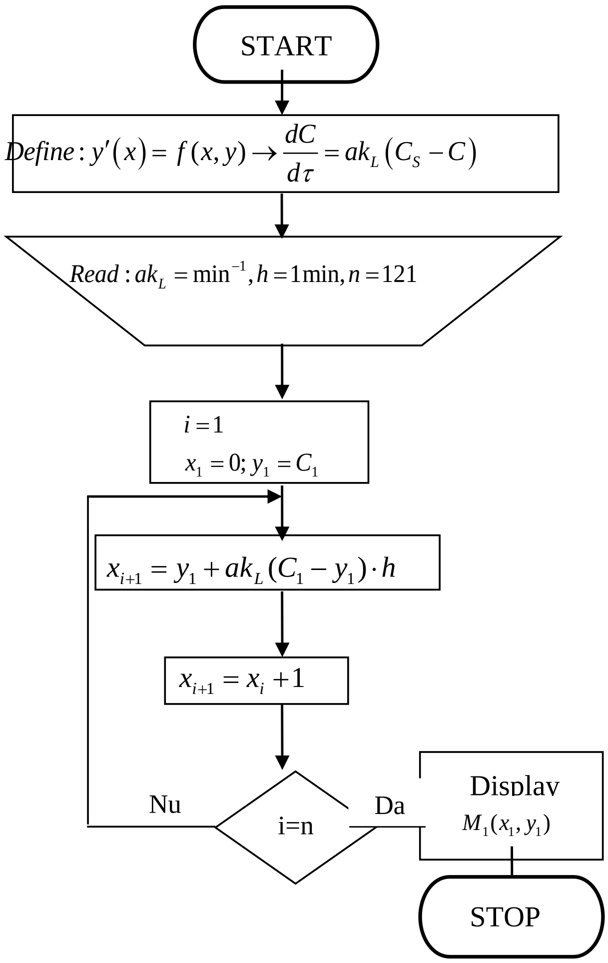

Fig. 2. Logical computation scheme for the numerical integration of the differential oxygen transfer rate equation

Following the computation program, curves 1 from figures 5 and 6 were constructed.
The scheme of the experimental installation and the operation of a fine bubble generator are shown in Figures 3 and 4.

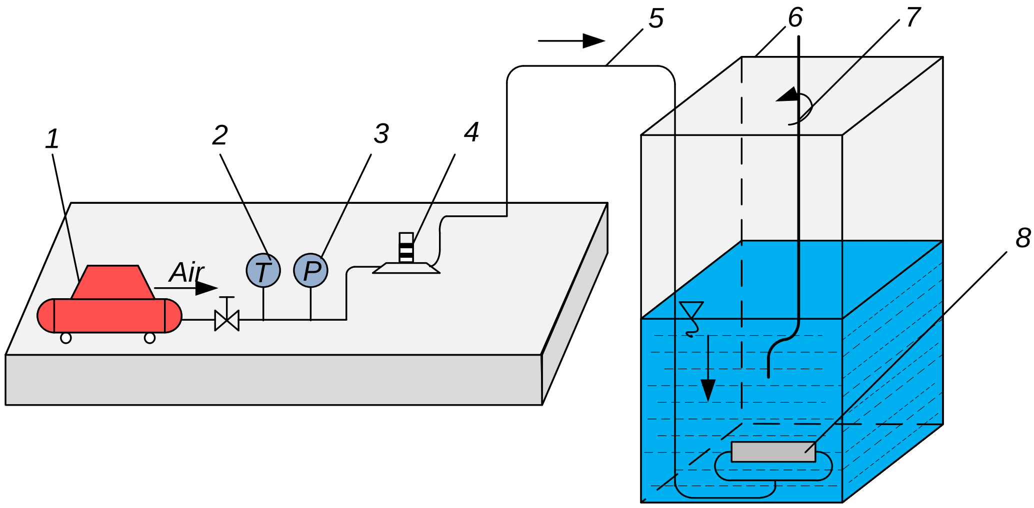

Fig. 3. Scheme of the installation for the introduction of atmospheric air into water:
1 - electrocompressor; 2 - digital thermometer; 3 - digital manometer; 4 - rotameter; 5 - air transport pipe; 6 - water tank; 7 - oxygenometer probe; 8 - fine bubble generator

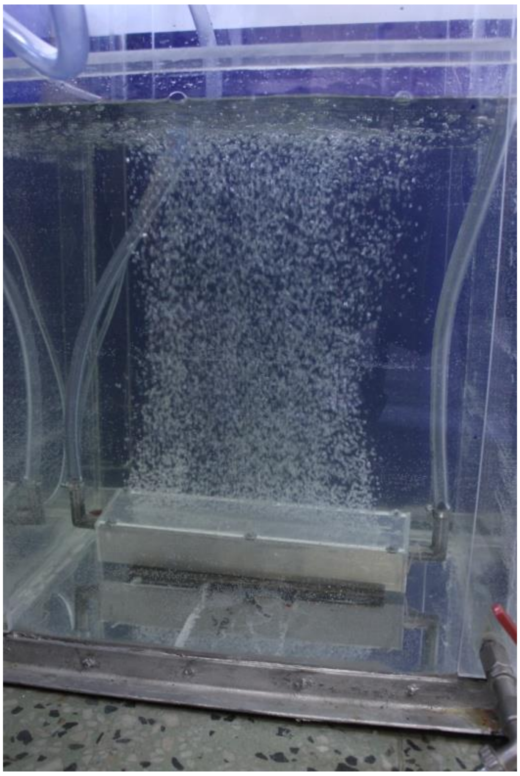

Fig. 4. Fine bubble generator in function

6. Researches methodology

Performing the measurements involves the following steps:
1. Check that the 132 orifices function, i.e., the atmospheric air is introduced into the bubble generator;
2. Fill the tank with water up to H = 500 mm H_2O;
3. Measure C_0, tH2O, t_{air};
4. Insert the fine bubble generator into the water tank and note the time (τ);
5. Every 15 minutes, take the fine bubble generator out of the tank and measure the dissolved oxygen concentration; subsequently, reinsert the fine bubbles generator in the water tank.

6. When a horizontal level of the function C = f (τ) is reached, the measurements stop, with the condition: C ≈ Cs;
7. From previous researches [14] [15][16], the concentration of dissolved oxygen in water tends to saturate after two hours. Therefore, the measurement of the oxygen concentration will be performed at the moments: 15, 30, 45, 60, 75, 90, 105, 120 minutes.
8. At the end of the measurements, clean the oxygenometer probe and drain the water from the tank.

Following the experimental measurements for version I, the data in Table 2 were obtained.

Table 2: Values of dissolved oxygen concentration in water as a function of time

τ [min]	0	15	30	45	60	75	90	105	120
$\dot{V}_{air}[dm^3/h]$	600	600	600	600	600	600	600	600	600
$\dot{V}_{IO_2} = 0.21 \cdot 600 = 126\,[dm^3/h]$	126	126	126	126	126	126	126	126	126
$\dot{V}_{O_2}$ from other sources	0	0	0	0	0	0	0	0	0
t_{H2O} [°C]	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7
t_{air} [°C]	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1
C_0 [mg/dm³]	5.84	5.84	5.84	5.84	5.84	5.84	5.84	5.84	5.84
C_s [mg/dm³]	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4
C [mg/dm³]	5.84	6.89	7.65	8.01	8.10	8.26	8.31	8.35	8.39

Based on the theoretical and experimental calculation results, curves 1 and 2 were drawn in Fig.5.

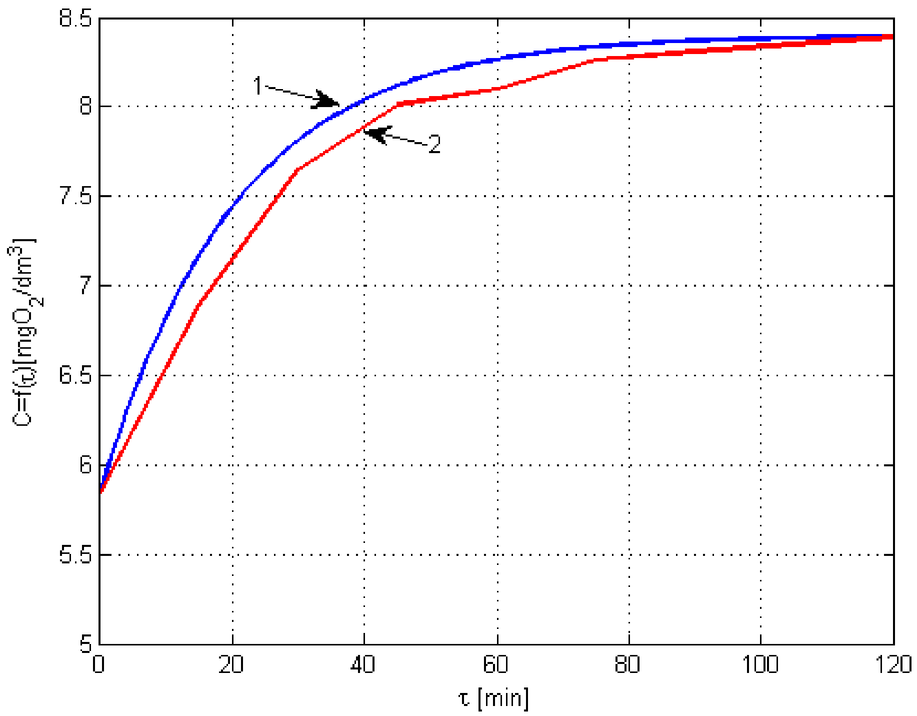

Fig. 5. Variation of the dissolved oxygen concentration in water over time for version I.
1 - curve drawn based on theoretical data (for Φ = 0.1);
2 - curve drawn based on experimental data (for Φ = 0.1)

Figure 5 shows that the differences between the two curves are very small, which demonstrates the correctness of the researches.

For version II (d_0 = 0.05 mm), following the experimental measurements, the data from table 3 were obtained:

Table 3: Values of dissolved oxygen concentration in water as a function of time

τ [min]	0	15	30	45	60	75	90	105	120
$\dot{V}_{air}[dm^3/h]$	600	600	600	600	600	600	600	600	600
$\dot{V}_{IO_2} = 0.21 \cdot 600 = 126\,[dm^3/h]$	126	126	126	126	126	126	126	126	126
$\dot{V}_{O_2}$ from other sources	0	0	0	0	0	0	0	0	0
t_{H2O} [°C]	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5
t_{air} [°C]	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5	23.5
C_0 [mg/dm³]	5.84	5.84	5.84	5.84	5.84	5.84	5.84	5.84	5.84
C_s [mg/dm³]	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2	9.2
C [mg/dm³]	5.84	7.9	8.2	8.35	8.53	8.71	8.75	8.85	9.00

Based on the theoretical and experimental calculation results, curves 1 and 2 were drawn in Fig. 6.

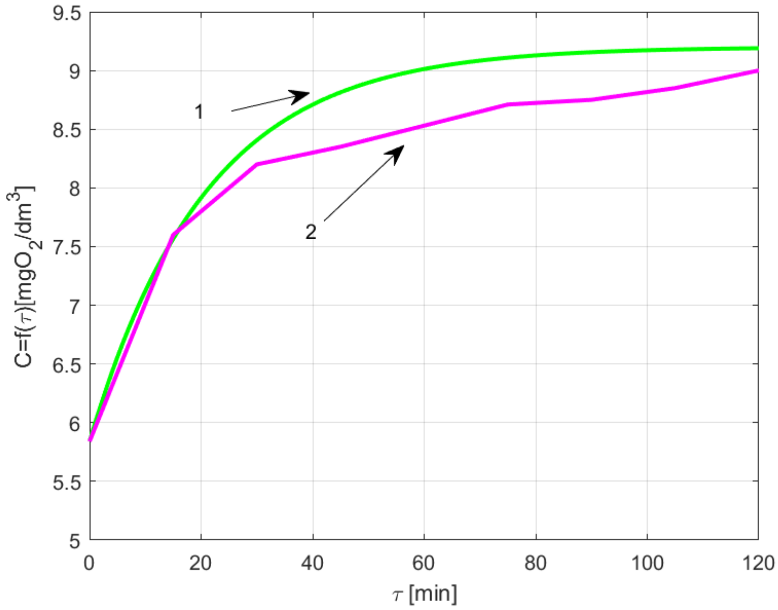

Fig. 6. Variation of dissolved oxygen concentration in water over time for version II
1 - curve drawn based on theoretical data Φ = 0.05;
2 - curve drawn based on experimental data Φ = 0.05

By comparing the experimental data obtained for the two versions, the following graph was obtained, shown in Figure 7:

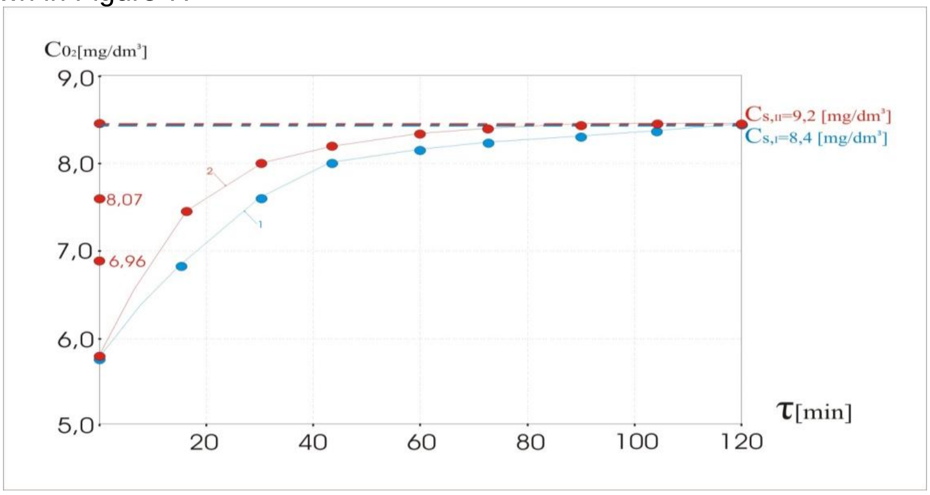

Fig. 7. The modification in time of the dissolved oxygen concentration in water C_{O2} = f (τ) for the two versions

7. Conclusions

- From the previous calculations, one can notice that as the orifice diameter in the perforated plate of the FBG increases, the diameter of the bubble immersed in water also increases. This leads to a change in the allure of the curve C = f (τ).
- The dissolved oxygen concentration in the water increases rapidly as the diameter of the air bubble that enters the water decreases.
- The realization of the fine bubble generator was made possible using micro technologies, which made it possible to make orifices with ø = 0.1 mm and ø = 0.05 mm, respectively, in the FBG perforated plate.
- The aim is to achieve a new stage, namely the use of nanotechnologies in the construction of water aeration equipment.

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