# GASDP Hybrid Model to Optimize the Minimum and Maximum Extractions of a Cascade Reservoir System

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Abstract: The determination of operating policies of a system of cascade dams for the purpose of electricity generation is a widely studied problem; the approach of the minimum and maximum extraction volumes between which the extraction policy must oscillate, taking into account the installed and physical capacity in the reservoirs, are data that are commonly set in the analysis. In the case of a cascading dam system, stochastic dynamic programming (SDP) has turned out to be an optimization algorithm that, with relatively little computational cost, solves the problem of obtaining an extraction policy for the filling state of the reservoir and for each stage of the year in which the problem is divided. The optimization process ends by the number of iterations or by meeting an established tolerance value, requiring approximately two hours to obtain an optimal policy. In this research, it was proposed to jointly use a simple genetic algorithm GA to obtain the values of the minimum (kmin) and maximum (kmax) extractions that can be used by stochastic dynamic programming to obtain an optimal operating policy, with the aim of minimizing the differences between the total benefits obtained between two successive years of application of the SDP. The one obtained with SDP was taken as the comparison policy and five tests were made with the GASDP hybrid algorithm; in four of the cases the algorithm was able to reduce the range of kmin and kmax values to initialize the SDP algorithm; in the fifth case, the kmin values were set and the kmax values were obtained. Despite achieving the objective of reducing the errors between the total accumulated benefits determined by the SPD, the policies found in tests 1 to 4 cause deficits in the system, a situation that does not happen with the policy obtained in test 5 in which there are no more spills or deficits as happens with the SDP policy and the calculation time with it is 59 minutes, the total maximum storage, sum of the two reservoirs, decreases by 0.4% with respect to the PDE policy, the average energy generated fortnightly drops 1.89 GWh. An inconvenience found in the GASPD hybridization was the calculation times, which grow from hours to days as the number of individuals and generations increases.

**Keywords:** Objective function; deficit; hybrid optimization; computational cost; Grijalva river dams; hybrid optimization; computational cost; Grijalva river dams

### 1. Introduction

The application of optimization tools to solve resource allocation problems in engineering issues has increased in importance to the extent that computer equipment continues to evolve; specifically, the issue of hydraulic exploitation and the determination of extraction policies in reservoirs and reservoir systems with different uses has been the subject of numerous investigations in recent decades [1,2,3,4,5,6].

The calculation times used by the different optimization algorithms are a criterion that helps to identify its efficiency; but sometimes it is worth the computational effort to obtain results in the simulations of the systems that are reflected in the reconciliation between the expected benefits and the occurrence of undesired events.

Stochastic dynamic programming is an algorithm that is applied with relative simplicity to reservoir operation problems to obtain optimal policies, exponentially decreasing operations as it is a sequential process, although it can be affected by the so-called curse of dimensionality in the case of a growth in the number of states, stages and decision or search variables that the problem has. [7,8,9,10,11].

Recently, research aims to hybridize optimization methods, seeking to highlight the benefits of each method involved [12,13,14].

In this study, it was proposed to combine a simple genetic algorithm (GA) and the stochastic dynamic programming (SDP) algorithm, calling it the SGASDP algorithm, to determine the values of maximum and minimum extractions, and obtain the optimal policies of an equivalent system of two dams. that work in cascade; the methodology was applied in the Grijalva river dam system. The article is organized in the following parts, the introduction indicated above, the methodology

used, the results and discussion, as well as the conclusions derived from the analysis.

### 2. Methodology

### GA

Simple genetic algorithms [15,16], were among the first bioinspired random algorithms, stand out for being robust and having convergence towards global optima. The algorithm begins with the random generation of an initial population of n individuals (chromosomes) containing the search variables, the performance of the best individual is tested through the evaluation of the objective function consisting of the maximization or minimization of a function preset, depending on the problem analysed; relative and population fitness are identified to use a method for selection of the best individuals; later some of said best individuals are selected for the cross with the roulette, universal stochastic or tournament method; the random cross can be at a single point or multipoint; the new individuals generated with the cross can mutate in one or more points; the population resulting from this last operator passes to the next generation and the process is repeated until reaching the number of generations given in the problem. The best individual in the last generation represents the optimal solution sought.

### SDP

Dynamic programming [8] solves the problem of obtaining the decision variables and trajectories that manage to optimize a process by considering a finite number of states, stages and sequential decisions that are applied in a large number of iterations or horizon. planning; when the decision variables are random, the transition probabilities of passing from an initial state to a final state intervene; the objective function that is solved with SDP may or may not be linear and considers the expected value of a benefit, in the case of the analysis of a system of hydroelectric dams that work in cascade, in addition to the fact that it is desired to maximize the expected benefit for electricity generation , terms that reduce said benefit can be included in the same objective function by imposing penalties in the event of undesired events of spills in the system or in the event of deficits (that is, the promise is not fulfilled). The SDP algorithm can be divided into two parts to avoid repetitive calculations and reduce process times [17,18]. In the case of a system of two dams that work in cascade, the algorithm can be separated into the following parts:

$$\phi_{n,k1,k2}(i_1,i_2) = \sum_{j_1=1}^{NS_1} q_{n,k1}(i_1,j_1) b_{n,k1}(i_1,j_1) + \sum_{j_2=1}^{NS_2} q_{n,k2}(i_2,j_2) b_{n,k1,k2}(i_1,j_1,i_2,j_2)$$
(1)

$$B_{n,k1,k2}(i_1,i_2) = \emptyset_{n,k1,k2} + \sum_{j_1=1}^{NS_J} \sum_{j_2=1}^{NS_J} q_{n,k1}(i_1,j_1) q_{n,k2}(i_2,j_2) B_{n+1(j_1,j_2)}^*$$
(2)

Where:

 $\phi_{n,k1,k2}(i_1,i_2)$  is the expected value of the immediate total benefit in stage n, given the initial conditions i1, i2 and the extractions k<sub>1</sub>, k<sub>2</sub>, of dams 1 and 2. These values are first calculated for all stages

 $q_{n,k1}(i_1, j_1)$  transition probabilities from state i to state j of dam 1, at stage n and given extraction k1.

 $b_{n,k1}(i_1, j_1)$  benefit at stage n, given a draw k of moving from an initial state i to a final state j1 of dam 1.

 $q_{n,k2}(i_2,j_2)$  transition probabilities from state i to state j of dam 2, at stage n and given extraction k2.

 $b_{n,k_1,k_2}(i_1, j_1, i_2, j_2)$  benefit in stage n, given extraction  $k_1$  and  $k_2$ , to go from initial state i1 to final state  $j_1$  in dam 1 and from state initial  $i_2$  to the final state  $j_2$  of dam 2.

 $B_{n,k_1,k_2}(i_1, i_1)$  total benefit in stage n, given the extraction  $k_1$  and  $k_2$  in dams 1 and 2, in the final state  $j_1$  and  $j_2$  of dams 1 and 2, respectively. These values are calculated in a second part of the algorithm.

 $B_{n+1(j_1,j_2)}^*$  optimal benefit at stage n+1, corresponding to the final state  $j_1$  and  $j_2$  of dams 1 and 2, respectively corresponding to the optimal extractions  $k_1^*$  and  $k_2^*$  in the dams 1 and 2.

The algorithm delivers in matrix form, for each stage n of the year, an extraction policy consisting of a matrix arrangement in whose rows the states of dam 1 are indicated and in the columns the states of dam 2 and in the intersection are established by unit of volume in the stage the extractions that will be made in the reservoir in the stage, depending on the filling levels of each reservoir at the beginning of the stage, Table 1 illustrates an example of said operating policy matrix.

S T A G E 2: OCT ΔV <sub>Stage2</sub> =100 hm <sup>3</sup>																
MALPASO STATES																
	ID	1	2	3	4	5	6	7	8	9	10	11	12	13		46
	1	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	2	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	3	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	4	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	5	304	304	304	304	304	304	304	304	304	304	304	304	304		318
S	6	304	304	304	304	304	304	304	304	304	304	304	304	304		318
ATE	7	304	304	304	304	304	304	304	304	304	304	304	304	304		318
A ST	8	304	304	304	304	304	304	304	304	304	304	304	304	304		318
TUR	9	304	304	304	304	304	304	304	304	304	304	304	304	304		318
GOS	10	304	304	304	304	304	304	304	304	304	304	304	304	304		318
A AN	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Ľ	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	22	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	23	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	24	304	304	304	304	304	304	304	304	304	304	304	304	304		318
	25	304	304	304	304	304	304	304	304	304	304	304	304	304		318

 Table 1: Example of extraction policy table. La Angostura and Malpaso Dams

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S T A G E 2: OCT ΔV <sub>Stage2</sub> =100 hm <sup>3</sup>														
MALPASO STATES														
ID	1	2	3	4	5	6	7	8	9	10	11	12	13	 46
26	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
27	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
28	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
29	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
30	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
31	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
32	304	304	304	304	304	304	304	304	304	304	304	304	304	 318
•	•	•	•	•	•	•	•	•	•	•	•	•	•	 •
-	-	•	•	•	•	•	•	•		•	•	•	•	 •
•	•	-	-	-	-	•	-	-		•	•	•	-	 -
65	120 4	120 4	120 4	120 5	120 6	120 7	120 8	120 9	121 0	121 3	121 3	121 3	121 3	 121 8

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# 2.1 GASDP algorithm

A hybrid model is proposed that takes advantage of the random nature of the simple genetic algorithm to obtain the maximum and minimum extractions in each stage of analysis using a stochastic dynamic programming algorithm applied in a system of two cascade dams, using minimization as objective function. of the differences of the total accumulated benefits found by the stochastic dynamic programming divided into two parts, the calculation of the immediate benefit in the stage and the calculation of the total benefit; the steps of the SGASDP are expressed below and in the flowchart given in Figure 1.

Start

Give the input data: Number of individuals: minimum and maximum extraction proposals (kmin and kmax).

Give the number of generations.

Randomly generates the initial population from the search intervals (based on SDP test 7). Start the iterations.

Evaluates the performance of individuals with the objective function.

OF= Minimize the difference of the sum of the total benefits between one year and another obtained with the SDP (executes the Cafitb and opdin optimization programs).

Select the best individuals with the roulette method.

Crossing processes are carried out, mutate individuals.

A new population is generated and passed on to the next generation.

The process is repeated until the number of generations is reached.

It saves the individual with the best performance in the last generation that contains the values of the minimum and maximum extractions of the optimal policy, the optimal policy is saved in files for each time interval that is being worked on (month, fortnight) and the policy in matrix form for each defined stage.

The optimal policy is simulated with the joint vessel operation simulation program. Ends the process.



Fig. 1. SGASDP method flowchart

## 2.1 Study site and data considered

In this investigation, the system of hydroelectric dams of the Grijalva River, Chiapas, was considered, which are arranged in a cascade (Figure 2), from upstream to downstream, said dams are: La Angostura, Chicoasén, Malpaso and Peñitas. Together they generate approximately 40% of hydroelectricity in Mexico, although in 2021 a generation of about 21% of the total annual hydroelectric energy generated was reported [19]. Due to the low regulation capacity of the Chicoasén and Peñitas dams, in order to obtain the optimal extraction policies, the system is analyzed in a simplified way as if there were only two dams: La Angostura and Malpaso, considering the contributions of potential energy that from Chicoasén to La Angostura and Peñitas to Malpaso; The total income from the La Angostura dam to Malpaso is also considered to take Chicoasén into account.



Fig. 2. Grijalva River Dams, Chis., Mexico. Source: Own design

For the purposes of using the SDP for each dam,  $\Delta V$  units of their useful capacity were considered as states of the problem, divided between the Minimum Water Level (mWL) that corresponds to state 1 and the Maximum Ordinary Water Level (MOWL) that corresponds to state NSi, with i=1, 2 (counter number of dams),  $\Delta V=200 \text{ hm}^3$  was assumed, which resulted in NS1=65 states for La Angostura and NS2=46 states for Malpaso. Groups of fortnights of the year were considered as stages of the problem according to the average values of fortnightly income, with which a total of 7 stages were defined: stage 1: the 2 fortnights of the months of November and December, stages 2, 3, 4 and 5: the 2 fortnights of the months of October, September, August and July, respectively, stage 6: the 2nd fortnight of May and the two fortnights of June, and stage 7: the 2 fortnights of the months of January, February, March and April, and the first half of May. For the use of the GA together with the SDP, the search intervals of the values of the minimum extractions (kmin) and maximum extractions (kmax) were proposed, taking into account the drinking water requirements in the populations downstream of the dam and the capacities installed in them for purposes of maximum available monthly generation. Different proposals for years of analysis of the SDP and also different number of generations used by the GA were defined, the selection method used was the roulette and with crossover probabilities of 0.7 and mutation of 0.7/length of the individual defined by the size of the binary string [20].

## 3. Results

This section presents the results obtained by applying the GASDP algorithm considering five tests called GA1SDP, GA3SDP, GA4SDP to GA5SDP that consider the conditions for optimization highlighted in columns 4 and 6 of Table 2, in addition to considering the kmin variable. In the GASDP search interval and the GA5SDP test in which kmin is fixed in the GASDP search interval, these policies were simulated, and the results compared against the values of test 7 using only SDP and was taken as a point of comparison because it is a policy that does not report spills or deficits when simulating with the historical record.

Table 2 summarizes the value reported by the objective function (OF) with the different GASDP policies tested, in addition to indicating the sum of differences in total benefits (Difan) and the number of years of iterations carried out by the SDP (Max iter) within the GASDP with said best policy.

**Table 2:** Comparison of results. Total benefitis differences (Difan), iterations (iter), tolerance (Tol). SDP and GASDP

Operation rule	OF GASDP	Difan	Max iter	Final iter	Tol <u>&lt;</u>	
Test 7 SDP		22.05756	100	100	1.00E-05	
GA1SDP	26.0702	26.07019	50	50	1.00E-01	
GA3SDP	3.0897	3.089722	50	36	5	
GA4SDP	0.0000	0.00E+00	100	82	1.00E-03	
GA5SDP	0.0000	0.00E+00	100	100	1.00E-03	
GA6SDP	20.6640	20.664	100	100	1.00E-03	

From Table 2 it is observed that the GA4SDP and GA5SDP tests achieve values equal to zero in the objective function, and in particular test 4 decreases the number of years simulated with SDP within the GASDP algorithm, the GAS6DP test has a slightly lower value. than the GA1SDP test and with double the number of iterations, but the tolerance used was smaller compared to the GA1SDP and GA3SDP tests.

Table 3 reports the calculation times that were taken in each test with the GASDP, based on the number of individuals proposed and the number of generations with which the GASDP was fed.

(h)	Number of individuals	Max number of generations	Calculation time, h, min	Calculation time, h	
GA1SDP 4		5	10 hours ,16 min	10.3	
GA3SDP	4	5	8 hours ,38 min	8.6	
GA4SDP	8	10	40 hours, 31 min	40.5	
GA5SDP	10	20	145 hours, 29 min	145.48	
GA6SDP	10	20	102 hours, 23 min	102.33	

Table 3: Calculation times according to number of individuals and generations. GASDP

Table 3 shows that doubling the number of individuals and generations almost quadruples the calculation time (GA1SDP test vs. GA4SDP test), and for 2.5 times the number of individuals and 4 times more the number of generations and doubling the maximum number of years in the SDP the calculation time is 14 times higher (GA5SDP vs. GA1SDP test). The foregoing is to be expected since with each pair of kmin and kmax values for each stage in which the analysis is considered, the GASDP performs the dynamic programming process to evaluate the performance of the individuals. When the value of kmin is fixed and with the same number of individuals and generations as the GA5SDP, it is observed that the calculation time grows approximately 10 times. Table 4 contains a summary of the simulation of the operation of the joint basin of the system of two cascade dams, highlighting the information on the total spills and deficits in the period of simulated years, for La Angostura. Malpaso and the sum of both; The values of the minimum and maximum initial storage of each dam, of the sum of both and the average fortnightly energy obtained in the simulation are also highlighted. In this case, the GA1SDP to GA5SDP tests were simulated both with the fortnightly volume data of the kmin and kmax extractions from base test 7, as well as with the kmin and kmax values obtained by the GASDP, the GA6SDP test was simulated only with the kmin and kmax values of the GASDP.

La Angostura									
Alte	rnative	Spill Deficit		Initial stor	rage (hm³)	Average energy/fortnight			
			hm <sup>3</sup>	Minimum	Maximum	GWh			
SDP	Test 7	0	0	1824.27	10308.70	279.97			
GASDP	GA1SDP	0	10577.9	0	9386.97	279.47			
	GA1SDP *	0	47033.21	0	8270.85	277.69			
	GA3SDP	0	123454.75	0	9386.97	275.58			
	GA3SDP*	0	185336.90	0	6766.22	275.19			
	GA4SDP	0	39440.46	0	9032.84	276.74			
	GA4SDP *	0	91514.92	0	7796.62	275.85			
	GA5SDP	0	59606.80	0	8513.75	276.91			
	GA5SDP *	0	113846.86	0	7204.58	276.14			
	AG6SDP*	0	0	802.56	10067.97	279.78			
				Malpaso					
Alte	rnative	Spill Deficit		Initial stor	rage (hm <sup>3</sup> )	Average energy/fortnight			
			hm <sup>3</sup>	Minimum	Maximum	GWh			
SDP	Test 7	0	0	1533.58	7824.95	197.59			
GASDP	GA1SDP	0	1021.51	0	8081.82	194.68			
	GA1SDP *	0	9842.21	0	8203.34	194.33			
	GA3SDP	0	2075.82	0	8581.56	196.97			
	GA3SDP*	0	4686.19	0	8494.11	197.08			
	GA4SDP	0	8907.42	0	7702	191.94			
	GA4SDP *	0	27745.64	0	7309.89	192.09			
	GA5SDP	0	12622.15	0	7587.82	193.52			
	GA5SDP *	0	34198.67	0.00	7564.22	192.94			
	GA6SDP*	0	0	1422.35	7992.24	195.89			
			-	Totals					
Alte	rnative	Spill	Deficit	Total initial s	storage (hm <sup>3</sup> )	Average total energy/fortnight			
		hm <sup>3</sup>		Minimum Maximum		GWh			
SDP	Test 7	0.00	0.00	3357.85	18133.65	477.56			
GASDP	GA1SDP	0.00	11599.41	0.00	17468.79	474.15			
	GA1SDP *	0.00	56875.42	0.00	16474.19	472.02			
	GA3SDP	0.00	125530.57	0.00	17968.53	472.55			
	GA3SDP*	0.00	190023.09	0.00	15260.33	472.27			
	GA4SDP	0.00	48347.88	0.00	16734.84	468.68			
	GA4SDP *	0.00	119260.56	0.00	15106.51	467.94			
	GA5SDP	0.00	72228.95	0.00	16101.57	470.43			
	GA5SDP *	0.00	148045.53	0.00	14768.80	469.08			
	GA6SDP*	0	0	2224.91	18060.21	475.67			
Notes:									
* It was ob	tained from the	e maxim	um of the total	initial storage f	ortnightly sum	of both dams			
<ul> <li>*** The averages La Angostura + Malpaso were added</li> </ul>									

Table 4: Summary of results of the joint reservoir operation simulation

The policies highlighted in Table 4 were obtained using the kmin and kmax passed to volume in the simulation program in each case and those indicated with white used in the simulation the kmin and kmax from the data file of policy 7. That is to say that the highlights are correct, and the whites have that combination of information when simulating

From Table 4 it stands out that the results GA1SDP to GA5SDP are policies that give extremely high spill and deficit conditions, and that condition worsens when the kmin and kmax values optimized with the GASDP are used in the simulation, so these policies are discarded. as an option to operate the analyzed dam system; On the contrary, with the GA6SDP policy, by fixing the kmin values in order to ensure the conditions of test 7 in minimum guaranteed volume, it is possible to obtain an operation policy without spills or deficits in the system's dams, it decreases a little the total minimum initial storage with respect to test 7 (by 33.74%), but also the maximum total initial storage with respect to test 7 (by 0.4%), and the total annual fortnightly energy decreases by about 2 GWh/fortnight. Therefore, the GA6SDP policy can be considered a policy that competes with that of test 7, in addition to the fact that, if the kmin and kmax data given by the GA6SDP policy are used and SDP is used to obtain said policy, the calculation decrease with respect to test 7 in about one hour.

This is that the time taken for hybridizing the optimization using GA and SDP does not seem to give an optimal policy with a slight improvement in the total maximum initial storage issue, but in return with lower minimum storages, less energy generated, but without spills and without deficit using lower maximum extraction values.

Figures 3 and 4 summarize the optimized values of kmin and kmáx, for each stage and each dam, using the GASDP algorithm, contrasting them with test 7. Figures 5 and 6 show the differences between the values of kmin and kmax of each test with respect to test 7, for each reservoir.



Fig. 3. Comparison of kmin used PDE and GASDP for both dams



Fig. 4. Comparison of kmáx used with SDP and GASDP for both dams

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Fig. 6. Differences vs test 7, kmax

### 4. Conclusions

The GA6SDP policy, calculated with the hybrid algorithm, allowed obtaining optimal operation policies with lower values for maximum extraction, without spills or deficits, although with a reduction in the average total energy generation in the system vs test 7 with SDP, but presented improvements in the value of the maximum storage registered, being lower by 0.4% than that obtained with SDP. Individually, the minimum initial storage in Malpaso decreased with the GA6SDp policy vs test 7 while the maximum increased slightly. In the case of La Angostura, the opposite happened in the maximum initial storage.

When using SDP with the kmin and kmax obtained with GA6 SDP, a reduction of almost one hour was observed in the calculation times with respect to that obtained in test 7, that is, in this sense, the advantage of using GA in hybrid form was observed. with SDP for the selection of the kmax.

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#### References

- [1] Hall, W. A., and N. Buras. "The dynamic programming approach to water resources development." *J. Geophys. Res.* 66 (1961): 517–520.
- [2] Nouh, Mamdouh. "Stochastic Approach to Reservoir Design and Management." *IFAC Proceedings Volumes* 13, no. 11 (1980): 329-332.
- [3] Simonović, S. P., and L. M. Miloradov. "Optimization in Water Resources Master Plans." *IFAC Proceedings Volumes* 18, no. 14 (1985): 221-225.
- [4] Chang, F.J., and L. Chen. "Real-Coded Genetic Algorithm for Rule-Based Flood Control Reservoir Management." *Water Resources Management* 12 (1998): 185–198.
- [5] Lin, Nay Myo, and Martine Rutten. "Optimal Operation of a Network of Multi-Purpose Reservoir: A Review." 12th International Conference on Hydroinformatics (HIC 2016) - Smart Water for the Future. Procedia Engineering 154 (2016): 1376 – 1384.

- [6] Dehghani, M., H. Riahi-Madvar, F. Hooshyaripor, A. Mosavi, S. Shamshirband, E. K. Zavadskas, and K. w. Chau. "Prediction of hydropower generation using Grey wolf optimization adaptive neuro-fuzzy inference system." *Energies* 12, no. 2 (2019): Article ID 289.
- [7] Bellman, Richard Ernest. Dynamic Programming. Princeton, New Jersey, Princeton Univ. Press, 1957.
- [8] Labadie, John W. "Optimal Operation of Multi-reservoir Systems: State of the Art Review." Water Resources Planning and Management 130, no. 2 (2004): 93-111.
- [9] Doraszelski, U., and K. L. Judd. "Avoiding the Curse of Dimensionality in Dynamic Stochastic Games." *Quantitative Economics* 3, no. 1 (March 2012): 53-93.
- [10] Zéphyr, Luckny, Pascal Lang, Bernard F. Lamond, and Pascal Côté. "Approximate Stochastic Dynamic Programming for Hydroelectric Production Planning." *European Journal of Operational Research* 262, no. 2 (October 2017): 586-601.
- [11] Saadat, M., and K. Asghari. "Feasibility Improved Stochastic Dynamic Programming for Optimization of Reservoir Operation." *Water Resources Management* 33 (2019): 3485–3498.
- [12] Chang, Fi-John, Shyh-Chi Hui, and Yen-Chang Chen. "Reservoir operation using grey fuzzy stochastic dynamic programming." *Hydrological Processes* 16 (March 2002): 2395–2408.
- [13] Li, He, Pan Liu, Shenglian Guo, Bo Ming, Lei Cheng, and Yanlai Zhou. "Hybrid Two-Stage Stochastic Methods Using Scenario-Based Forecasts for Reservoir Refill Operations." *Journal of Water Resources Planning and Management* 144, no. 12 (December 2018): 04018080 1-04018080 11.
- [14] Babamiri, Omid, Arash Azari, and Safar Marofi. "An integrated fuzzy optimization and simulation method for optimal quality-quantity operation of a reservoir-river system." Water Supply 22, no. 4 (2022): 4207–4229.
- [15] Holland, John H. Adaptation in Natural and Artificial Systems. Ann Arbor, MI, The University of Michigan Press, 1975.
- [16] Goldberg, David E. *Genetic Algorithms in Search, Optimization and Machine Learning*. Reading, MA, Addison-Wesley Publishing Company, 1989.
- [17] Arganis, M. L., and R. Domínguez. "Hydropower System Management Considering the Minimum Outflow." *American Journal of Environmental Sciences* 4, no. 3 (2008): 178-184.
- [18] Mendoza Ramírez, Rosalva, Ramón Dominguez Mora, Maritza Liliana Arganis Juárez, and Eliseo Carrizosa Elizondo. "Quantification of excess and deficit volumes in guide curves to determine operating policies in a hydroelectric system in Mexico / Cuantificación de los volúmenes de exceso y déficit en curvas guía para determinar políticas de operación en un sistema hidroeléctrico en México." Proceedings of the XXVIII Latin American Congress of Hydraulics, IAHR, UCA, Buenos Aires, Argentina, September 18-21, 2018.
- [19] Observatorio de Inteligencia del Sector Energético (OISE). Intelligence, Smartdata. 2022. "Energía Hidroeléctrica." Accessed May 12, 2022. https://www.oise.mx/hidraulica.
- [20] The MathWorks. MATLAB Reference Guide. The MathWorks, Inc., 1992.