

## Algorithm in Python for Simulating Hydraulic Network for Educational Purposes

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**Abstract:** *In order to make learning and fixing topics about water distribution networks simpler for students, by allowing quick tests, comparisons with results obtained by hand, and evaluation of the different head losses and nodal pressures generated by each specific situation of input values, an algorithm developed in Python language is presented. After a literature review on the subject of computational models in hydraulic systems and their application in the area of water distribution networks, the steps for the execution of the model are discussed and presented, with some illustrative screens. Finally, a test is carried out to assess the good functioning of the Python algorithm, by comparing its resolution of a random network with the solution by the SCALER software.*

**Keywords:** *Water distribution networks, Python programming, hydraulic models, education in engineering*

### 1. Introduction

Water supply system is defined by a set of works, equipment and services intended to supply drinking water to a community for the purposes of domestic consumption, public services and industrial consumption [1]. In general, it is composed of water source, intake, adduction, treatment, reservoir, distribution network, pumping or booster stations. A water distribution network is the unit of the supply system that brings water to the points of consumption (buildings, industries, etc.). It is formed by a set of pipes and special parts arranged conveniently in order to guarantee the good service of the consumption points.

The water supplied by the system must be in sufficient quantity and of the best quality, from a physical, chemical and bacteriological point of view. For the implementation of a water supply system, it is necessary to prepare studies and projects with a view to defining the works to be undertaken. These works must have their capacity determined not only for the current needs, but also for the future service of the community, with construction being planned in stages.

### 2. Overview of Computer Models in Hydraulic Systems

According to [2, 3], over the last few decades, a huge amount of effort has gone into building computer models for use in water resource planning and management. In all elements of water management, powerful generic software packages are becoming increasingly crucial. With recent improvements in computer technology, virtually everyone working in the water resources and environmental fields now has easy access to desktop computers with all of the hardware capabilities required to run the vast array of available models. New and different technologies are introduced every day in an overwhelming way. Access to globalized information through the internet, communication through emails, the production of increasingly faster processors, the increase in data storage capacity and the development of increasingly robust computer systems, are just a few good examples of this great evolution.

One of the most significant professional problems and possibilities confronting water and environmental practitioners and academics today is to utilize and leverage the fast rising sources of available data and computing capacity. Sensor expansion, large-scale and extensive data

acquisition, increasingly sophisticated modeling tools, digital infrastructure, the internet of things, and the rollout of 5G wireless networks will allow for the development of far more mutualistic relationships between rural residents, municipalities, urban citizens, and businesses [4]. A significant modeling application is the analysis of municipal water distribution networks.

In water distribution networks, hydraulic modeling is used to forecast hydraulic characteristics such as water velocity, flow rate, and pressure. Predictions can be made at various places throughout the network, as well as at the chosen seasons, dates, and periods. As a result, hydraulic characteristics may be predicted in space and time [5].

In the early days of water distribution computer modeling, simulations were primarily used by engineers to solve design problems, to plan fully functional water distribution systems [6]. The use of computer programs became standard practice, initially, because engineers may focus on design decisions because automated calculations save them from tiresome, iterative computations. Second, because models can account for considerably more of the complexity of real-world systems than manual calculations, they boost the engineer's confidence that the design will function after it is implemented. Finally, the simplicity and speed with which models may be employed allows engineers to investigate many more choices under a variety of scenarios, resulting in more cost-effective and resilient solutions.

There is no one strategy to utilize models that is completely right. Reference [7] describes how the application of a model for design goals varies depending on whether the model is used for master planning, preliminary design, subdivision development, or system rehabilitation. As detailed in the study, each model type has a distinct aim and set of attributes.

Models have become more sophisticated and easier to use as software technology has advanced and, because of that, operational workers have adopted computer simulations as a tool to assist them in keeping the distribution system functioning properly. As a result, models began to be utilized to address ongoing issues, examine suggested operational improvements, and plan for unexpected situations. Instead of relying on trial-and-error adjustments in the actual system, the operator may discover the reasons of system issues and propose alternatives that will work the first time by comparing model results with field operations [6].

### **3. Computer Applications in Education on Water Networks and Related Topics**

According to [8], some of the most important characteristics of educational applications include ease of use, simplicity, information quality, and a lower cost for a functioning license. Because typical commercial software is both expensive and complex, free and/or open source code may be a better option for academic purposes.

Reference [9] provides free and helpful software for water distribution networks (WDNs). The software is free and open source, and it makes use of spreadsheet functions and options as well as MS Excel VBA programming. Because of the extensive mathematical processes required, the design of WDNs is one of the most difficult undertakings, particularly for engineering students. Several commercial software packages for the WDN project are prohibitively expensive for students, who frequently require open source software. According to the author, while some open source applications, such as EPANET, are accessible, students are frequently required to develop scripts in order to conduct a simulation with them. Furthermore, EPANET utilizes a different solution approach (global gradient algorithm), which frequently conflicts with the instructional objectives connected with the Hardy-Cross methodology. As a result, there is a scarcity of simple, open source programs ideal for training. The study's software is supposed to fill this need and provide a reliable source for engineers who cannot buy a commercial application.

An extensive set of computer tools for executing an active teaching-learning strategy on an undergraduate hydraulic engineering course in the subject units of pressure flow and free surface flow is presented in [10]. The tools include a variety of easy Excel based programs to help students learn basic principles, as well as a series of practical activities that employ the free professional simulators EPANET and IBER to familiarize students with the tools used in real life. The new technique resulted in more student satisfaction and engagement, as well as more contact with teachers and classmates. To ease their adaptation to different courses and/or degrees, the apps and practical sessions are easily transferable and freely accessible to the community.

References [11, 12] describe the creation of MS Excel applications for optimizing the process parameters in environmental engineering education. The software codes were written in Visual Basic for Applications version 7.0. A variety of test cases were also supplied to evaluate the tool's efficacy in the sectors of air pollution management, water treatment, anaerobic treatment, and water distribution networks for steady-state and extended period simulations. The computed coefficients of determination for each test instance were 0.98 and above. The results indicated that the MS Excel application gives satisfactory rapid and reliable results and may be utilized safely for optimization work in environmental investigations.

For both instructional and practical reasons, a computer program to solve water distribution networks (WDNs) is essential. To resolve WDN, [13] apply three approaches, including the h-based Newton-Raphson method, the finite element method, and the gradient algorithm. These are built with MATLAB and an Excel spreadsheet. The fundamentals of this computer program were discussed step by step using codes because the educational aspects of the system were the major emphasis of the paper. The codes and a software application are supplied with the hope that many instructors and candidates would evaluate them for both educational and practical uses in this engineering discipline.

The same authors of [13] describe in [14] a step-by-step application of Hardy-Cross, Linear Theory, and Q-based Newton-Raphson techniques for addressing water distribution networks employing MATLAB and Excel spreadsheet. These approaches are used to evaluate a basic pipe network in order to focus more on the educational elements of software programs.

Reference [15] presents an instructional software program designed to teach municipal engineering students real-time state modeling of water distribution networks. The state variable values were calculated using the weighted-least-square model and the Davidon-Fletcher-Powell method with real-time data. Network loading, state simulation, and data management were all elements of the software package. Delphi 7.0 was used to code the software and the appropriate graphical user interface. Two evaluation methodologies were used to demonstrate the software tool's effectiveness in teaching and learning. Furthermore, the favorable outcomes spurred software developers and professors to create or at least employ computer-aided instructional tools. In [16], a new educational methodology called Implementation of Mathematical, Experimental, and Computer-based Education is proposed to investigate the synergistic impact of traditional mathematical solution procedure, visual implementation, and MATLAB-based modeling on engineering students' conceptions and quality of learning. The water discharge problem (a specific application of Bernoulli's principle and Toricelli's law) was selected as an illustration of application within the scope of the study. The paper provided in detail the empirical results of three distinct tank models (vertical cylindrical, conical, and spherical) and the solution methods created in MATLAB, as well as novel equations generated for each lab scale system experimental results. With the involvement of 84 learners, the efficiency of the suggested strategy was statistically tested. The performance of the applied approach was assessed as 97.73 percent, 67.36 percent, and 82.55 percent, accordingly, based on mean values collected from all participants.

In engineering education, online training systems are becoming increasingly popular. Education and management tasks can be merged into a single system by merging the pedagogical form of online learning with laboratory information management systems. A work described in [17] sought to create a collaborative education and administration system for a PHP-based water hydraulics laboratory. Throughout all experimental courses, the system provides a simple and multi-functional educational and administration platform for students and teachers. Attitudes of students were investigated and assessed. The findings demonstrated that the suggested approach might improve educational performance and learning efficacy.

#### **4. Methodology**

The network chosen for the simulations has a branched topology, as schematized in Figure 1. In view of the simple educational nature of the work, a very simple configuration network was purposely adopted. The purpose of this expedient was to focus the students' observations exclusively on the variations in the results (head losses and nodal pressures) caused by the different input parameters in the algorithm, without dispersion of their attention, and possible

demotivation, due to more in-depth questions. The network has 06 sections, of which 03 are branches with dead ends, and 01 section (A-B) with no distribution along its length. Such a profile allows several input values to be combined and tested. The pressure at the upstream point (A) must be set by the user.

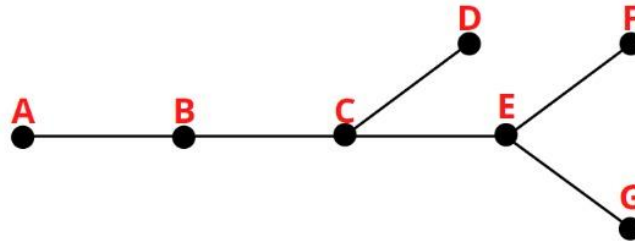


Fig. 1. Hydraulic network

The programming was developed in the Python language. Some libraries were used for data analysis, always aiming to adapt the best data processing model and computational routines. In this sphere, the first library used was Numpy, where it was possible to work with matrices and lists. Afterwards, the Pandas library was used, which includes more resources, through Data Frames. With the two mentioned libraries, it was possible to analyze a dataset without having to resort to huge “for” and “if” loops, facilitating the coding work and making the program better optimized. Figure 2 shows a partial screen of the respective listing.

```

#AB
TrechoDF['CP Jusante'] = [0,0,0,0,0,TrechoDF['CP Montante']][5]-TrechoDF['Perda de Carga Total [m]'][5]
#BC
TrechoDF['CP Montante'] = [0,0,0,0,TrechoDF['CP Jusante']][5],x]
TrechoDF['CP Jusante'] = [0,0,0,0,TrechoDF['CP Montante']][4]-TrechoDF['Perda de Carga Total [m]'][4],TrechoDF['CP Montante']][5]-TrechoDF['Perda de Carga Total [m]']
#CD
TrechoDF['CP Montante'] = [0,0,0,TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][5],x]
TrechoDF['CP Jusante'] = [0,0,0,TrechoDF['CP Montante']][3]-TrechoDF['Perda de Carga Total [m]'][3],TrechoDF['CP Montante']][4]-TrechoDF['Perda de Carga Total [m]']
#CE
TrechoDF['CP Montante'] = [0,0,TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][5],x]
TrechoDF['CP Jusante'] = [0,0,TrechoDF['CP Montante']][2]-TrechoDF['Perda de Carga Total [m]'][2],TrechoDF['CP Montante']][3]-TrechoDF['Perda de Carga Total [m]']
#EF
TrechoDF['CP Montante'] = [0,TrechoDF['CP Jusante']][2],TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][5],x]
TrechoDF['CP Jusante'] = [0,TrechoDF['CP Montante']][1]-TrechoDF['Perda de Carga Total [m]'][1],TrechoDF['CP Montante']][2]-TrechoDF['Perda de Carga Total [m]']
#EG
TrechoDF['CP Montante'] = [TrechoDF['CP Jusante']][2],TrechoDF['CP Jusante']][2],TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][4],TrechoDF['CP Jusante']][5],x]
TrechoDF['CP Jusante'] = [TrechoDF['CP Montante']][0]-TrechoDF['Perda de Carga Total [m]'][0],TrechoDF['CP Montante']][1]-TrechoDF['Perda de Carga Total [m]'][1],1
TrechoDF

```

Fig. 2. Python algorithm partial listing screen

As per [18], Python is one of the most common languages among the community in the area, whether among teachers, data analysts and programming beginners. This fact can be explained through a significant number of libraries and frameworks to solve various real-world problems. Some reasons that made Python reach this level are ease, increased productivity, wide community and popularity, and versatile use.

The positive impact of programming that helps code review processes and understanding what each line is compiling is evident. In this field, the use of descriptive programming, as used, cooperates in the development of the entire algorithm, being able to locate each specific piece of programming [19].

## 5. Results and Discussion

Input and output data screens are shown in Figures 3 and 4, respectively. The input data to the simulator algorithm are the linear distributed flow rate, lengths of the stretches, diameters of each section (as this is a simulation problem, not a design one), terrain elevations of the notable points (nodes) and pressure at the network's initial node. The output data are the reference flow rates, the head losses, the piezometric (energetic) heads, and the pressures on the nodes. Head losses are determined automatically by routine, through the Hazen-Williams equation, with a roughness

coefficient C of 140, by default, referring to the PVC material for the pipes. The value of C can be changed if other pipe materials are used.

```

▶ Digite a vazão em marcha: 0.0015
▶ Digite a pressão no ponto A: 55
▶ Digite o comprimento to trecho EG (m): 150
▶ Digite a Cota Topográfica à Montante do trecho EG (m): 731
▶ Digite a Cota Topográfica à Jusante do trecho EG (m): 750
▶ Digite o Diâmetro do Trecho EG(mm²): 50

▶ Digite o comprimento to trecho EF (m): 100
▶ Digite a Cota Topográfica à Montante do trecho EF (m): 731
▶ Digite a Cota Topográfica à Jusante do trecho EF (m): 738
▶ Digite o Diâmetro do Trecho EF(mm²): 50

▶ Digite o comprimento to trecho CE (m): 250
▶ Digite a Cota Topográfica à Montante do trecho CE (m): 725
▶ Digite a Cota Topográfica à Jusante do trecho CE (m): 731
▶ Digite o Diâmetro do Trecho CE(mm²): 50

▶ Digite o comprimento to trecho CD (m): 220
▶ Digite a Cota Topográfica à Montante do trecho CD (m): 725
▶ Digite a Cota Topográfica à Jusante do trecho CD (m): 741
▶ Digite o Diâmetro do Trecho CD(mm²): 50

▶ Digite o comprimento to trecho BC (m): 280
▶ Digite a Cota Topográfica à Montante do trecho BC (m): 719
▶ Digite a Cota Topográfica à Jusante do trecho BC (m): 725
▶ Digite o Diâmetro do Trecho BC(mm²): 75

▶ Digite o comprimento to trecho AB (m): 300
▶ Digite a Cota Topográfica à Montante do trecho AB (m): 711
▶ Digite a Cota Topográfica à Jusante do trecho AB (m): 719
▶ Digite o Diâmetro do Trecho AB(mm²): 75

```

Fig. 3. Input data screen

| Trecho | Montante | Jusante | Comprimento (m) | CT Montante | CT Jusante | Diâmetro (mm²) | Ponto seco? | Marcha | Vazão Jusante | Vazão Montante | Vazão Fictícia | Perda de Carga Unitária [m/m] | Perda de Carga Total [m] | CP Montante | CP Jusante | Pressão Montante | Pressão Jusante |           |
|--------|----------|---------|-----------------|-------------|------------|----------------|-------------|--------|---------------|----------------|----------------|-------------------------------|--------------------------|-------------|------------|------------------|-----------------|-----------|
| 0      | EG       | E       | G               | 150.0       | 731.0      | 750.0          | 50.0        | Sim    | 0.225         | 0.000          | 0.225          | 0.1125                        | 0.000122                 | 0.018355    | 764.351261 | 764.332906       | 33.351261       | 14.332906 |
| 1      | EF       | E       | F               | 100.0       | 731.0      | 750.0          | 50.0        | Sim    | 0.150         | 0.000          | 0.150          | 0.0750                        | 0.000058                 | 0.005780    | 764.351261 | 764.345481       | 33.351261       | 14.345481 |
| 2      | CE       | C       | E               | 250.0       | 725.0      | 731.0          | 50.0        | Não    | 0.375         | 0.375          | 0.750          | 0.5625                        | 0.002403                 | 0.600752    | 764.952013 | 764.351261       | 39.952013       | 33.351261 |
| 3      | CD       | C       | D               | 220.0       | 725.0      | 741.0          | 50.0        | Sim    | 0.330         | 0.000          | 0.330          | 0.1650                        | 0.000249                 | 0.054676    | 764.952013 | 764.897337       | 39.952013       | 23.897337 |
| 4      | BC       | B       | C               | 280.0       | 719.0      | 725.0          | 75.0        | Não    | 0.420         | 1.080          | 1.500          | 1.2900                        | 0.001549                 | 0.433724    | 765.385737 | 764.952013       | 46.385737       | 39.952013 |
| 5      | AB       | A       | B               | 300.0       | 711.0      | 719.0          | 75.0        | Não    | 0.000         | 1.500          | 1.500          | 1.5000                        | 0.002048                 | 0.614263    | 766.000000 | 765.385737       | 55.000000       | 46.385737 |

Fig. 4. Output data screen

The developed algorithm allows tests by students, with comparisons with solutions made by hand, with more direct and clear assimilation and verification between the varied impacts of the situations raised, approached and simulated by themselves, using different data.

### 5.1 Comparison Test

In order to verify the proper functioning of the proposed algorithm, it was contrasted with the SCALER software [20, 21]. For this purpose, the data for the network of the present work were arbitrated, and are presented in Table 1.

The length values of sections and diameters were taken randomly, however within a reasonableness of real situations. The pressure at the upstream node (A) was set as 55.000 mH<sub>2</sub>O, and the flow distributed along the sections was 0.0015 L/s/m, resulting from the total flow rate demanded by the network divided by its useful length (i.e., the length in which there is distribution).

Table 1: Hydraulic network data

| Branch | Length (m) | Diameter (mm) | Upstream node terrain elevation (m) | Downstream node terrain elevation (m) |
|--------|------------|---------------|-------------------------------------|---------------------------------------|
| A-B    | 300        | 75            | 711.000                             | 719.000                               |
| B-C    | 280        | 75            | 719.000                             | 725.000                               |
| C-D    | 220        | 50            | 725.000                             | 741.000                               |
| C-E    | 250        | 50            | 725.000                             | 731.000                               |
| E-F    | 100        | 50            | 731.000                             | 750.000                               |
| E-G    | 150        | 50            | 731.000                             | 750.000                               |

By using Python algorithm, the values of nodal pressures in nodes A, B, C, D, E, F, and G were, respectively, 55.000 (pressure set), 46.386, 39.952, 23.897, 33.351, 14.345, and 14.333 mH<sub>2</sub>O. Solving the network, with the same data, both manually and through the SCALER software, the same nodal pressure values were obtained as those generated by the Python algorithm. In this way, it was possible to verify its correct operation.

## 6. Conclusions

A computational algorithm for simulating water distribution networks in Python language was introduced. This has the function of contributing to the increase in the understanding of concepts on the topic by students, as it provides agility to the testing process and verification of results due to the different configurations of input data. The accuracy of the algorithm was validated by a simple situation and it is intended to be applied to larger networks, including networks with closed loop and distribution along the stretches, simultaneously.

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## References

- [1] Azevedo Netto, J. M., M. F. Fernandez, R. Araujo, and A. E. Ito. *Hydraulics Manual* (Manual de Hidráulica). São Paulo, Blucher, 2013.
- [2] Wurbs, R. A. *Computer Models for Water Resources Planning and Management*. College Station, Texas A&M University, 1994.
- [3] Kimura, A. *Computing Applied to Reinforced Concrete Structures* (Informática Aplicada a Estruturas de Concreto Armado). São Paulo, Oficina de Textos, 2018.
- [4] Wagener, T., D. Savic, D. Butler, R. Ahmadian, T. Arnot, J. Dawes, S. Djordjevic et al. “Hydroinformatics education – the water informatics in science and engineering (WISE) centre for doctoral training.” *Hydrology and Earth System Sciences* 25, no. 1 (May 2021): 2721–2738. <https://doi.org/10.5194/hess-25-2721-2021>.
- [5] Muhammetoglu, A., H. Muhammetoglu, A. Adigüzel, Ö. İritaş, and Y. Karaaslan. “Management of water losses in water supply and distribution networks in Turkey.” *Turkish Journal of Water Science and Management* 2, no. 1 (January 2018): 58–75. <https://doi.org/10.31807/tjwsm.354298>.
- [6] Walski, T. M., D. V. Chase, D. A. Savic, W. Grayman, S. Beckwith, and E. Koelle. *Advanced Water Distribution Modeling and Management*. Bentley Institute Press, 2004.
- [7] Walski, T. M. “Optimization and pipe sizing decisions.” *Journal of Water Resources Planning and Management* 121, no. 4 (July/August 1995): 340–343. [https://doi.org/10.1061/\(ASCE\)0733-9496\(1995\)121:4\(340\)](https://doi.org/10.1061/(ASCE)0733-9496(1995)121:4(340)).
- [8] Drumea, A. “Education in development of electronic modules using free and open source software tools.” *HIDRAULICA Magazine*, no. 3-4 (2012): 54–60. [https://hidraulica.fluidas.ro/2012/3\\_4/54\\_60.pdf](https://hidraulica.fluidas.ro/2012/3_4/54_60.pdf).
- [9] Gokyay, O. “An easy MS Excel software to use for water distribution system design: A real case distribution network design solution.” *Journal of Applied Water Engineering and Research* 8, no. 4 (October 2020): 1–8. <https://doi.org/10.1080/23249676.2020.1831975>.
- [10] Bermúdez, M., J. Puertas, and L. Cea. “Introducing Excel spreadsheet calculations and numerical simulations with professional software into an undergraduate hydraulic engineering course.”

- Computer Applications in Engineering Education* 28, no. 1 (January 2020): 193–206. <https://doi.org/10.1002/cae.22185>.
- [11] Demir, S., A. Karadeniz, H. C. Yörüklü, and N. M. Demir. “An MS Excel tool for parameter estimation by multivariate nonlinear regression in environmental engineering education.” *Sigma Journal of Engineering and Natural Sciences* 35, no. 2 (2017): 265–273. <https://sigma.yildiz.edu.tr/article/560>.
- [12] Demir, S., N. M. Demir, and A. Karadeniz. “An MS Excel tool for water distribution network design in environmental engineering education.” *Computer Applications in Engineering Education* 26, no. 2 (August 2017): 203–214. <https://doi.org/10.1002/cae.21870>.
- [13] Niazkar, M., and S. H. Afzali. “Analysis of water distribution networks using MATLAB and Excel spreadsheet: h-based methods.” *Computer Applications in Engineering Education* 25, no. 1 (January 2017): 129–141. <https://doi.org/10.1002/cae.21786>.
- [14] Niazkar, M., and S. H. Afzali. “Analysis of water distribution networks using MATLAB and Excel spreadsheet: Q-based methods.” *Computer Applications in Engineering Education* 25, no. 2 (March 2017): 277–289. <https://doi.org/10.1002/cae.21796>.
- [15] Zhang, H., X. Zhou, L. Wang, K. Wang, and W. Wang. “Development of a software tool for teaching real-time state simulation of water distribution networks.” *Computer Applications in Engineering Education* 26, no. 3 (May 2018): 577–588. <https://doi.org/10.1002/cae.21909>.
- [16] Yetilmezsoy, K. “IMECE - Implementation of mathematical, experimental, and computer-based education: A special application of fluid mechanics for civil and environmental engineering students.” *Computer Applications in Engineering Education* 25, no. 5 (September 2017): 833–860. <https://doi.org/10.1002/cae.21871>.
- [17] Wang, H., H. Xu, Q. Li, and Y. Fu. “PHP-based collaborative education and management system for water hydraulic laboratory.” *Computer Applications in Engineering Education* 26, no. 2 (March 2018): 259–271. <https://doi.org/10.1002/cae.21882>.
- [18] Matthes, E. *Python Crash Course: A Hands-On, Project-Based Introduction to Programming* (Curso Intensivo de Python: Uma introdução prática e baseada em projetos à programação). Translated into Portuguese by L. A. Kinoshita. São Paulo, Novatec Editora, 2016.
- [19] McKinney, W. *Python for Data Analysis: Data Wrangling with Pandas, Numpy, and IPython*. Sebastopol, O'Reilly Media, 2017.
- [20] Ignácio, J. P. C., M. V. Nascimento, P. H. G. Oliveira, R. Platz, and H. S. Pizzo. “SCALER - software for sizing water distribution networks (SCALER - software para dimensionamento de redes de distribuição de água).” *Brazilian Journal of Development* 7, no. 7 (July 2021): 71854–71877. <https://doi.org/10.34117/bjdv7n7-387>.
- [21] Pizzo, H. S., J. P. C. Ignácio, and M. V. Nascimento. “Review of water network analysis and validation of SCALER hydraulic simulator.” *Journal of Civil Engineering Frontiers* 3, no. 1 (January/June 2022): 1–11. <https://doi.org/10.38094/jocef30142>.