Generation of Non-Linear Equations to Approximate the Normalized Annual Agricultural Production Curve, Using Drought Indexes

Eng. **Valdemar TORRES ESPINOSA[1]**, PhD. **Maritza Liliana ARGANIZ JUÁREZ[2,3,*]**

[1] Escuela Superior de Ingeniería y Arquitectura, Instituto Politécnico Nacional. Gustavo A. Madero. C. P. 07738. Ciudad de México, CDMX., México

[2] Instituto de Ingeniería, Universidad Nacional Autónoma de México. Ciudad Universitaria. C. P. 04510. Ciudad de México, CDMX., México

[3] Facultad de Ingeniería, Universidad Nacional Autónoma de México. Ciudad Universitaria. C. P. 04510. Ciudad de México, CDMX., México

* Corresponding author's e-mail address: marganisj@iingen.unam.mx

Abstract: *Extreme droughts are one of the biggest problems for agricultural and livestock activities throughout Mexico. These events have caused a decrease in income for producers due to the increase in input costs, which in turn causes an increase in the price of products and food for the poorest populations. In this research equations that allow us to estimate a normalized value of agricultural production in each state of Mexico were obtained. The equations were generated with optimization of their parameters using genetic algorithms and were taken as data from agricultural production records and drought indexes registered in the country. These, in turn, depend on the return period of drought events. These models can be used to estimate agricultural production given a drought with a given return period, which will help to take preventive actions against this problem.*

Keywords: *Drought, genetic algorithm, agricultural production, regionalization, economic losses, drought indexes*

1. Introduction

Drought is a meteorological event that is defined as the decrease or absence of rainfall compared to the annual rate [1]. This climatological phenomenon is present all over the world and is occurring with increasing frequency. Unlike other climatological phenomena, drought is an event that occurs slowly and inconspicuously [2]. For this reason, it is considered one of the most devastating natural disasters worldwide due to its long-term environmental and socioeconomic impact. The direct and indirect effects caused by this phenomenon are also increasing, with negative repercussions on different sectors such as water supply systems, the natural environment, agricultural, livestock and industrial production, among others [3]. The severity of the effects depends on the level of development, population density, demand for water and other natural resources, technological development, and the political system [4].

This meteorological phenomenon is considered to cause the greatest economic damage to humanity [2]. During the last 30 years, 470 drought-related disasters have been estimated around the world [5]. In economic terms, during the last 20 years, droughts have been the cause of the loss of 5 billion USD worldwide [6]. Approximately 1.4 billion people worldwide work in the agricultural sector, and for all of them droughts mean an obstacle to achieving a life of well-being [7].

In the year 2020, in the state of California, U. S., economic losses occurred around 15 billion USD due to droughts and wildfires. In Guatemala, El Salvador, Honduras and Nicaragua about 6.4 million people lost their crops due to drought events [8]. European Commission, et al. 2020 [9] estimated in 2020 that the annual losses due to drought in the European Union and the United Kingdom are around 9 billion euros, with Spain (1.5 billion euros/year), Italy (1.4 billion euros/year), and France (1.4 billion euros/year) showing the highest losses.

Droughts can occur at any time and place; however, there are specific areas of the Earth with greater susceptibility to the phenomenon, determined by their geographic location, based on latitude [10]. Mexico has a large part of its territory in the strip of high northern latitude pressure, so

52% of its territory is classified as arid or semi-arid [1]. During the drought period from 1970 to 1978 that Mexico suffered, 30% of the Mexican territory had gone through the most severe drought period in history, and due to this event, there were deficiencies in the storage of water in several dams in the country, so there were several restrictions for sowing in irrigated areas. The economic losses of this event were estimated at 5 billion Mexican pesos [11]. These devastating events have been intensifying in recent years because of the climatological phenomenon known as "La Niña", which has spread over the last few years, with 2021 being the most severe episode of drought in the country since 2012 [12].
In nature, nothing is constant and predictable. When a natural event reaches its extreme conditions, it can be a risk for the inhabitants of a certain environment. In order to reduce the effects of any catastrophe, it is necessary to anticipate the behavior of these phenomena. In 2015, the study [13] obtained normalized equation models that allow determining the annual agricultural production in the states of Mexico for this same purpose and, like this study, generated equations using the records of drought indexes in the country and its agricultural production (but in Mexican pesos) with the genetic programming methodology. This study updates the equations obtained in [13] using more recent records of drought indexes and agricultural production (in tons), using the genetic algorithm methodology as a function of the return period.

2. Methods

2.1. Genetic Algorithm (GA)

Genetic algorithms (GA) [14] are stochastic global search methods that use a metaphor of natural biological evolution and aim to find the optimal solution to a problem by applying the principle of survival of the fittest to produce increasingly better approximations to a solution.
In each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and reproducing them through operators taken from natural genetics. This process leads to the evolution of populations of individuals that are better adapted to their environment than the individuals from which they were created, just as in natural adaptation.
Individuals, or current approximations, are coded as strings, or chromosomes, composed on some alphabet, so that genotypes (chromosomal values are uniquely mapped to the decision variable (phenotypic) domain. The most commonly used representation in GAs is the binary alphabet {0, 1}, although other representations can be used, e.g., ternary, integer, real-valued, to name a few.
Examining the chromosome string in isolation yields no information about the problem we are trying to solve. Only by decoding the chromosome into its phenotypic values can any meaning be applied to the representation.
Decoding the chromosome representation allows us to evaluate the performance or fitness of individual members of a population. This is done through an objective function that characterizes the performance of an individual in the problem domain. Thus, the objective function establishes the basis for the selection of pairs of individuals that will mate during reproduction.
During the reproduction phase, each individual is assigned a fitness value derived from its raw performance measure given by the objective function. This value is used in selection to bias toward fitter individuals. Highly fit individuals have a high probability of being selected for mating.
Once individuals have been assigned a fitness value, they can be chosen from the population to be crossed or mutated to generate a new population of processed individuals. Following this, the recombination operator intervenes to exchange genetic information between pairs or larger groups of individuals. After recombination, the mutation operator slightly transforms an individual, can avoid convergence and changes the search direction. Then, population replacement establishes the criteria for survivors at each generational iteration, depending on the selection and crossing methods used [15].

2.2. Data set

The data used in the study were the agricultural production records, obtained from the SIAP platform (*Servicio de Información Agroalimentaria y Pesquera*) [16], and the records of the drought indexes were obtained from the Meteorological Drought Monitor platform of CONAGUA (*Comisión Nacional del Agua*) [1] and from the study of [13].

3. Results and Discussion

The drought indexes were obtained from the Mexican Drought Monitor (MSM) of the National Meteorological Service (SMN). Data for the period December 2013 and July 2022 were recorded from the platform. Data prior to December 2013 were obtained from [13]. The information obtained from the MSM indicates the biweekly drought indexes. From this record, the annual average of the data was obtained. These enveloped percentages were reordered from highest to lowest as shown in Table 1.

Table 1: Drought indexes in Mexico

No.	Without affection	D0 to D4	D1 to D4	D2 to D4	D3 to D4	D4
1	76.30	75.99	61.25	45.74	27.31	7.34
2	75.75	62.13	38.84	22.39	7.95	2.42
3	73.24	57.67	38.51	19.61	6.50	0.66
4	70.70	53.84	30.84	15.53	5.23	0.65
5	70.33	52.97	30.73	12.28	3.37	0.57
6	68.37	47.68	28.04	10.87	3.14	0.15
7	65.56	47.30	26.94	9.73	1.93	0.14
8	65.43	47.17	23.76	9.59	1.85	0.14
9	59.28	46.51	22.09	8.82	1.71	0.14
10	54.39	45.61	17.87	7.41	1.58	0.09
11	53.49	40.72	17.64	6.62	1.57	0.09
12	52.83	34.57	14.16	4.68	1.06	0.07
13	52.70	34.44	11.81	3.34	0.95	0.01
14	52.32	31.63	10.57	2.31	0.66	0.00
15	47.03	29.67	8.59	2.29	0.51	0.00
16	46.17	29.30	8.28	1.93	0.39	0.00
17	42.33	26.77	7.12	1.71	0.33	0.00
18	37.88	24.26	7.01	0.99	0.12	0.00
19	24.01	23.70	6.86	0.53	0.01	0.00

The categorization of droughts, according to CONAGUA, is as follows:

- Abnormally Dry (D0): not a drought category, but a dry condition indicating the beginning or end of a drought period. Immediate impacts ranging from delaying/limiting crop or pasture growth to the point where crops or pastures may not fully recover. There is a risk of fire.
- Moderate Drought (D1): Perceptible damage to crops and pastures, a high risk of fire, and water body levels begin to drop. A voluntary restriction of water use is suggested.
- Severe Drought (D2): Losses in agricultural production are now likely, where water is usually scarce. Water consumption restrictions should be imposed.
- Extreme Drought (D3): Losses in agricultural production are greater, the risk of forest fires is extreme and scarcity forces general reductions in consumption.
- Exceptional Drought (D4): Emergency situation due to total water shortage in reservoirs, streams and wells, agricultural production with extraordinary losses and exceptional risk of fires.

From the Statistical Yearbook of Agricultural Production of the Agrifood and Fisheries Information

Service (SIAP) the record of annual agricultural production at the state level was obtained in tons from 2003 to 2021. The production values rearranged from highest to lowest are shown in Table 2.

With the data in Table 2, the mean ($\bar{X}$), standard deviation (σ) and coefficient of variation (Cv) were obtained for each state with the same equations used in [13], which are shown below (Eqs 1 to 3).

Mean.

$$\bar{X} = \sum_{i=i}^{n} \frac{X_i}{n} \tag{1}$$

Standard deviation.

$$\sigma = \sqrt{\frac{\sum_{i=i}^{n} (X_i - \bar{X})^2}{n}} \tag{2}$$

Coefficient of variation.

$$Cv = \frac{\sigma}{\bar{X}} \tag{3}$$

Where X_i is a given agricultural production value and n is the number of agricultural production values recorded.

Six groups of states were formed according to the coefficient of variation. The intervals of the groups formed are shown in Table 3.

The agricultural production values in Table 2 were normalized by dividing each production value by the calculated mean. For example, for the state of Veracruz, the highest normalized value is: 30.073 / 26.609 = 1.13. The normalized values are shown in Table 4.

Table 2: Annual statewide agricultural production record in millions of tons

State	1	2	3	4	5	6	7	8	9	10	11
AS	2.758	2.757	2.712	2.699	2.683	2.663	2.580	2.518	2.517	2.509	2.508
BC	4.214	4.028	3.994	3.858	2.507	2.474	2.458	2.428	2.409	2.362	2.344
BS	0.744	0.705	0.679	0.659	0.644	0.632	0.612	0.591	0.591	0.561	0.523
CC	2.218	1.985	1.981	1.841	1.824	1.817	1.627	1.524	1.494	1.129	1.087
CS	14.182	13.138	12.527	12.311	12.213	12.065	11.952	11.895	11.685	11.602	10.769
CH	11.178	11.048	10.223	10.020	9.969	9.569	9.486	9.329	9.179	7.835	6.431
CL	5.896	5.806	5.781	5.729	5.708	5.689	5.650	5.584	5.570	5.568	5.512
CM	3.773	3.756	3.738	3.659	3.617	3.583	3.549	3.478	3.454	3.410	3.407
DF	0.485	0.456	0.434	0.432	0.419	0.417	0.416	0.411	0.397	0.385	0.383
DG	9.117	8.593	8.563	8.504	8.351	8.257	7.883	7.674	7.489	7.157	7.092
GT	9.953	9.947	9.944	9.874	9.816	9.671	9.388	9.078	9.065	8.990	8.979
GR	5.779	5.724	5.677	5.674	5.541	5.420	5.414	5.238	5.094	4.947	4.798
HG	7.781	7.674	7.639	7.601	7.584	7.442	7.373	7.356	7.298	7.120	7.033
JC	36.295	35.548	35.537	34.394	31.409	31.152	28.452	26.752	25.732	25.041	25.003
MC	9.122	8.864	8.648	8.631	8.516	8.435	8.331	8.271	8.249	8.133	7.783
MN	11.532	11.460	11.450	10.986	10.802	10.296	9.846	9.694	9.583	9.543	9.304
MS	3.833	3.736	3.583	3.523	3.409	3.385	3.341	3.300	3.252	3.194	3.179
NT	6.347	6.243	6.201	6.157	6.145	6.131	6.103	6.016	6.010	5.953	5.744
NL	3.943	3.567	3.547	3.520	3.461	3.293	3.253	3.146	3.124	3.040	2.977
OC	20.108	19.551	19.500	19.477	19.336	18.771	18.440	16.787	15.215	15.144	14.906
PL	7.396	7.323	7.239	7.077	7.058	7.027	6.987	6.959	6.623	6.245	5.976

QO	2.250	2.166	2.164	2.135	2.121	2.095	2.040	2.013	1.956	1.931	1.912
QR	2.226	2.180	2.168	2.070	1.982	1.871	1.871	1.856	1.829	1.819	1.723
SP	11.636	11.189	10.897	10.749	10.515	10.146	9.940	8.765	7.493	7.027	7.005
SL	13.415	12.860	12.703	12.240	12.226	12.174	11.804	11.674	11.141	11.050	11.003
SR	6.997	6.918	6.616	6.484	6.210	5.958	5.692	5.594	5.563	5.490	5.441
TC	4.008	3.859	3.777	3.584	3.459	3.367	3.293	3.275	3.198	3.171	2.960
TS	10.465	9.420	9.406	9.395	9.324	9.050	8.898	8.717	8.575	8.455	8.443
TL	1.516	1.478	1.430	1.423	1.387	1.355	1.347	1.338	1.334	1.323	1.310
VZ	30.073	29.919	29.289	29.007	28.267	28.251	27.640	27.593	27.239	25.990	25.733
YN	13.234	5.716	5.682	5.601	5.589	5.533	5.462	5.382	5.338	5.300	5.285
ZS	7.282	7.089	7.063	6.903	6.856	6.817	6.489	6.307	5.857	5.298	5.242

Table 2: Annual statewide agricultural production record in millions of tons (Continuation)

State	12	13	14	15	16	17	18	19	Mean	σ	Cv
AS	2.487	2.451	2.424	2.317	2.295	2.192	2.145	2.122	2.49	0.20	0.08
BC	2.335	2.298	2.285	2.271	2.183	2.141	1.847	1.777	2.64	0.76	0.29
BS	0.460	0.441	0.438	0.436	0.430	0.429	0.429	0.415	0.55	0.11	0.20
CC	0.988	0.973	0.956	0.915	0.859	0.822	0.791	0.679	1.34	0.49	0.37
CS	10.467	10.187	10.032	9.223	8.761	8.605	6.414	6.151	10.75	2.15	0.20
CH	6.397	6.298	5.981	5.737	5.130	5.086	4.836	4.539	7.80	2.30	0.29
CL	5.410	5.364	5.329	5.318	5.271	5.000	4.394	4.236	5.41	0.44	0.08
CM	3.339	3.211	3.150	3.020	2.967	2.906	2.299	2.258	3.29	0.44	0.13
DF	0.375	0.373	0.367	0.365	0.361	0.359	0.357	0.352	0.40	0.04	0.09
DG	7.021	6.778	6.732	6.709	5.729	5.116	4.949	4.818	7.19	1.30	0.18
GT	8.817	8.701	8.522	8.510	8.471	8.358	8.227	8.023	9.07	0.64	0.07
GR	4.788	4.563	4.506	4.271	4.227	4.080	2.914	2.860	4.82	0.87	0.18
HG	7.003	6.981	6.688	6.553	6.490	6.222	6.189	6.089	7.06	0.54	0.08
JC	24.771	23.767	23.627	22.998	22.898	22.457	21.698	21.311	27.31	5.13	0.19
MC	7.332	7.285	7.266	7.082	6.867	6.802	5.692	5.597	7.73	1.01	0.13
MN	8.962	8.880	8.842	8.775	8.399	8.395	8.237	8.205	9.64	1.14	0.12
MS	3.128	3.039	3.027	3.022	2.991	2.860	2.812	2.444	3.21	0.34	0.10
NT	5.689	5.442	5.013	4.734	4.662	3.967	3.937	3.563	5.48	0.89	0.16
NL	2.883	2.827	2.771	2.736	2.659	2.638	0.963	0.706	2.90	0.81	0.28
OC	14.569	14.388	13.880	13.662	11.273	8.113	6.510	6.477	15.06	4.38	0.29
PL	5.912	5.882	5.639	5.600	5.514	5.435	5.342	5.299	6.34	0.76	0.12
QO	1.912	1.847	1.843	1.776	1.249	1.218	1.204	1.155	1.84	0.36	0.20
QR	1.633	1.619	1.595	1.439	1.387	1.279	1.187	1.171	1.73	0.33	0.19
SP	6.948	6.812	6.427	6.425	6.248	6.222	6.172	5.741	8.23	2.08	0.25
SL	10.990	10.924	10.194	10.161	9.940	9.675	9.139	8.102	11.13	1.36	0.12
SR	5.196	5.052	4.963	4.843	4.685	3.887	3.716	3.156	5.39	1.05	0.20
TC	2.903	2.830	2.744	2.627	2.605	2.466	2.434	2.417	3.10	0.50	0.16
TS	8.344	8.332	8.217	8.147	8.056	7.964	6.192	5.715	8.48	1.09	0.13
TL	1.309	1.292	1.265	1.243	1.182	1.140	1.116	0.748	1.29	0.17	0.13

VZ	25.405	25.286	24.998	24.873	24.602	24.418	24.054	22.948	26.61	2.14	0.08
YN	5.096	5.070	4.971	4.677	4.552	4.471	0.578	0.460	5.16	2.48	0.48
ZS	3.982	3.744	3.546	3.394	3.274	3.246	2.652	2.351	5.13	1.74	0.34

Table 3: Groups of states according to the coefficient of variation Cv

Group	Cv Interval
1	0.07 - 0.10
2	0.12 - 0.16
3	0.18 - 0.20
4	0.25 - 0.29
5	0.34 - 0.37
6	0.48

An average of the 19 normalized records was determined for each group of states shown in Table 3. Table 5 shows the averages of the normalized records for each group.
Correlation plots of the normalized value of annual production vs. the different drought indexes were made for each group of states. The correlation graphs, their trend lines, their equation and the coefficients of determination of these lines were obtained with the Excel tool. This is done to identify with which type of drought a better correlation is obtained according to the group of states.
Groups 1, 2, 3 and 4 had a better correlation (a higher value in the coefficient of determination) with drought indexes from D2 to D4, group 5 had a better correlation with drought indexes from D1 to D4, and group 6 had a better correlation with drought indexes from D0 to D4. For these drought types, correlation graphs were made of the enveloping percentage of drought vs. the return period in years.
The return period was obtained with the Weibull equation (n+1)/m, where n is the size of the annual series and m is the number of ordered data.
The equations of the trend lines generated from the correlation plots were generated with the Excel tool, and the coefficients of the polynomial models were obtained with Excel Solver and AG. The equations obtained are shown below (Eqs 4 to 6).

For D0 a D4.

$$D_i = 14.222\ Ln(T_D) + 29.181 \tag{4}$$

For D1 a D4.

$$D_i = 14.88\ Ln(T_D) + 6.93 \tag{5}$$

For D2 a D4.

$$D_i = -0.1044{T_D}^2 + 3.942T_D - 1.363 \tag{6}$$

Where D_i is the drought indexes and T_D is the return period in years.

Table 4: Standardized record of agricultural production

State	1	2	3	4	5	6	7	8	9	10
AS	1.11	1.11	1.09	1.08	1.08	1.07	1.04	1.01	1.01	1.01
BC	1.59	1.52	1.51	1.46	0.95	0.94	0.93	0.92	0.91	0.89
BS	1.36	1.29	1.24	1.20	1.17	1.15	1.12	1.08	1.08	1.02
CC	1.65	1.48	1.48	1.37	1.36	1.35	1.21	1.13	1.11	0.84
CS	1.32	1.22	1.17	1.15	1.14	1.12	1.11	1.11	1.09	1.08
CH	1.43	1.42	1.31	1.28	1.28	1.23	1.22	1.20	1.18	1.00
CL	1.09	1.07	1.07	1.06	1.05	1.05	1.04	1.03	1.03	1.03
CM	1.15	1.14	1.13	1.11	1.10	1.09	1.08	1.06	1.05	1.04
DF	1.22	1.15	1.09	1.09	1.06	1.05	1.05	1.04	1.00	0.97
DG	1.27	1.20	1.19	1.18	1.16	1.15	1.10	1.07	1.04	1.00
GT	1.10	1.10	1.10	1.09	1.08	1.07	1.03	1.00	1.00	0.99
GR	1.20	1.19	1.18	1.18	1.15	1.13	1.12	1.09	1.06	1.03
HG	1.10	1.09	1.08	1.08	1.07	1.05	1.04	1.04	1.03	1.01
JC	1.33	1.30	1.30	1.26	1.15	1.14	1.04	0.98	0.94	0.92
MC	1.18	1.15	1.12	1.12	1.10	1.09	1.08	1.07	1.07	1.05
MN	1.20	1.19	1.19	1.14	1.12	1.07	1.02	1.01	0.99	0.99
MS	1.19	1.16	1.12	1.10	1.06	1.05	1.04	1.03	1.01	0.99
NT	1.16	1.14	1.13	1.12	1.12	1.12	1.11	1.10	1.10	1.09
NL	1.36	1.23	1.22	1.21	1.19	1.14	1.12	1.09	1.08	1.05
OC	1.34	1.30	1.29	1.29	1.28	1.25	1.22	1.11	1.01	1.01
PL	1.17	1.15	1.14	1.12	1.11	1.11	1.10	1.10	1.04	0.98
QO	1.22	1.18	1.18	1.16	1.15	1.14	1.11	1.09	1.06	1.05
QR	1.29	1.26	1.25	1.20	1.14	1.08	1.08	1.07	1.06	1.05
SP	1.41	1.36	1.32	1.31	1.28	1.23	1.21	1.07	0.91	0.85
SL	1.21	1.16	1.14	1.10	1.10	1.09	1.06	1.05	1.00	0.99
SR	1.30	1.28	1.23	1.20	1.15	1.10	1.06	1.04	1.03	1.02
TC	1.29	1.24	1.22	1.15	1.11	1.08	1.06	1.06	1.03	1.02
TS	1.23	1.11	1.11	1.11	1.10	1.07	1.05	1.03	1.01	1.00
TL	1.17	1.14	1.11	1.10	1.07	1.05	1.04	1.04	1.03	1.02
VZ	1.13	1.12	1.10	1.09	1.06	1.06	1.04	1.04	1.02	0.98
YN	2.57	1.11	1.10	1.09	1.08	1.07	1.06	1.04	1.03	1.03
ZS	1.42	1.38	1.38	1.35	1.34	1.33	1.27	1.23	1.14	1.03

Table 4: Standardized record of agricultural production (Continuation)

State	11	12	13	14	15	16	17	18	19
AS	1.01	1.00	0.98	0.97	0.93	0.92	0.88	0.86	0.85
BC	0.89	0.88	0.87	0.86	0.86	0.83	0.81	0.70	0.67
BS	0.95	0.84	0.80	0.80	0.79	0.78	0.78	0.78	0.76
CC	0.81	0.74	0.72	0.71	0.68	0.64	0.61	0.59	0.51
CS	1.00	0.97	0.95	0.93	0.86	0.82	0.80	0.60	0.57
CH	0.82	0.82	0.81	0.77	0.74	0.66	0.65	0.62	0.58
CL	1.02	1.00	0.99	0.98	0.98	0.97	0.92	0.81	0.78
CM	1.03	1.01	0.98	0.96	0.92	0.90	0.88	0.70	0.69
DF	0.97	0.94	0.94	0.92	0.92	0.91	0.91	0.90	0.89
DG	0.99	0.98	0.94	0.94	0.93	0.80	0.71	0.69	0.67
GT	0.99	0.97	0.96	0.94	0.94	0.93	0.92	0.91	0.88
GR	1.00	0.99	0.95	0.94	0.89	0.88	0.85	0.61	0.59
HG	1.00	0.99	0.99	0.95	0.93	0.92	0.88	0.88	0.86
JC	0.92	0.91	0.87	0.87	0.84	0.84	0.82	0.79	0.78
MC	1.01	0.95	0.94	0.94	0.92	0.89	0.88	0.74	0.72
MN	0.96	0.93	0.92	0.92	0.91	0.87	0.87	0.85	0.85
MS	0.99	0.97	0.95	0.94	0.94	0.93	0.89	0.88	0.76
NT	1.05	1.04	0.99	0.92	0.86	0.85	0.72	0.72	0.65
NL	1.03	1.00	0.98	0.96	0.94	0.92	0.91	0.33	0.24
OC	0.99	0.97	0.96	0.92	0.91	0.75	0.54	0.43	0.43
PL	0.94	0.93	0.93	0.89	0.88	0.87	0.86	0.84	0.84
QO	1.04	1.04	1.00	1.00	0.96	0.68	0.66	0.65	0.63
QR	0.99	0.94	0.93	0.92	0.83	0.80	0.74	0.69	0.68
SP	0.85	0.84	0.83	0.78	0.78	0.76	0.76	0.75	0.70
SL	0.99	0.99	0.98	0.92	0.91	0.89	0.87	0.82	0.73
SR	1.01	0.96	0.94	0.92	0.90	0.87	0.72	0.69	0.59
TC	0.95	0.94	0.91	0.88	0.85	0.84	0.79	0.78	0.78
TS	1.00	0.98	0.98	0.97	0.96	0.95	0.94	0.73	0.67
TL	1.01	1.01	1.00	0.98	0.96	0.92	0.88	0.86	0.58
VZ	0.97	0.95	0.95	0.94	0.93	0.92	0.92	0.90	0.86
YN	1.02	0.99	0.98	0.96	0.91	0.88	0.87	0.11	0.09
ZS	1.02	0.78	0.73	0.69	0.66	0.64	0.63	0.52	0.46

Table 5: Normalized registers of the groups of states

No.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	GROUP 5	GROUP 6
1	1.13	1.20	1.29	1.43	1.54	2.57
2	1.11	1.16	1.24	1.37	1.43	1.11
3	1.09	1.14	1.22	1.33	1.43	1.10
4	1.08	1.12	1.19	1.31	1.36	1.09
5	1.07	1.10	1.15	1.20	1.35	1.08
6	1.06	1.09	1.13	1.16	1.34	1.07
7	1.04	1.07	1.09	1.14	1.24	1.06
8	1.03	1.06	1.07	1.08	1.18	1.04
9	1.01	1.04	1.05	1.02	1.13	1.03
10	1.00	1.02	1.02	0.96	0.94	1.03
11	0.99	0.99	0.99	0.92	0.92	1.02
12	0.97	0.98	0.95	0.90	0.76	0.99
13	0.97	0.96	0.92	0.89	0.73	0.98
14	0.95	0.93	0.92	0.86	0.70	0.96
15	0.94	0.91	0.88	0.85	0.67	0.91
16	0.93	0.89	0.81	0.78	0.64	0.88
17	0.90	0.85	0.76	0.73	0.62	0.87
18	0.88	0.78	0.69	0.57	0.56	0.11
19	0.84	0.72	0.66	0.52	0.49	0.09

With equations 4, 5 and 6, the drought index record was updated using the same T_D values that were used to initially generate the equations. New correlation graphs of the normalized value of annual production vs. the new drought indexes were made from these data. With these graphs, the equation models of the trend lines were obtained with the Excel tool. The coefficients of the equation models were obtained with the GA. It was also possible to obtain the equation models with the Solver tool, but the results of this tool always showed a lower coefficient of determination than those obtained with the genetic algorithm. The equations obtained by GA are shown below (Eqs 7 to 12).

Group 1.

$$A_P = 0.075Ln(D_{iu}) + 0.861 \quad (7)$$

Group 2.

$$A_P = 0.121Ln(D_{iu}) + 0.776 \quad (8)$$

Group 3.

$$A_P = 0.17Ln(D_{iu}) + 0.685 \quad (9)$$

Group 4.

$$A_P = 0.2399Ln(D_{iu}) + 0.555 \quad (10)$$

Group 5.

$$A_P = -7.34 \cdot 10^{-4} {D_{iu}}^2 + 0.066D_{iu} + 0.048 \quad (11)$$

Group 6.

$$A_P = -5.5 \cdot 10^{-6} {D_{iu}}^3 + 7.7 \cdot 10^{-4} {D_{iu}}^2 + 3 \cdot 10^{-5} D_{iu} \quad (12)$$

Where A_P is the normalized value of annual production and D_{iu} is the updated drought index.
Equations 7, 8, 9, 10, 11 and 12 had the variable D_{iu} substituted by the equations that determine the drought indexes according to the return period. The substitution is made so that from T_D the normalized value of the annual production can be obtained.
The simplified equations are shown below (Eqs 13 to 18).

Group 1.

$$A_P = 0.075 Ln(-0.1044 {T_D}^2 + 3.942 T_D - 1.363) + 0.861 \quad (13)$$

Group 2.

$$A_P = 0.121 Ln\left(-0.1044 {T_D}^2 + 3.942 T_D - 1.363\right) + 0.776 \quad (14)$$

Group 3.

$$A_P = 0.17 Ln\left(-0.1044 {T_D}^2 + 3.942 T_D - 1.363\right) + 0.685 \quad (15)$$

Group 4.

$$A_P = 0.2399 Ln\left(-0.1044 {T_D}^2 + 3.942 T_D - 1.363\right) + 0.555 \quad (16)$$

Group 5.

$$A_P = -0.1625 Ln(T_D)^2 + 0.8307 Ln(T_D) + 0.4701 \quad (17)$$

Group 6.

$$A_P = -0.0158 Ln(T_D)^3 + 0.0584 Ln(T_D)^2 + 0.4397 Ln(T_D) + 0.5199 \quad (18)$$

Where A_P is the normalized value of annual production and T_D is the return period in years.

3.1. Example of application of the obtained equations

For the explanation of the application of the equations obtained in this work, the state of Veracruz will be taken as an example. According to Table 2, Veracruz has a Cv of 0.08. Then, according to the group formation defined in Table 3, Veracruz is part of group 1, since its Cv falls within the interval of 0.07-0.1.

If Veracruz presents a drought in a 19-year return period, applying equation 13, the normalized production is expected to be 1.13 (Table 6). If historically the average production has been 26,609,785 tons, then the agricultural production will be the result of equation 13 multiplied by the historical average production. That is (26,609,785 tons)*(1.13)= 30,069,057 tons (Table 7).

Table 6: Result of eq. 13. Agricultural production in Veracruz

T_D (In years)	A_P (Registered)	A_P (Calculated)
19	1.13	1.13

Table 7: Agricultural production expected in Veracruz for a 19-year T_D drought

T_D (In years)	A_P (Registered)	A_P (Calculated)
19	1.13	1.13

4. Conclusions

Droughts are meteorological events that cannot be avoided and cannot be determined with total certainty, which makes the consequences of these events inevitable, but it is possible to control the severity of the repercussions if their effects can be foreseen. The usefulness of the equations obtained in this research is in providing an estimate of the expected value of agricultural production in the face of drought events. With this information, it is possible to take actions in the face of the economic effects that droughts can cause in the agricultural sector.

The methodology used in this study made it possible to determine a similar behavior in agricultural production in the states, and with this it was possible to develop equations for different groups of states that had similarities in their agricultural production. This same methodology can be used to determine the behavior of other meteorological events. The standardization of the information recorded for agricultural production can be substituted by records of other meteorological phenomena.

Acknowledgments

This study has benefited from the public data obtained from the CONAGUA and SIAP. The authors would like to thank the Instituto de Ingeniería for their assistance and support.

References

[1] Esparza, Miguel. "Drought and water scarcity in Mexico. Current situation and future prospects." / "La sequía y la escasez de agua en México. Situación actual y perspectivas futuras." *Secuencia*, no. 89 (2014): 195-219. http://dx.doi.org/10.18234/secuencia.v0i89.1231.
[2] CENAPRED. *Droughts Fascicles* / *Fascículos Sequías*. D.F., Ministry of the Interior / Secretaría de Gobernación (SEGOB), 2007.
[3] Vatter, Juliane. *Drought Risk. The Global Thirst for Water in the Era of Climate Crisis*. Berlin, WWF Germany, 2019.
[4] Eriyagama, Nishadi, Vladimir Smakhtin, and Nilantha Gamage. *Mapping drought patterns and impacts: a global perspective*. Colombo, International Water Management Institute, 2009.
[5] UNDRR. "Number of Climate-related Disasters Around the World (1980-2011)." *The United Nations Office for Disaster Risk Reduction*, June 13, 2012. Accessed March 22, 2023. https://www.preventionweb.net/files/20120613_ClimateDisaster1980-2011.pdf.
[6] Real-Rangel, Roberto A. "Monitoring of droughts in Mexico through multivariate indices." / "Monitoreo de sequías en México a través de índices multivariados." MSCE thesis, National Autonomous University of Mexico / Universidad Nacional Autónoma de México, CDMX, 2016.
[7] Meza, Isabel, Stefan Siebert, Petra Döll, Jürgen Kusche, Claudia Herbert, Ehsan Eyshi Rezaei, Hamideh Nouri, Helena Gerdener, Eklavyya Popat, Janna Frischen, Gustavo Naumann, Jürgen V. Vogt, Yvonne Walz, Zita Sebesvari, and Michael Hagenlocher. "Global-scale drought risk assessment for agricultural systems." *Natural Hazards and Earth System Sciences*, no. 20 (2020): 695-712. https://doi.org/10.5194/nhess-20-695-2020.
[8] UNCCD. *Drought in numbers*. New York, UNCCD, 2022.
[9] European Commission, Joint Research Centre. C. Cammalleri, G. Naumann, L. Mentaschi, G. Formetta, G. Forzieri, S. Gosling, B. Bisselink, A. De Roo, and L. Feyen. "Global warming and drought impacts in the EU." *JRC PRESETA IV project: Task 7*, (2020) https://data.europa.eu/doi/10.2760/597045.
[10] CENAPRED. *Diagnosis of hazards and identification of disaster risks in Mexico* / *Diagnóstico de peligros y identificación de riesgos de desastres en México*. D.F., Ministry of the Interior / Secretaría de Gobernación (SEGOB), 2001.
[11] Florescano Mayet, Enrique, Jaime Sancho y Cervera, and David Pérez Gavilán Arias. "Droughts in Mexico: history, characteristics and effects." / "Las sequías en México: historia, características y efectos." *Comercio Exterior* 30, no. 7 (1980): 747-757.
[12] Bank of Mexico / Banco de México. "Drought in Mexico and its potential impact on economic activity"/ "Sequía en México y su potencial impacto en la actividad económica" *Bank of Mexico Quarterly Report* / *Banxico Informe Trimestral*, (2022): 30-35.
[13] Drust-Nacarino, Ariadne Sofía, Maritza Liliana Arganis-Juaréz, Rodolfo Silva-Casarín, Edgar Gerardo Mendoza-Baldwin, and Óscar Arturo Fuentes-Mariles. "Drought and Genetic Programming to Approach Annual Agriculture Production Normalized Curves". *Revista Facultad de Ingeniería Universidad de Antioquia*, no. 77 (December 16, 2015): 63–74. https://doi.org/10.17533/udea.redin.n77a09.

[14] Goldberg, David E. *Genetic Algorithms in Search, Optimization and Machine Learning.* Massachusetts, Addison-Wesley Publishing Company, 1989.
[15] Chipperfield, Andrew, Peter Fleming, Hartmut Pohlheim, and Carlos Fonseca. *Genetic Algorithm Toolbox User´s Guide.* Sheffield, University of Sheffield, Department of Automatic Control and Systems Engineering, 1994.
[16] SIAP. “Statistical Yearbook of Agricultural Production” / "Anuario Estadístico de la Producción Agrícola." *Closure of agricultural production* / *Cierre de la producción agrícola*, June 13, 2022. Accessed August 2, 2022. https://nube.siap.gob.mx/cierreagricola/
[17] CONAGUA. "Mexico Drought Monitor”/ “Monitor de sequía de México." *Monitor de sequía de México (MSM)*, 2014. Accessed August 2, 2022. https://smn.conagua.gob.mx/es/climatologia/monitor-de-sequia/monitor-de-sequia-en-mexico.