Numerical Aspects of Fluid Peristaltic Circulation Model

Associate professor **Fănel Dorel ȘCHEAUA**[1,*]

[1] Dunarea de Jos University of Galati, MECMET Research Center

* fanel.scheaua@ugal.ro

Abstract: *The peristaltic model of flow describes the movement of fluids through channels or tubes, highlighting the natural process of peristalsis observed in biological systems, such as the gastrointestinal tract, blood vessels and lymphatic systems. Peristalsis is characterized by rhythmic, wave-like contractions of muscle layers that propel fluids forward. In the human body, this mechanism is crucial for the transport of various substances, of interest being the mechanisms by which this process occurs. In this paper, a description of the fluid movement typology with a mathematical model and a numerical analysis are presented in order to highlight the main parameters involved and their effects in the flow. The results obtained highlight the main possibilities of intervention and the changes that occur in the flow phenomenon in case when certain involved parameters values are modified.*

Keywords: *Fluid flow, peristaltic flow, fluid type, vessel elasticity, mathematical model, numerical analysis*

1. Introduction

The peristaltic flow typology represents a fundamental mechanism of fluid transport in nature, being found in various biological systems such as the digestive system, urethras, or lymphatic vessels, being taken up and used further even in industrial field for applications involving peristaltic pumps for the circulation and transport of liquids. The movement is characterized by undulating movements of a flexible channel or tube walls within which the flow takes place, which ultimately has the ability to create a directed flow without requiring internal mechanical components.

Peristaltic fluid flow is driven by cyclic contractions and relaxations of the channel walls, which generate peristaltic waves. These waves create a pressure difference that causes the fluid to move in a specific direction, usually in the longitudinal direction of the channel in which the flow occurs.

The main characteristics of peristaltic flow are given by the periodicity with which the movement of the channel walls occurs, since the channel walls move in a repetitive manner, often described by a sinusoidal function.

The flow also occurs in a non-mixing flow because the fluid is pushed forward without significant mixing, which is essential for the transport of sensitive fluids such as blood.

This ensures a flow that is not dependent on valves because this mechanism does not require valves to prevent reflux, the movement of peristaltic waves ensuring directionality.

The flexibility of the channel walls is a main characteristic in that peristaltic tubes can transport fluids of various viscosities, including suspensions and non-Newtonian fluids.

The peristaltic flow type is characteristic mainly of biological systems represented by the digestive system by transporting food through the oesophagus, stomach and intestines which is accomplished by peristaltic contractions, the urinary system where the urethras use peristaltic waves to conduct urine from the kidneys and also the circulatory system where peristaltic movements help transport blood and lymph, in the body necessary to transport nutrients, oxygen, carbon dioxide, especially in small lymphatic vessels.

The advantages of this type of movement are presented in biology and technology by preventing contamination within peristaltic pumps, where the fluid remains isolated within the tube, thus preventing contact with the pump components, and regarding transport safety elements, peristalsis has a delicate action on fluids and implicitly does not intervene to affect the chemical composition, offering increased adaptability because it works for fluid typologies with different specific values of viscosity and composition.

Practical applications of the peristaltic flow typology involve the use of peristaltic pumps used in laboratories and medicine for performing infusions or transporting sterile liquids, various

simulations in biomedical engineering for design activities of various implants or medical devices, as well as for irrigation systems and fluid transport in industrial processes.
The limitations related to the type of peristaltic flow are related to the fact that it presents decreases in efficiency at high speed values or at high pressures.
For the case of very viscous fluids, to ensure continuity of flow, waves with higher amplitudes are needed in order to maintain the flow, as well as possible wear of the channel walls or flexible tubes, especially in mechanical applications.

2. Physical and mathematical model of peristaltic flow

The physical and mathematical model of peristaltic flow considers the occurrence of a generated peristaltic wave, which can be described by means of sinusoidal equations considering a circular channel of radius *(r)*, an emitted contraction of amplitude *(a)*, a wavelength (λ) and an angular wave frequency (ω):

$$r(x,t) = r_0 + a\sin\left(2\pi\frac{x}{\lambda} - \omega t\right) \qquad (2.1)$$

The flow regime within peristaltic flow can be in the laminar or turbulent range, depending on the Reynolds Number (Re), which describes the ratio between inertial and viscous forces.
Regarding the viscosity values of the fluid, it should be highlighted that Newtonian fluids such as water and non-Newtonian fluids such as blood exhibit different behaviours.
The fluid pressure and flow rate are directly dependent on the frequency of the peristaltic waves *(f)*, the amplitude of the contractions *(a)*, as well as the resistance of the channel with reference to the dependence on the viscosity of the fluid.
Fluid flow is described by the Navier-Stokes equations and the continuity equation [1].
The continuity equation ensures the conservation of mass, where the velocity components *(u, v)* are considered in two main axial *(x)* and radial *(r)* directions:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial (rv)}{\partial r} = 0 \qquad (2.2)$$

The Navier-Stokes equation in cylindrical coordinates describes the conservation of momentum for a viscous fluid, highlighting the values of pressure *(p)*, dynamic viscosity *(μ)* and density of the fluid *(ρ)* [1]:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\right] \qquad (2.3)$$

The introduction of approximations to simplify the calculation must be taken into account, defining initially a laminar fluid flow, with dominant viscous stresses, while inertial forces are negligible (Re≪1), maintaining a small value for the wave amplitude $(a << r_0)$, which allows the linear approximation of the wave, as well as a periodic oscillating flow where it is assumed that the velocity and pressure exhibit periodic variations.
In order to obtain an approximate solution for the fluid velocity, it is assumed that the axial velocity *(u)* depends only on the radius *(r)* and time *(t)*, making it possible to calculate the velocity using a relationship similar to Poiseuille's law [1-5]:

$$u(x,r,t) = \frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left(r_{xt}^2 - r^2\right) \qquad (2.4)$$

Where (r_{xt}) is the radius of the channel at position *(x)* and time *(t).*

The volumetric flow rate $Q(x,t)$ circulating through the channel is obtained by integrating the velocity over the cross section [1-7]:

$$Q(x,t) = \int_0^r 2\pi r u(x,r,t) dr \tag{2.5}$$

Performing the velocity substitution, we obtain:

$$Q(x,t) = -\frac{\pi}{8\mu}\frac{\partial p}{\partial x} r_{x,t}^4 \tag{2.6}$$

The pressure required to maintain the flow is determined based on the pressure gradient:

$$\Delta p = \frac{8}{\pi}\frac{\mu l_c Q}{r^4} \tag{2.7}$$

where (l_c) represents the length of the channel segment considered [8-10].
The main effects of the parameters on the flow typology are represented by direct actions of the wave amplitude (a), since high values of the amplitude have the possibility of increasing the volumetric flow circulated, but can generate increased hydrodynamic resistances.
Higher frequencies lead to faster transport, but can increase the energy required to perform the pumping process, while dynamic viscosity (μ) has an influence related to the need to generate a higher pressure to maintain a constant circulation flow, valid for fluids with higher dynamic viscosity.
For turbulent regime, peristaltic flow presents a higher complexity degree compared to the laminar domain, by involving nonlinear interactions between inertial and viscous forces, as well as the formation of turbulent structures (vortices). In this case, the model must take into account the characteristics of turbulence, such as random velocity fluctuations and large pressure gradients. The information of peristaltic flow in turbulent regime is related to the Reynolds Number that exceeds the critical value characteristic of the laminar regime. The flow becomes turbulent when the Reynolds Number (Re) exceeds the critical value (Re>2000, approximately, for cylindrical channels) [5-11]:

$$\mathrm{Re} = \frac{1}{\mu}(\rho \cdot \bar{u} \cdot r) \tag{2.8}$$

where $(\bar{u})$ represents the average flow velocity and (μ) dynamic viscosity.

Within the structure of turbulent flow, eddies (vortices) are formed in the flow, which leads to a loss of energy through dissipation, having a direct impact on the propagation of the peristaltic wave, in the sense that it introduces additional instabilities in the flow, amplifying turbulent fluctuations. For the turbulent flow regime, the wave contribution to the net flow depends on the frequency (f), amplitude (a) and the degree of dissipation.

Turbulent flow is described by the time-averaged Navier-Stokes equations (RANS – Reynolds-Averaged Navier-Stokes) combined with a turbulence model:

$$\rho\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial r}\right) = -\frac{\partial \bar{p}}{\partial x} + \mu\left[\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \bar{u}}{\partial r}\right)\right] - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{xr}}{\partial r} \tag{2.9}$$

where $(\bar{u})$ is average velocity in the *(x)* direction, (τ_{xx}) is Reynolds stress in the *(x)* direction (velocity fluctuations) and (τ_{xr}) is Reynolds stress in the *(x)* and *(r)* directions, [1, 6-12].

For Reynolds stress modelling, turbulence models are used to close the system of equations, such as the $(k-\varepsilon)$ model containing the turbulent energy model *(k)* and the turbulent energy dissipation rate (ε):

$$(k):\ \frac{\partial k}{\partial t}+\bar{u}\frac{\partial k}{\partial x}=\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j}-\varepsilon+\nu_t\nabla^2 k$$
$$(\varepsilon):\ \frac{\partial \varepsilon}{\partial t}+\bar{u}\frac{\partial \varepsilon}{\partial x}=C_1\frac{\varepsilon}{k}\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j}-C_2\frac{\varepsilon^2}{k}+\nu_t\nabla^2\varepsilon \tag{2.10}$$

Another approach is given by the $(k-\omega)$ model which considers the use of specific vortices $\left(\omega=\frac{\varepsilon}{k}\right)$ instead of the dissipation rate.

The model for peristalsis in the turbulent regime takes into account the displacement of the walls, and the equations are solved with dynamic boundary conditions, which primarily consider the displacement of the channel wall, thus periodically modifying the channel radius according to the peristaltic wave function:

$$r(x,t)=r_0+a\sin(kx-\omega t) \tag{2.11}$$

The boundary conditions must be established primarily at the walls, where the radial velocity of the fluid must coincide with the wall velocity:

$$v(r,t)=v(r=r(x,t),t)=\frac{\partial r(x,t)}{\partial t} \tag{2.12}$$

as well as at the flow channel axis level through the radial derivative of the axial velocity which is zero (symmetry):

$$\left.\frac{\partial u}{\partial r}\right|_{r=0}=0 \tag{2.13}$$

The turbulent regime volumetric flow as averaged volumetric flow rate $(\bar{Q})$ is directly dependent on the turbulent velocity and the peristaltic shape of the channel:

$$\bar{Q}=\int_0^{r(x,t)}2\pi r\bar{u}(x,r,t)dr \tag{2.14}$$

In turbulent regime, the axial velocity $(\bar{u})$ shows a flatter distribution than that in the laminar regime, precisely due to the achievement of turbulent mixing within the fluid [13-15].

3. Numerical analysis for peristaltic fluid flow

For numerical simulation of peristaltic flow in turbulent regime, usually CFD (Computational Fluid Dynamics) methods are used, which enable RANS solution iterations with software such as OpenFOAM, ANSYS Fluent or COMSOL Multiphysics.
The problem of peristaltic flow is described by the Navier-Stokes equations for an incompressible fluid, in the context of a time-varying flow domain with a sinusoidal wall or other type of periodic motion.
Usually, the channel wall is assumed to follow a sinusoidal motion, and the fluid is subjected to a non-Newtonian model.
To solve this problem numerically, methods such as finite difference, finite volume, or finite element can be used. A finite difference approach will be used to solve the partial equations in space and time.

To simulate the deformation of a vessel wall under the influence of peristaltic fluid flow, a numerical approach is considered that calculates the interaction between the fluid pressure and the shear forces acting on the channel wall.
The pressure action is introduced as sinusoidal variable in time and axially, while the elasticity properties of channel wall are considered, completing a basic model used to identify solutions related to the pressure distribution and channel wall deformation where the fluid flow takes place (figure 1).

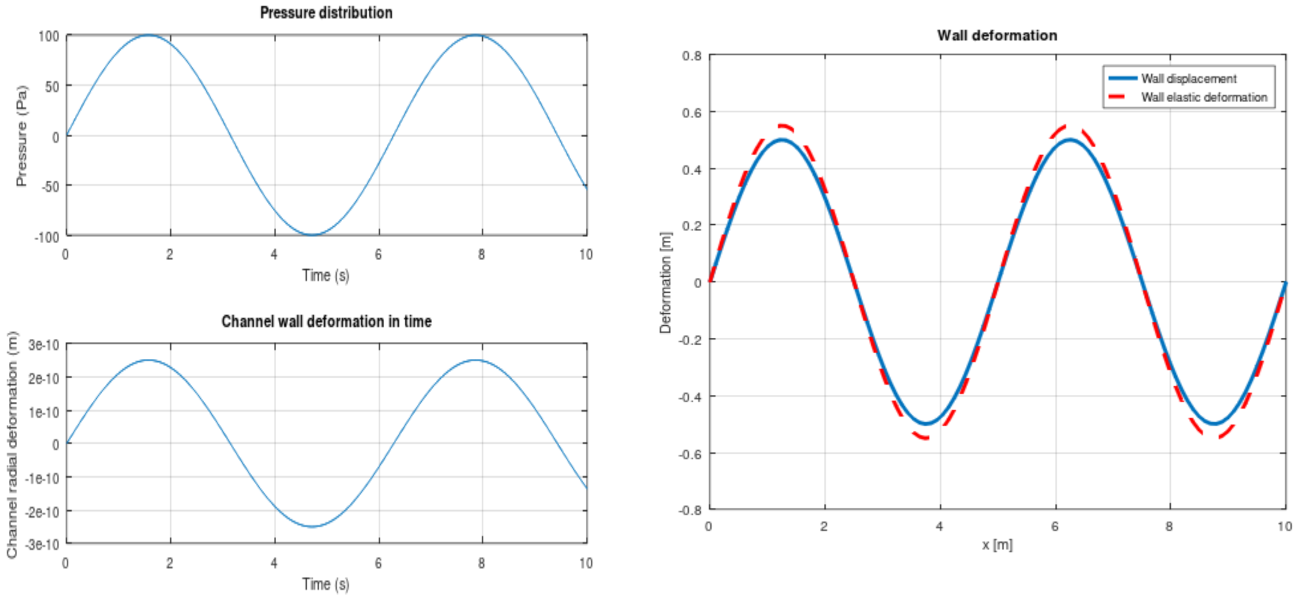

Fig. 1. The pressure distribution and channel wall time and space deformation

The numerical analysis is highlighting the proper deformation values of the channel walls depending on the change in the viscosity values of the fluid.
It must be taken into account that the channel in which the peristaltic flow takes place undergo elastic deformations under the action of the internal pressure generated by the flow.
The channel in which the flow takes place is of circular cross-section, aiming to establish the flow rate as a function of viscosity and to determine the pressure value necessary to maintain the flow.
As the viscosity of the fluid increases, the pressure required to maintain flow will increase and this will cause greater channel deformation, an effect that is visualized by a change in the channel radius.
The internal pressure is calculated as a function of viscosity and further the channel deformation limit will be established as a function of this pressure. The basic parameters of the numerical model are presented in Table 1.

Table 1. Numerical parameters

Crt. No.	Parameter	Value
1.	Channel radius	5.00 (mm)
2.	Channel length	1 (m)
3.	Volumetric flow rate	5 ml/s
4.	Elasticity modulus of the channel	1e7 (Pa)
5.	Fluid viscosity range	0.001-0.003 mPa.s

The channel deformation is calculated assuming that the channel in which the flow occurs behaves as an elastic material, involving the modulus of elasticity.
The results obtained are presented in the form of diagrams representing the evolution of viscosity as a function of the shear rate, then the pressure values as a function of viscosity, as well as the deformation of the vessel radius as a function of viscosity, highlighting how the increase in viscosity determines a greater deformation of the channel.
It can be seen that as the viscosity increases, the required pressure necessary to maintain a constant flow rate will be higher in value.

The deformation of the channel will increase as the internal pressure rises, which reflects the fact that increased viscosity (especially at low shear) will lead to a higher pressure and, implicitly, to a more significant deformation of the channel.
This basic model demonstrates how an increase in fluid viscosity values can lead to higher internal pressure and implicitly, to a deformation of the channel wall, with results presented in figure 2.

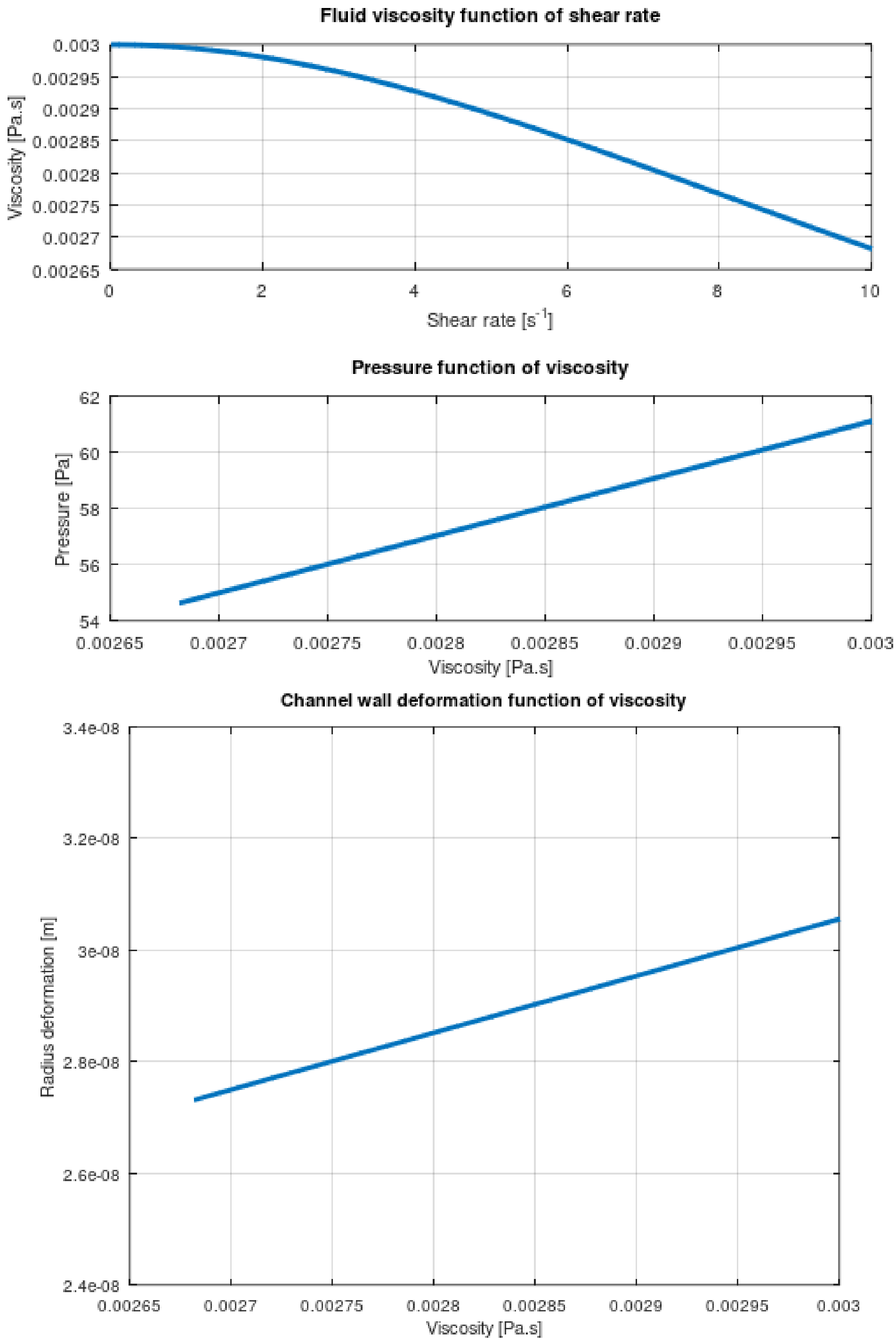

Fig. 2. The channel wall elastic deformation function on viscosity and pressure effects

In reality, blood vessels exhibit a more complex behaviour, being subjected to compression and tension effects that require more advanced numerical models, but this model can provide a basic understanding of the fluid viscosity effects on circulation.
Also, the modelling process for peristaltic flow in turbulent regime requires a combination of Navier-Stokes equations, turbulence models and numerical methods, essential for understanding complex

phenomena in biology (e.g. turbulent flow in large blood vessels) and technology (industrial peristaltic pumps).

4. Conclusions

Numerical modeling of peristaltic flow is a powerful method to understand the behavior of complex fluids, such as blood circulation in biological systems and further in the engineering field.
Specific approaches have been used over time and the conclusions and perspectives are based on theoretical, practical and numerical aspects.
The method of modeling peristaltic flow based on fluid viscosity, shear rate and channel wall deformation provides a realistic perspective on fluid transport in biological systems, such as blood circulation.
The influence of viscosity and rheological behavior of the fluid is highlighted by non-Newtonian models, which capture well the variation of viscosity as a function of shear rate.
In regions with a high velocity gradient (near the walls), the apparent viscosity decreases, which improves transport, while in the center of the vessel, where the shear rate is low, the increase in viscosity leads to stagnant zones.
Blood, as a non-Newtonian fluid, exhibits shear-thinning behavior (decrease in viscosity at high shear rates), which optimizes transport in large arteries and numerical models that include shear-dependent viscosity better predict the real fluid dynamics.
The shear stress distribution shows that the maximum shear stress occurs at the channel walls and significantly influences their deformation.
In the central regions of the vessel, the stress is low, which creates slow-flowing zones, favorable for the sedimentation of particles (e.g., blood cells).
High shear rate prevents flow stagnation and optimizes the exchange of substances, while low shear rate can favor the formation of thrombi or the sedimentation of particles in slow-flow regions.
The impact of vessel wall deformation shows the fluid-structure interaction mode. The flexible vessel walls play a crucial role in generating peristaltic flow, being responsible for inducing pressure waves and the wall deformation introduces complex phenomena such as the propagation of elastic waves and the variation of the cross-section with time.
The amplitude and frequency of the waves determine the velocity and volume of the transported fluid flow. Stiffer walls reduce the transport efficiency, while very flexible walls can induce flow instabilities.
In channels with deformable walls, the flow is significantly influenced by the interaction between the shear stress and the elastic properties of the walls. Large deformations create separation or reflux zones, reducing the transport efficiency.
Numerical results show that peristaltic flow models integrate shear rate-dependent viscosity, shear stress, wall deformation and provide a realistic description of fluid flow.
Practical applications of flow modeling are particularly important in the medical field, where models can be used to analyze the efficiency of peristaltic pumps in medical devices or to understand thrombus formation in blood vessels and further in the engineering field, where complex fluid transport systems in industrial environments benefit from such models for flow optimization.
The numerical model of peristaltic flow based on viscosity, shear rate and wall deformation provides an advanced understanding of complex fluid transport. This reveals critical interactions between fluid properties, wall mechanics and flow conditions.
The applications of these models are vast, ranging from understanding biological processes to optimizing technological fluid transport systems.

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