Considerations about Vibratory Processes of Hydrostatic Systems with Rotary Engines

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Abstract: *This paper examines transient regimes in mobile technological equipment that initiate oscillatory processes within hydrostatic systems driven by rotary engines. It provides a detailed analysis of the excitation sources responsible for these dynamic behaviors, particularly within hydraulic circuits that operate the working tool. The study highlights the dynamic characteristics of these systems, including pressure resonance observed during rapid acceleration (when the pump shaft's angular velocity approaches the system's natural frequency), command instability, and the resulting effects on the motion precision of the working tool. Additionally, the conditions for the occurrence of magnitude and phase resonances in the driving pressure are evaluated.*

Keywords: *Hydrostatic system, working regime, vibrations, rotary engine, analysis*

1. Introduction

As known, the dynamic instability phenomenon in hydrostatically driven equipment manifests through sudden variations in hydraulic pressure without an obvious cause. This instability often occurs in new equipment when the hydrostatic drive system has not been thoroughly analyzed for dynamic behavior. The instant pressure variations are linked to sudden load increases on the work components, which may suggest the presence of dynamic phenomena like mechanical resonances [1,2]. In this context, the volumetric pump of the hydraulic system acts as a source of harmonic perturbations. The pulsations in the pump's output flow can lead to pressure fluctuations, which resemble resonance phenomena seen in mechanical systems with harmonic excitations. Some studies [3,4] have identified similarities between the behavior of rotating mechanical systems with viscous damping and hydraulic systems with rotary motors.
In the field of hydraulic oscillation research, Wylie [5,6] made significant contributions by advancing the impedance method for determining hydraulic resonance within systems. In 1970, Chaudhry [7] introduced an alternative approach to predict the frequency response of hydraulic systems, while also working to refine and organize oscillatory flow equations using the transfer matrix method. Suo and Wylie [8] further explored hydraulic oscillations in pressurized systems, offering additional improvements to hydraulic resonance analysis. To address transient events in complex piping systems, Kim [9] applied the impedance matrix method, incorporating initial conditions and time-history data.
In this paper the author aims to highlight the conditions under which such resonance phenomena in hydrostatic systems arise. When resonance phenomenon occurs, it results in a pressure increase beyond the system's stable state, often accompanied by an increase in noise, unintended activation of hydraulic safety valves, and dynamic stress on system components, potentially affecting their durability and performance.

2. Theoretical background

Let's proposed a hydrostatic drive system that consists of rotative motor, which acts the working tool of a technological equipment, like vibratory roller (Figure 1). Thus, during the startup process of a vibratory roller, fluid flow in the pipelines becomes obstructed, causing an impact on the hydraulic vibration system. The load applied to the motor creates resistance to the hydraulic oil flow, resulting in a reduction in motor speed. According to fluid dynamics theory, a decrease in fluid flow rate inevitably leads to an increase in system pressure. The greater the drop-in flow rate, the higher the resulting pressure rise. As the total energy of the fluid is converted into pressure energy, the hydraulic system experiences rapid pressure fluctuations, leading to the occurrence of instantaneous high pressure and hydraulic shocks.

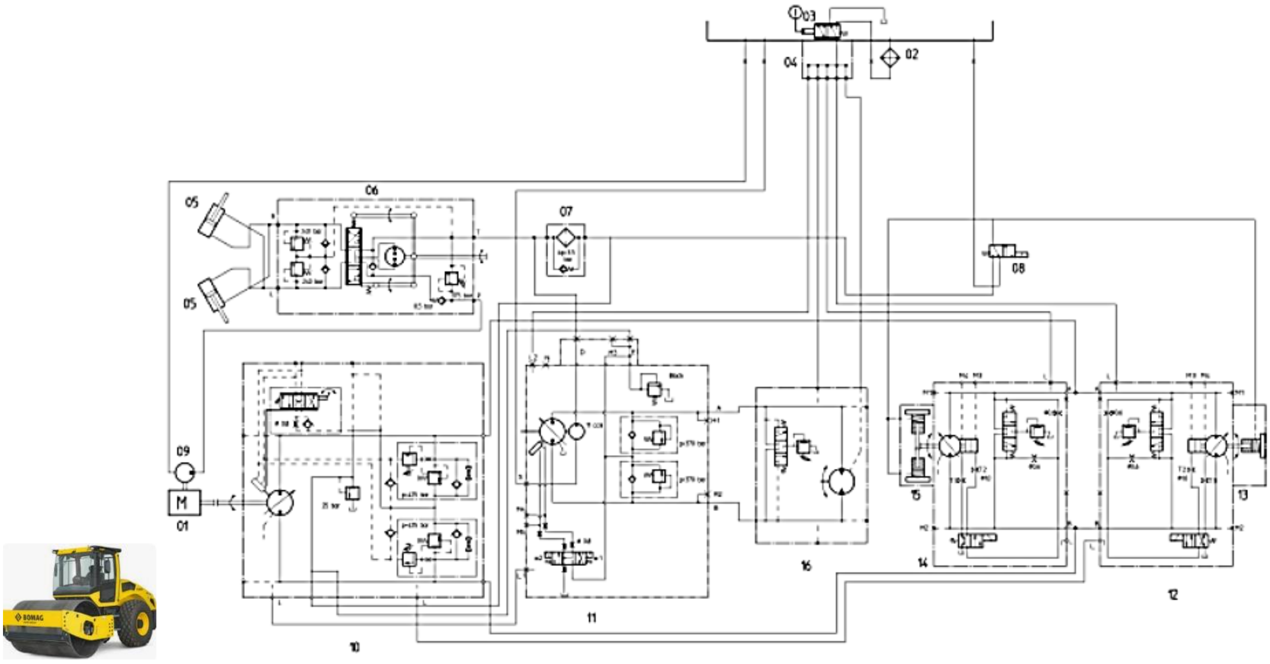

Fig. 1. Single drum vibratory roller hydraulic circuit (Bomag Bw 213)

For all types of hydraulic circuits (both open and closed systems), the main components include a hydraulic pump, control valves, actuators (such as cylinders or rotary motors), reservoirs, filters, and piping or hoses, as shown in Figure 2. These components work together to ensure the efficient transfer of hydraulic energy to perform various tasks. This diagram will form the basis for the development of the study in this paper.

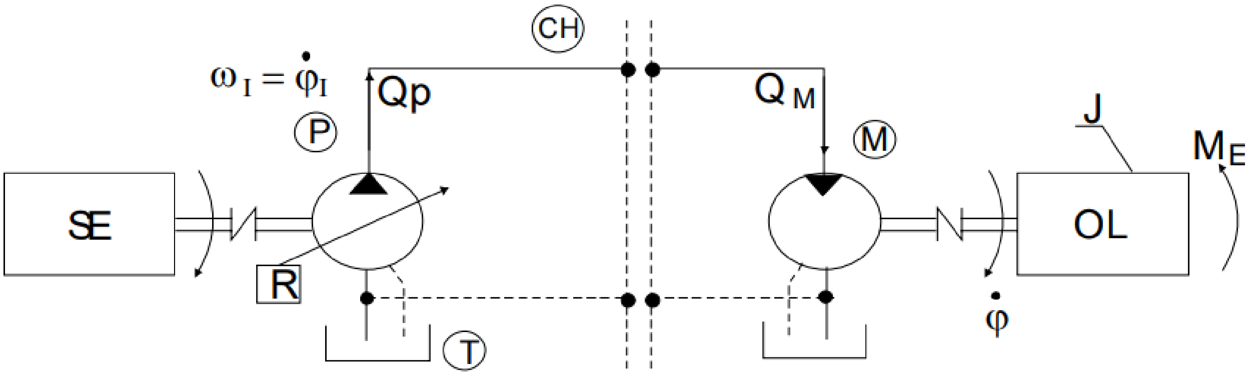

Fig. 2. The drive system model:
SE - energy source; P - pump; CH - hydraulic circuit; M – hydraulic motor; OL - rotative working tool;
R - pump debit regulator; T - hydraulic tank.

The expression for evaluating the output flow (Q_P) from the hydraulic circuit is [10]

$$Q_P = \frac{V_p \omega_I}{2\pi} = \frac{V_{OM} \omega_E}{2\pi} + \alpha_{MR} p + \beta_{MR} \dot{p}, \qquad (1)$$

where V_{OM} represents the volume of the hydraulic agent content in the rotative engine, ω_E is the rotational speed at the axle of the hydrostatic engine (for the working element), α_{MR} is the coefficient of the volumetric losses of the hydraulic agent from the system (proportional to the pressure p), and β_{MR} is the coefficient that define the hydraulic capacity of the circuit (proportional to the derivative of pressure p with respect to time).

In the circuit, the pump *P* and the hydraulic motor *M* can be considered, of the constructive type with axial pistons, with an inclined block or in disc construction. If the aim is to highlight the effects of the pulsation of the flow rate of the pump Q_P for a set of *m* pistons being in the pumping action, it can be evaluated with expression:

$$Q_P = 2rA_p sin\alpha \sum_{i=1}^{m} sin\varphi_i, \tag{2}$$

where α is the angle of pump block or disk, *r* is the disposition radius of the pump pistons, A_p is the piston area, and φ_i represents the angular motion at the spindle pump. In typical cases of axial piston pumps with seven pistons (commonly used in the drive circuits of vibratory rollers), the pump flow rate can be expressed as:

$$Q_p = \frac{q_0 \omega_I}{2\pi} \sin(\omega_I t + \varphi_0), \tag{3}$$

where

$$q_0 = 2rA_p sin\alpha \cdot \frac{sin\frac{m\pi}{7}}{sin\frac{\pi}{7}}, \tag{4}$$

and

$$\varphi_0 = \frac{(m-1)\pi}{7}. \tag{5}$$

Therefore, the equation of dynamic balance at the motor shaft, deduced by the kineto-static method, is [10]

$$M_I = J\ddot{\varphi} = \frac{V_{0M}\dot{\varphi}}{2\pi} - \chi_{MR}\dot{\varphi} - M_E, \tag{6}$$

where J represents the mechanical moment of inertia of all movable elements (reduced to the axis of the rotary engine), φ is the angular motion at the engine shaft, χ_{MR} is the adjustment factor of the cylinder volume of the motor, and M_E is the resistance moment developed to the working tool by the external medium (which must be kept constant regardless of the roller's response to the continuous variation in the degree of soil compaction). To simplify the mathematical model, moment losses caused by dynamic flow forces, Coulomb friction forces, and aerodynamic friction forces, etc., are neglected.

3. Mathematical modeling

Processing the terms of the Eqns. (1), (3) and (6) leads to the mathematical modelling of the hydrostatic circuit with rotary motor.

$$\ddot{p} + a_1\dot{p} + a_2 p = a_3 M_E + a_4 \omega_I^2 cos(\omega_I t + \varphi_0) + a_5 \omega_I sin(\omega_I + \varphi_0), \tag{7}$$

where $a_1 = \frac{\alpha_{MR}}{\beta_{MR}} + \frac{\chi_{MR}}{J}$, $a_2 = \frac{1}{J\beta_{MR}}\left(\frac{V_{0M}^2}{4\pi^2} + \alpha_{MR}\chi_{MR}\right)$, $a_3 = \frac{V_{0M}}{2\pi J\beta_{MR}}$, $a_4 = \frac{q_0}{2\pi\beta_{MR}}$, $a_5 = \frac{\chi_{MR} q_0}{2\pi J\beta_{MR}}$.

By incorporating the expressions for the dynamic factors and simplifying the terms on the right-hand side of the preceding equation, the resulting expression is obtained as follows [11]

$$\ddot{p} + 2\zeta_{MR}\omega_{MR}\dot{p} + \omega_{MR}^2 p = H_{MR} + \Pi_{MR} sin(\omega_I t + \Phi), \tag{8}$$

with the description of the involved parameters in the Table 1.

In Eq. (8) it is considered transient flows that represents the intermediate flow conditions when the flow is changed from one steady state to another. Thus, depending upon the characteristics of the system and of the excitation, a disturbance in a piping system may be amplified with time instead of decaying and may result in severe pressure and flow oscillations (phenomenon called resonance). In addition, the periodic dynamic force of the excitation generated by the vibration system incorporated into the roller's drum causes the pressure and flow in the entire system to oscillate at the period of the excitation, being named steady-oscillatory flow.

Table 1: Parameters identification from Eq. (8)

Description of the parameters	Symbol	Formula
Natural pulsation of the hydro-mechanic system	ω_{MR}	$\frac{V_{oM}}{2\pi}\sqrt{\frac{2}{JC_h}}\,[s^{-1}]$
Damping factor	ζ_{MR}	$\sqrt{\frac{J}{C_h}}\frac{2\alpha_{MR}+\chi_{MR}}{\frac{V_{0M}}{\pi}\sqrt{2}}\ [-]$
Disturbing factor due to the resistant torque at the working tool	H_{MR}	$\frac{V_{0M}}{2\pi}\frac{M_E}{J}\frac{2}{C_h}\ [N/m^2s^2]$
Excitation magnitude	Π_{MR}	$\frac{q_0\omega_I^2}{\pi C_h}\sqrt{1+\left(\frac{\chi_{MR}}{J\omega_I}\right)^2}\ [N/m^2s^2]$
Initial phase of the excitation	ϕ	$\phi=\varphi_0+arctg\frac{J\omega_I}{\chi_{MR}}\ [rad]$

An analytical solution of the differential equation (8) has the expression

$$p=p_1+p_2=p_0e^{-\zeta_{MR}\omega_{MR}t}sin\left(\omega_{MR}\sqrt{1-\zeta_{MR}}t\right)+p_s+p_0sin(\omega_It+\phi_0) \tag{9}$$

but, for the phenomenon analyzed in this paper presents interest only pressure that described the behavior of the working steady state, and then we used the analytical solution as the next form

$$p=p_s+p_r\sin(\omega_It+\phi_0) \tag{10}$$

where p_s is the static state system pressure and, respectively, p_r is the overpressure due to the resonance phenomenon.

$$p_s=\frac{2\pi M_E}{V_{0M}}, \tag{11}$$

$$p_r=\frac{q_0}{\pi C_h}\frac{\mu^2\sqrt{1+\varepsilon^2}}{\sqrt{(1-\mu^2)^2+(2\zeta_{MR}\mu)^2}}. \tag{12}$$

We denote the relative damping of the hydro-mechanic system as $\varepsilon=\frac{\chi_{MR}}{J\omega_I}$ and the initial phase of the pressure as

$$\phi_0=arctg\frac{tg\varphi_0[\varepsilon(1-\mu^2)+2\zeta_{MR}\mu]+(1-\mu^2)-2\varepsilon\zeta_{MR}\mu}{\varepsilon(1-\mu^2)+2\zeta_{MR}\mu tg\varphi_0[(1-\mu^2)-2\varepsilon\zeta_{MR}\mu]}. \tag{13}$$

4. Numerical scenarios simulation

The pressure resonance phenomenon, as highlighted by Eq. (12), corresponds to a similar phenomenon observed in mechanical systems. It is characterized by a sudden increase in pressure beyond the steady-state working value when the angular speed of the pump spindle matches the natural pulsation of the hydro-mechanical system, respectively: $\omega_I=\omega_{MR}$ when $\mu=1$ and $\zeta_{MR}=1$.
The pressure amplification is incorporated into the evidence through the magnitude factor

$$\Omega_p=\frac{p_r}{p_0}=\frac{\mu^2\sqrt{1+\varepsilon^2}}{\sqrt{(1-\mu^2)^2+4\zeta_{MR}^2\mu^2}}. \tag{14}$$

where $p_0=q_0/(\pi C_h)$ represents the static pressure developed by a single piston in the pump.

Since the denominator in Eqn. (13) is the sum of squares, the magnitude factor p_1 remains finite for any value of the relative pulsation μ. Pressure magnitude resonance occurs when the Ω_p factor reaches its maximum value. The resonance pulsation value is determined by nullifying the derivative of the p_1 magnitude factor with respect to μ, leading to the following result:

$$\omega_I = \omega_{MR} \frac{1}{\sqrt{1 - 2\zeta_{MR}^2}} \text{ for } \zeta < \frac{1}{\sqrt{2}}. \tag{15}$$

In this case, the magnitude factor of the pressure resonance is

$$\Omega_{prez} = \frac{1}{2\zeta_{MR}} \sqrt{\frac{1+\varepsilon^2}{1 - 2\zeta_{MR}^2}}. \tag{16}$$

For μ=1, as derived from Eqn. (13), the phase resonance can be defined as

$$\phi_0 = arctg \frac{a_0 - \varepsilon}{1 + a_0\varepsilon}, \tag{17}$$

independent of the damping factor ζ_{MR}, but influenced by the relative damping ε and the a_o factor. Figures 3 to 5 illustrate the variation laws for the p_1 magnitude factor, the magnitude factor at resonance, and the phase resonance, all corresponding to different values of the specific parameters: $\mu = 0 - 3$; $\varepsilon = 0 - 1{,}5$; $\zeta_{MR} = 0 - 0.5$; $a_0 = arctg(5\pi/7)$.

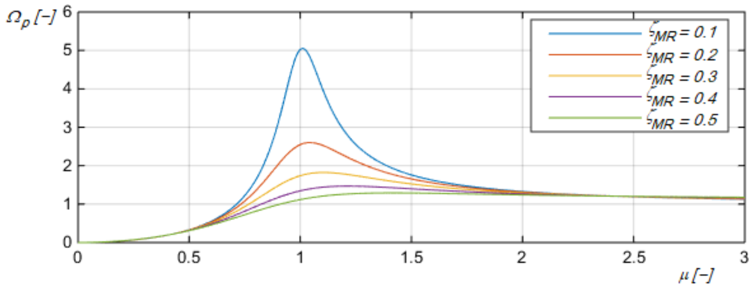

Fig. 3. The variation of $\Omega_{prez} - \mu$

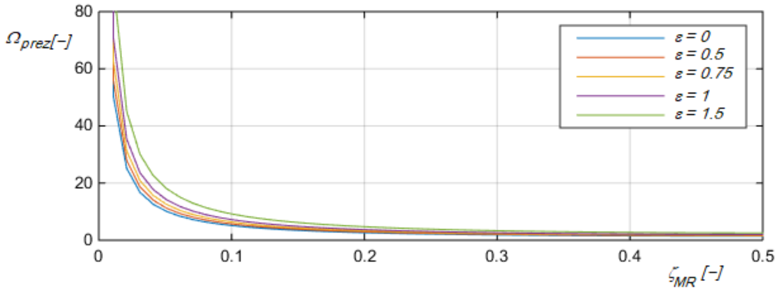

Fig. 4. The variation of $\Omega_{prez} - \zeta_{MR}$

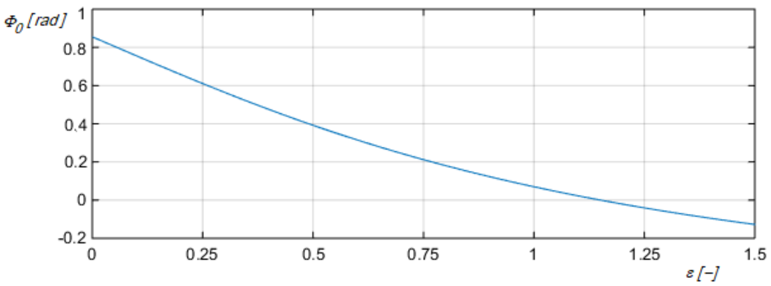

Fig. 5. The variation of $\phi_0 - \varepsilon$.

5. Conclusions

Based on the analysis of the "pressure resonance phenomenon" in hydraulic driving systems with rotary motors, the following conclusions can be drawn:

a) The observed phenomenon highlights the functional instability of hydraulic driving systems with rotary motors, particularly in predominant dynamic operating states, such as movement-driving systems for technological equipment and working tools. This instability arises due to variations in the hydraulic pump flow of the system. The primary source of this instability cannot be effectively identified using conventional methods provided by automated system analysis theory, which are commonly applied for stability evaluations of such driving systems.
b) The analysis reveals that the system's pressure gain factor can exceed twice the value of the static pressure for typical damping factor values (ζ_{MR}: 0.1 to 0.3), which are common in standard hydraulic driving system components (see Figure 3). In a system operating with stable pressure values of 250–300 bar, the resonance phenomenon can briefly generate peak pressure values ranging from 750 to 1500 bar. These extreme pressure levels pose significant risks, leading to overloading of hydraulic components, reduced lifespan and durability, and ultimately, premature failure of the hydraulic system components.
c) Another effect of the analyzed phenomenon is the increased acoustic level of the pump. During very short time intervals (0.01–0.02 seconds), the pump operates under pronounced overloads, approximately 3 to 4 times the nominal load. During these periods, the system's overload protection elements are activated, further contributing to the rise in noise levels.
d) The resonance gain factor (Ω_{prez}), shown in Figure 4, representing the maximum pressure magnitude of the system, can easily reach values 20 to 50 times greater than the static pressure for damping factors below 0.1. These values show slight variation with the relative damping (ε) of the hydro-mechanical system. This highlights the resonance phenomenon, bringing it into the typical operating range of a hydraulic driving system.
e) The phase resonance illustrated in Figure 5 are minimal impact on the operating state of the system.

The study concludes by highlighting the instability phenomenon in hydraulic driving systems caused by flow variations within system components. Effective methods to reduce or eliminate this harmful phenomenon will be proposed in future studies conducted by the author.

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