Kinematic Analysis and Geometric Synthesis of Elevator Mechanisms with Non-Articulated X-Bars Mounted on Mobile Robots

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Abstract: X-shaped (scissor) articulated bar lifts take up less space in areas of use, such as warehouses. By folding, when not in use, these scissor lifts lower to their minimum level. The kinematic analysis of the mechanisms of these elevators involves the method of analytical calculation of the displacements of the main characteristic points of the dyadic chains. In the case of elevators with non-articulated bars in X, a problem of geometric synthesis of the quadrilateral deltoid type mechanism is presented and solved.

Keywords: Mobile robots, kinematic analysis, elevator mechanisms, kinematic chain

1. Introduction

The single-level X-joint lifting mechanism (fig. 1) operates in the vertical plane and is actuated by a piston-type actuator (*a*) and an oscillating cylinder (*b*).



Fig. 1. Kinematic diagram of the X-bar elevator mechanism, upper / open position (*a*) and lower / closed position (*b*)

The technological resistance force acts as the weight of the manipulated object (fig. 1), and the driving force acts in the cylinder b on the piston a, in the direction AOA (fig. 1a).

The construction diagram shown in the color picture (fig. 2) shows an elevator with X-shaped articulated bars, which is placed on a 4-wheel mobile robot cart. This elevator has 2 levels and uses linear guides for translational couplings at the base and at the level of the upper platform. In other constructive variants of elevators (fig. 3) both bars are articulated at the base (the chassis of the mobile robot), but are not connected to each other, as in the previous variants (fig. 1 and 2).



Fig. 2. Construction diagram (picture) of an X elevator with two levels

As a rule, the actuator of this type of elevator (fig. 3) is mechanical with a screw and nut, being articulated at the upper ends of the two non-articulated X-bars.



Fig. 3. Constructive diagram (a) and kinematic diagrams (b) of two-level non-articulated X-bar elevator.

2. Kinematic analysis of X-bar elevator mechanisms

2.1. Lifting mechanisms with articulated bars in X

For comparison, let's first analyze the plane elevator with bars articulated in X (fig. 4), where the first two bars have equal lengths ($A_0B=BC$). The kinematic diagram (fig. 4a) shows a closed contour in the form of an isosceles triangle A_0BC with a variable base A_0C , which is obtained with the help of a roto-translational kinematic couple C (2,0) [1, 2, 3, 4].



Fig. 4. Kinematic diagram of the mechanism with bars 1 and 2 articulated in X

By doubling the length of bars 1 ($A_0B=BE=I_1$) and 2 ($BC=BD=I_2$) the letter X is obtained, thus the points D and E are kept at the same level. Bar 3 represents the horizontal platform, which takes the weight of the materials handled in the warehouse, being articulated at D at bar 2 and resting at point E at bar 1 (fig. 4). With the movement of piston a in cylinder b (fig. 4a), bar 1 rotates in the trigonometric sense, while bar 2 rotates with the angle ϕ_2 (measured clockwise). Thus, the bar 1 rotates with respect to the fixed joint A_0 in the direct direction from ϕ_{1min} (lower position) to ϕ_{1max} (upper position).

For the X-bar elevator with a single level (fig. 4a), the height of the platform 3 is deduced from the formula

$$h_3 = 2l_1 \sin \phi_1 = 2l_2 \sin \phi_2; \quad l_1 = l_2$$
 (2.1)

where the notations where used (fig. 4): $A_0B = BE = l_1$; $BC = CD = l_2$.

It is mentioned that the maximum value of the angle ϕ_1 (ϕ_{1max}) is imposed by the limitation of the displacement deviation of point B (x_B) from the vertical (fig. 4*b*).

For $A_0A = s_0$ is obtained $A_0C = x_{Cmax}$, and for ϕ_{1max} and $A_0A = s_{max}$ results $A_0C = x_{Cmin}$.

It is observed that the maximum displacement of piston a in cylinder b (fig. 4a) is $h=s_{max} - s_0$.

2.2. Kinematic analysis of elevators with non-articulated X bars

In these elevators (fig. 5a) both X bars are articulated at the base, but they are not articulated between them, which structurally distinguishes them from the elevators that were previously analyzed (fig. 4).



Fig. 5. Kinematic diagram of the elevator with non-articulated bars in X

As a topological structure, these planar mechanisms of elevators with non-articulated bars in X are composed of an articulated quadrilateral rocker - rocker type (fig. 5) with the two rockers (1 and 3) crossed in X, without being articulated between them.

The planar articulated quadrilateral is the closed kinematic chain $A_0ABB_0A_0$, to whom (fig. 5b): $A_0A=AB$ and $BB_0=A_0B_0$, respectively $I_1=I_2$ and $I_3=I_0$.

In the upper part of the kinematic scheme of the lifting mechanism is the platform 4 which takes the force of the weight of the handled object.

The mobility of the mechanism is verified by the fact that bar 4 is equivalent to a dyadic chain of type (R, R+T), with a joint in B (3,4) and a roto-translational couple in C (1,4).

Kinematic analysis of the quadrilateral mechanism involves writing the closing vector equation of the closed loop (fig. 5*b*):

$$\overrightarrow{A_0A} + \overrightarrow{AB} = \overrightarrow{A_0B_0} + \overrightarrow{B_0B}$$
(2.2)

or, using the mentioned notations:

$$\bar{l}_1 + \bar{l}_2 = \bar{l}_0 + \bar{l}_3$$
 respectively $\bar{l}_3 - \bar{l}_2 = \bar{l}_1 - \bar{l}_0$ (2.3)

The scalar projection equations are obtained from (2.3) and have the expressions:

$$l_{3}\cos(\pi - \phi_{3}) - l_{2}\cos\phi_{2} = l_{1}\cos\phi_{1} - l_{0}$$

$$l_{3}\sin(\pi - \phi_{3}) - l_{2}\sin\phi_{2} = l_{1}\sin\phi_{1}$$
(2.4)

Assuming that the angle is known (as an independent parameter), the system (2.4) consists of two nonlinear scalar equations with two unknowns (ϕ_2, ϕ_3).

To solve this system of two trigonometric equations (in *sin* and *cos*) one of the unknowns is isolated (e.g. ϕ_2), obtaining the following expressions:

$$l_{3}\cos(\pi - \phi_{3}) - l_{1}\cos\phi_{1} + l_{0} = l_{2}\cos\phi_{2}$$

$$l_{3}\sin(\pi - \phi_{3}) - l_{1}\sin\phi_{1} = l_{2}\sin\phi_{2}$$
(2.5)

Squaring the two equations and adding them yields the equation with one unknown ϕ_3

$$[l_3\cos(\pi - \phi_3) - l_1\cos\phi_1 + l_0]^2 + [l_3\sin(\pi - \phi_3) - l_1\sin\phi_1]^2 = l_2^2$$
(2.6)

or, more conveniently:

$$[-l_3\cos\phi_3 - l_1\cos\phi_1 + l_0]^2 + [l_3\sin\phi_3 - l_1\sin\phi_1]^2 = l_2^2$$
(2.7)

A trigonometric equation of the known form was obtained [1, 2]

$$a_3 \sin \phi_3 + b_3 \cos \phi_3 + c_3 = 0 \tag{2.8}$$

In equation (2.8) the three coefficients have the expressions:

$$a_{3} = 2l_{1}l_{3}\sin\phi_{1}; \quad b_{3} = 2l_{3}(l_{0} - l_{1}\cos\phi_{1}); c_{3} = 2l_{0}l_{1}\cos\phi_{1} + l_{2}^{2} - l_{1}^{2} - l_{3}^{2} - l_{0}^{2}$$
(2.9a)

To solve equation (2.8) the trigonometric functions are replaced sin and cos with tg function:

$$\sin\phi_3 = \frac{2tg(\phi_3/2)}{1+tg^2(\phi_3/2)}; \quad \cos\phi_3 = \frac{1-tg^2(\phi_3/2)}{1+tg^2(\phi_3/2)}$$
(2.9b)

Using the notation $tg(\phi_3/2) = t$, the equation (2.8) is written

$$a_3 \frac{2t}{1+t^2} + b_3 \frac{1-t^2}{1+t^2} + c_3 = 0$$
(2.9c)

This is an algebraic equation of degree 2, which results in the form

$$(b_3 - c_3) \cdot t^2 - 2a_3 \cdot t - (b_3 + c_3) = 0$$
(2.9d)

The two solutions of equation (2.9d) are expressed by the formula

$$t = \frac{a_3 \pm \sqrt{a_3^2 + b_3^2 - c_3^2}}{b_3 - c_3} \tag{2.9e}$$

Taking into account the notation introduced in equation (2.9c), the expression for the ϕ_3 angle is deduced:

$$\phi_3 = 2arctg(t) = 2arctg\left[\frac{a_3 \pm \sqrt{a_3^2 + b_3^2 - c_3^2}}{b_3 - c_3}\right]$$
(2.9f)

It is noted that only one of the two solutions (2.9f) corresponds to reality ($\phi_3 < \pi/2$). As a numerical example, given the lengths of the quadrilateral (fig. 5*b*) for $\phi_1 = 30^\circ$, is deduced the ϕ_3 angle, first calculating the coefficients a_3 , b_3 , c_3 defined by equations (2.9a):

 $a_{3} = 2l_{1}l_{3}\sin\phi_{1} = 2 \cdot 0.35 \cdot 1 \cdot \sin 30^{\circ} = 0.35$ $b_{3} = 2l_{3}(l_{0} - l_{1}\cos\phi_{1}) = 2 \cdot 1 \cdot (1 - 0.35 \cdot \cos 30^{\circ}) = 2 - 0.35\sqrt{3} = 1.4$ $c_{3} = 2l_{0}l_{1}\cos\phi_{1} + l_{2}^{2} - l_{1}^{2} - l_{3}^{2} - l_{0}^{2} = 2 \cdot 1 \cdot 0.35 \cdot 0.5\sqrt{3} + 0.35^{2} - 0.35^{2} - 1^{2} - 1^{2} = -1.4$

With these numerical values of the variable coefficients entered in the formula (1.9f) is obtained:

$$\phi_3 = 2 \operatorname{arctg} \frac{0.7}{2.8} = 2 \operatorname{arctg} (0.25) = 2 \cdot 14^\circ = 28^\circ$$
 (2.9k)

The difference between $\phi_1 = 30^\circ$ and $\phi_3 = 28^\circ$ is 2°, which is convenient.

3. Structural and kinematic analysis of non-articulated bar multi-storey elevator in X

This overloaded elevator in X (fig. 6*a*) is obtained from the single-level kinematic scheme (fig. 5) by amplifying it with two dyadic chains LD (4,5) and LD (6,7).





It should be noted that bar 7 is the platform on which the object manipulated at a certain level is placed to be placed in another area of the warehouse.

The mobility of the articulated plane mechanism with multi-storey structure (fig. 6) is checked immediately with the help of the formula [1, 2]:

$$M_3 = 3n - 2C_5 - C_4 \tag{3.1}$$

where the following parameters are identified (fig. 6):

n = 7kinematic elements;

 $C_5 = 10$ 5th class kinematic couples (with 5 restrictions);

 $C_4 = 0$ 4th class kinematic couples (with 4 restrictions);

By substituting these numerical values in formula (3.1) it is deduced:

$$M_3 = 3 \cdot 7 - 2 \cdot 10 - 0 = 1 \tag{3.2}$$

The obtained result shows that the multi-storey elevator (fig. 6a) can be operated with a single driving element, for example by bar 1 positioned by the angle ϕ_1 .

The actuator can be with a pneumatic cylinder (articulated at base 0 and bar 1), but it can also be operated with a mechanical screw actuator that is articulated at points B and C (fig. 6*b*).

In the last variant of the lifting mechanism with mechanical screw actuator (fig. 6*b*), its mobility is deduced with the general formula [2] for complex mechanisms [5, 6]:

$$M_b = \sum_{1}^{5} mC_m - \sum_{2}^{6} rN_r \tag{3.3}$$

The following parameters are identified in the mobility formula (3.3):

 $m = 1, 2, \dots, 5$ represents the mobility of a kinematic couple;

 C_m is the number of mobile kinematic couples m;

r = 2,3,...,6 is the rank of an independent closed kinematic contour;

 N_r is the number of independent closed kinematic contours.

For the mechanism of the complex multi-story elevator (fig. 6b) the following numerical values are identified: m = 1 corresponds to planar joints (0,1), (0,3), (1,2), (2,3), (3,4), (4,5), (1,5), (1,6), (4,7), (6,7), (8,9), (1,9), (8,10), (3,10): accordingly $C_1 = 14$;

r = 3 corresponds to independent closed kinematic chains (0,1,2,3,0), (1,2,4,5), (4,5,6,7);

r = 4 is the rank of the independent closed kinematic contour (1,2,10,8,9).

Substituting these numerical values in formula (3.3) results $M_b = 1 \cdot 14 - (3 \cdot 3 + 4 \cdot 1) = 1$ Kinematic analysis of the multi-storey elevator mechanism includes the calculation of the height of the ceiling 7 according to the ϕ_1 angle(fig. 15*a*).

In the first stage of the kinematic analysis, the angle is calculated from the trigonometric equation (2.8), its value being close to that of the ϕ_1 angle.

In the second stage, we consider the dyadic chain LD(4,5) represented by the triangle BDC (fig. 7) where the coordinates of points B and C are obtained with the formulas:



Fig. 7. The kinematic diagram of the over-storey elevator

$$x_B = l_0 + l_3 \cos(\pi - \phi_3)$$

$$y_B = l_3 \sin(\pi - \phi_3)$$
(3.4)

$$\begin{aligned} x_C &= l_1 \cos \phi_1 \\ y_C &= l_1 \sin \phi_1 \end{aligned} \tag{3.5}$$

The closed vector equation of the closed contour DBC (fig. 7a) is:

$$\overrightarrow{BD} - \overrightarrow{CD} = \overrightarrow{BC} \tag{3.6}$$

The equations of the projections on the orthogonal coordinate axes are:

$$l_{4}\cos\phi_{4} - l_{5}\cos\phi_{5} = x_{C} - x_{B}$$

$$l_{4}\sin\phi_{4} - l_{5}\sin\phi_{5} = y_{C} - y_{B}$$
(3.7)

From the system of two scalar equations (3.7) one of the two unknowns is deduced, by the method of elimination ϕ_4 and ϕ_5 , from a trigonometric equation of the form (2.8):

$$a_i \sin \phi_i + b_i \cos \phi_i + c_i = 0, \quad i = 4,5$$
 (3.8)

In the third stage, the kinematics of the dyadic chain LD(6,7) from the triangle CEF is solved, where the coordinates of points C are known, from the expression (3.5) and of F belonging to bar 4 (fig. 7a):

$$x_{F} = x_{B} + l'_{4} \cos \phi_{4} y_{F} = y_{B} + l'_{4} \sin \phi_{4}$$
(3.9)

The non-linear scalar equations are written from the CEF triangle(fig. 7a):

$$l_{6}\cos\phi_{6} - l_{7}\cos\phi_{7} = x_{F} - x_{C}$$

$$l_{6}\sin\phi_{6} - l_{7}\sin\phi_{7} = y_{F} - y_{C}$$
(3.10)

This system of two equations with two unknowns is solved by eliminating one of the unknowns in turn, obtaining a trigonometric equation of the form (3.8)

$$a_i \sin \phi_i + b_i \cos \phi_i + c_i = 0, \quad i = 6,7$$
 (3.11)

If bars 4 and 6 have equal lengths the angles and are equal and it follows that bar 7 is horizontal (fig. 7*a*), so $\phi_7 = 180^\circ$. It should be emphasized that the screw actuator kinematic chain (8,9,10), which is articulated at points B and C (fig. 7*b*), it has zero mobility, being a neutral kinematic chain.

4. Geometric synthesis of the quadrilateral mechanism of the elevator with non-articulated bars in X

For this articulated quadrilateral mechanism (fig. 8) it is required to determine the geometric condition for positioning the crank and the rocker at equal angles ($\phi_1 = \phi_3$).



Fig. 8. Kinematic schemes of planar mechanisms with bars and gears

A first solution (fig. 8*a*) consists in the use of two equal toothed sectors, in external gearing, with bar 1 and bar 3 solidarized to each toothed sector.

The gear is external, so the toothed sectors rotate in the opposite direction, so the angles of rotation are equal ($\phi_1 = \phi_3$). Therefore, points B and C are located at the same height ($y_B = y_C$). The screw actuator can be used to actuate this elevator mechanism with non-articulated bars (fig. 8*c*), it being mounted at points B and C through the respective joints (fig. 9*a*).



Fig. 9. Kinematic diagrams of screw actuator assembly

Similarly proceed with the assembly of the screw actuator (4,5,6) to the articulated quadrilateral mechanism with crank 1, connecting rod 2 and rocker arm 3 (fig. 9*b*).

After mounting the screw actuator, in both versions (fig. 9), by turning the screw 4, the joints B and C come closer, when the angles ϕ_1 , ϕ_3 of bars 1 and 3 are increasing.

Similarly, if screw 4 is turned in the opposite direction, the distance between points B and C increases. so that the angles ϕ_1 and ϕ_3 are decreasing.

After assembling the screw actuator (fig. 10), a kinematic chain consisting of two dyads linked in series can be added by overlapping LD(7,8) and LD(9,10).



Fig. 10. Kinematic diagram for assembling the variants of overstory mechanisms with screw actuator

5. The geometric synthesis of the quadrilateral mechanism to achieve the condition $\phi_1 = \phi_3$

Consider the kinematic diagram (fig. 11) of an articulated quadrilateral mechanism, deltoid type, where bars 1 and 2 have equal lengths $(l_1 = l_2)$; also, the length of bar 3 (rocker) is equal to the length of fixed bar 0 $(l_3 = l_0)$.



Fig. 11. Kinematic diagram of the quadrilateral deltoid mechanism

For the sides of the articulated quadrilateral (fig. 11*a*) the following notations are used:

$$A_0B_0 = l_0; \ A_0A = l_1; \ AB = l_2; \ BB_0 = l_3$$
 (5.1)

The particular case of the analyzed concave deltoid type mechanism is considered:

$$A_0 A = AB = l_1; A_0 B_0 = BB_0 = l_3$$
(5.2a)

It should be noted that the solution with convex deltoid type quadrilateral $A_0A'BB_0$ (fig. 11a) cannot be accepted because the opposite sides A_0A' and BB_0 do not intersect.

Therefore, only the variant of the concave deltoid quadrilateral will be further analyzed, in order to carry out the synthesis of the concave quadrilateral mechanism that achieves the equality of the angles ϕ_1 and ϕ_3 .

The geometric synthesis problem of the concave deltoid quadrilateral (fig. 11*a*) refers to determining the x-ratio between the lengths of bars 1 and 3 ($x=I_1/I_3$) from the condition $\phi_1 = \phi_3$.

The vector contour of the concave articulated quadrilateral is considered (fig. 11*b*), for which the closing vector equation is:

$$\overrightarrow{A_0A} + \overrightarrow{AB} = \overrightarrow{A_0B_0} + \overrightarrow{B_0B}$$
(5.2b)

or, using the notations (5.2a), the vector equation is written

$$\vec{l}_1 + \vec{l}_2 = \vec{l}_0 + \vec{l}_3; \quad \vec{l}_3 - \vec{l}_2 = \vec{l}_1 - \vec{l}_0$$
 (5.3)

The scalar projection equations are derived as follows:

$$l_{3}\cos(\pi - \phi_{3}) - l_{2}\cos\phi_{2} = l_{1}\cos\phi_{1} - l_{0}$$

$$l_{3}\sin(\pi - \phi_{3}) - l_{2}\sin\phi_{2} = l_{1}\sin\phi_{1}$$
(5.4)

For the particular case (5.2a) the scalar equations (5.4) are written:

$$l_{3}\cos(\pi - \phi_{3}) - l_{1}\cos\phi_{2} = l_{1}\cos\phi_{1} - l_{3}$$

$$l_{3}\sin(\pi - \phi_{3}) - l_{1}\sin\phi_{2} = l_{1}\sin\phi_{1}$$
(5.5)

Substituting $l_1 = x \cdot l_3$ the equations (5.5) become:

$$-\cos\phi_3 - x \cdot \cos\phi_2 = x \cdot \cos\phi_1 - 1$$

$$\sin\phi_3 - x \cdot \sin\phi_2 = x \cdot \sin\phi_1$$
(5.6)

The angle is removed ϕ_2 isolating it on the right side of the equations:

$$1 - \cos \phi_3 - x \cdot \cos \phi_1 = x \cdot \cos \phi_2$$

$$\sin \phi_3 - x \cdot \sin \phi_1 = x \cdot \sin \phi_2$$
(5.7)

Squaring these equations, adding and equating $\phi_3 = \phi_1$, results for *x*, the expression

$$x = \frac{1 - \cos \phi_1}{\cos \phi_1 - \cos 2\phi_1} \tag{5.8}$$

It is observed that the ratio value $x = l_1/l_3$ is a non-linear function of the angle ϕ_1 , which shows that the equality of the two angles ϕ_1 and ϕ_3 it can only be achieved for one position of the mechanism. Practically, for an elevator with articulated bars (fig. 19), the sharp positioning angles of bars 1 and 3 do not exceed the value of 60°.

For $\phi_1 = 60^{\circ}$ is obtained

$$x = \frac{1 - \cos 60^{\circ}}{\cos 60^{\circ} - \cos 120^{\circ}} = \frac{1 - 1/2}{1/2 - (-1/2)} = 1/2 = 0.5$$
(5.9)

Similarly, for $\phi_1 = 45^\circ$, 30°, 20°, 10°, 5° the ratio values are deduced *x*:

$$x = 0.41; \ 0.36; \ 0.34; \ 0.337; \ 0.335$$
 (5.10)

The obtained result shows that, for ϕ_{1max} , maximum ratio variation x is

$$\Delta x = xmin_{max} \tag{5.11}$$

But, for ϕ_{1max} , observing (5.10), the maximum deviation of the ratio x decreases to the value

$$\Delta x = xmin_{max} \tag{5.12}$$

Therefore, for the last case analyzed ϕ_{1max} it can be admitted that the optimal average value of the ratio x is: x = 0.35.

6. Conclusions

The planar articulated mechanisms of the X-bar elevators are kinematically analyzed by the analytical method, highlighting the vertical movements as in goods warehouses.

For scissor [7, 8, 9] X-bar elevators, the kinematic analysis is simpler than for non-articulated X-bar elevators. For elevators with non-articulated bars in X, the kinematic study includes the identification of independent kinematic contours, the writing of the closing vector equations and the nonlinear scalar equations of projections on the two conveniently chosen axes.

The paper ends with the formulation of a geometric synthesis problem for the optimization of a quadrilateral deltoid type mechanism.

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