The Volumetric Efficiency of Water Ram

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Abstract: *In the present work the research problem related to the low and limited efficiency of hydraulic rams is addressed, for whose solution the behavior physical process of the volumetric efficiency of the multi-pulsor hydraulic ram was taken as an object of investigation, and as a field of action: The mathematical process to determine the relation of the volume of the air chamber with the volumetric efficiency of the multipulsor hydraulic ram. The modeling of the field of action is carried out with the nonlinear regression tool and the least square method, closely related to the steps to develop an investigation, finding the solution with the help of Microsoft Excel software. The objective of the investigation: To model the volumetric efficiency of the hydraulic ram in direct and inversely proportional relation with the volume of the air chamber, and of the pumping pipe respectively.*

Keywords: *Hydraulic ram, air chamber, hammers water, volumetric efficiency, pumping pipe, hydraulic friction*

1. Introduction

The hydraulic ram pump is a device that uses the energy of a waterfall to generate power and continuously raise a percentage of the incoming water to a height greater than the drop height [1]. Its operation is based on the cyclical phenomenon known as "water hammer," which is repeated indefinitely if there is no dissipating force. Previous work has already defined the influence of the relationship between the dimensions of the supply pipes and the impulse valves on the efficiency of the ram [1]; the research problem is objectively related to the low and limited efficiency of ram pumps, and the solution to improve it. The efficiency of the hydraulic ram is relatively lower than that of other pumps [2]. The documentary analysis carried out revealed that in one hundred and fifty portable document formats, only three of them carried out experiments related to the air chamber, and in a fourth we found a basis for considering the following scientific research assumption: the volume of the air chamber should be approximately equal to the volume of the pumping pipe in the practice. Mathematical modeling is carried out by following the steps of an investigation [3], where the physical process of the volumetric efficiency of the multi-drive hydraulic ram is considered as the object of research, and the mathematical process to determine the relationship between volumetric efficiency and the volume of the air chamber is considered as the field of action. The object of research is corroborated by the experimental method, and the field of action is corroborated by the mathematical method of least squares. It is one of the best methods for obtaining such formulas [4].
The objective is to model the relationship between volumetric hydraulic efficiency and the ram's air chamber volume, keeping the other parameters constant. Volumetric hydraulic efficiency is calculated from the results of the experimental work. By maintaining the same length of the pumping or discharge pipe, with the variation of the volume of the air chamber, the fraction between the volumes of the air chamber and the discharge or pumping pipe Vc/Vb also varies. This led to the scientifically groundbreaking conclusion that the volume of the air chamber is 38 percent larger than the volume of the pumping pipe. The research focuses on three key elements: the tabulation of experimental results, the mathematical analysis, and their characteristics. The Microsoft Excel 2010 software spreadsheet is used as a solution and for modeling the procedure [5].

2. Experimental study for the evaluation of the volumetric efficiency of the Multi-Drive Hydraulic Ram

The study was conducted using the experimental method in the Rio Carpintero community, Santiago de Cuba. The continuous source of water from the river feeds a reservoir [6]. The dam of more than 10 m^3 requires the use of a ram feed pipe of 0.101 and 8 m in diameter and length respectively, and the distance to the distribution reservoir is 0.050 and 145 m in diameter and length. As a more economical alternative to feed the hydraulic ram, the siphon mechanism was used (Figures 1 and 2).
The reservoir without water is shown in Figure 1.

Fig. 1. Reservoir for feeding the ram

Reservoir filled with water is shown in Figure 2.

Fig. 2. Reservoir for feeding the ram

The Water ram designed and manufactured CITA-4 supports a maximum feed flow of 0.004 m^3/s permissible in a pipe with a diameter of 0.101 m. The efficiency, calculated by the relationship between the height of the discharge and impulse pipe [7], turned out to be 5, which is equivalent to 67%. In Water ram hydraulic pumps, losses or loads in the feed and pumping pipes are another important condition to take into account for the design [8].
Calculated from the Darcy equation, [9] in particular turned out to be 0.025 and 0.8 m, respectively. The experimental work consisted of evaluating the characteristics of volumetric hydraulic efficiency. Five chambers with different volumes were tested. By maintaining the same pumping or discharge pipe, the ratio between the volumes of the air chamber and the discharge or pumping pipe, Vc/Vb, also varies with the variation in the volume of the air chamber.

2.1. Procedures for carrying out the experiments

a) Mount the 0.016 m^3 air chamber on the multi-drive ram (Figure 3).
b) Keep the impulse valves closed.
c) Operate the shut-off valve and frequently press the impulse valves until the ram pumps.
d) Measure the spill and discharge or pumping flow rates using the volumetric method. Tabulate the values.
e) Repeat steps ***a*** through ***d*** for the remaining chambers (Figures 4 and 5).
First air chamber inserted into the experiment - Figure 3.

Fig. 3. 0.016 m^3 air chamber

Third air chamber inserted in the experiment - Figure 4.

Fig. 4. 0.064 m^3 air chamber

Fifth air chamber inserted in the experiment - Figure 5.

Fig. 5. 0.130 m^3 air chamber

The results of the experiments, carried out under real conditions on a test bench for hydraulic rams in the Rio Carpintero community of the Santiago de Cuba municipality, are shown in Table 1.

Table1: Behavior of the air chamber volume and volumetric efficiency

V_c (m³)	V_b(m³)	q (10^{-3}m³/s)	Q+q(10^{-3}m³/s)	V_c/V_b	η_v %
0.016	0.058	0.04	3.34	0.27	1.1
0.032	0.058	0.05	3.35	0.55	1.4
0.060	0.058	0.065	3.365	1.03	1.9
0.105	0.058	0.06	3.36	1.8	1.7
0.130	0.058	0.05	3.35	2.24	1.4

Source: Author

Nomenclature
Vc - Volume of the air chamber (m³)
Vb –Volume of the pumping line (m³)
q - Pumping flow rate (m³/s)
Q+q - Feed flow rate (m³/s)
Vc/Vb - Dimensionless volume ratio
Ƞv - Volumetric efficiency (%)

Economic and site conditions allowed for the installation of the ram and the measurement of parameters - The pumping line volume includes two sections of pipe with restricted diameters, which increases losses (Table 2).

Table 2: Parameters of the feed and pumping pipe

H (m)	h (m)	L (m)	H_r (m)	L_b 1" (m)	h_r 1" (m)	L_b¾" (m)	h_r ¾" (m)	L_b (m)	h_r (m)	V_b (m³)
3	15	10.30	1.027	80.7	0.48	66	1.2	146.7	1.68	0.058

Source: Author

Nomenclature
H - Feed height m
h - Pumping height m
L - Feed pipe length m
Hr - Feed pipe losses m
Lb - Pumping pipe length m
hr - Pumping pipe losses m
Vb - Pumping pipe volume m³

3. Analysis of research using the traditional method

Mathematical Modeling is carried out by following the steps [3] (The research in particular conforms to the steps):

a) The research problem. Hydraulic rams have a relatively low and limited volumetric efficiency compared to other hydraulic pumps. The objective is to model the volumetric efficiency of the hydraulic ram in relation to the volume of the air chamber and the volume of the pumping pipe. The results are: the theoretical contribution is a mathematical model; the practical contribution is a technological procedure for optimizing resources in the development of technological charts; and the scientific innovation is an integrative formula for the physical quantities involved in the relationship between volumetric efficiency and the volume of the air chamber.

b) Knowledge that contributes to research. Fluid mechanics, with Bernoulli's and Torricelli's laws, Poisson's equation, N. Zhukovski's principle, the concepts of efficiency, water hammer, turbulent flow, losses, load and flow.

c) Formulation of the problem situation in mathematical terms. The performance of renewable energy technologies behaves as a convex parabola. The parabolic or quadratic model is given by $Y = a + bx + c\, X^2$ where *a* remains the intersection with the Y-axis, and *b* and *c* are related to the slope and the rate of change of the curve respectively.

d) The solution is carried out using the nonlinear regression model and the least square method.

The quadratic model is represented by the general second degree equation (1).

Scope of action: Mathematical process that relates volumetric efficiency to the volume of the air chamber.

$$Y = ax^2 + bx + c \qquad (1)$$

Where *c* is still the intercept with the Y-axis, and (*b* and *a*) are related to the slope and the rate of change.

The prediction equation of the parabola that fits a set of *n* points of the form (x_i, y_i), i= 1,2,...,n , has the equation:

$$\boldsymbol{\hat{Y} = \hat{a}x^2 + \hat{b}\,x + \hat{c}} \qquad (2)$$

Prediction model - where $\hat{a}$, $\hat{b}$, $\hat{c}$ are the least square estimators of *a*, *b* and *c*, determined by solving the normal system of the least square method [4].

$$nc + b\sum_{i=1}^{n} x_i + a\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} y_i \qquad (3)$$

$$\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i^2 + a\sum_{i=1}^{n} x_i^3 = \sum_{i=1}^{n} x_i y_i \qquad (4)$$

$$c\sum_{i=i}^{n} x_i^2 + b\sum_{i=1}^{n} x_i^3 + a\sum_{i=1}^{n} x_i^4 = \sum_{i=1}^{n} x_i^2 y_i \qquad (5)$$

Table 3: Volumetric efficiency calculated from the mathematical model

	η_v	(V_c/V_b)	$(V_c/V_b)^2$	$(V_c/V_b)^3$	$(V_c/V_b)^4$	$\eta_v *(V_c/V_b)$	$\eta_v *(V_c/V_b)^2$
	1.1	0.27	0.07	0.019	0.005	0.29	0.08
	1.4	0.55	0.30	0.16	0.09	0.77	0.42
	1.9	1.03	1.06	1.09	1.12	1.9	2.01
	1.7	1.8	3.24	5.8	10.4	3.06	5.5
	1.4	2.24	5.01	11.23	25.17	3.13	7.01
addition	7.5	5.89	9.68	18.2	36.7	9.15	15.02
Average	1.5	1.17	1.9	3.6	7.3	1.83	3

Source: Author

Covariance is a statistical measure that can indicate the linear relationship or association between variables. X and Y. ($X=(V_c/V_b)$), and ($Y= \eta_v$)
Substituting the values in equation (6)

$$C_{ov} = \frac{\sum_i^n (X_i - \bar{X})(Y_i - \bar{Y})}{n} = \overline{XY} - \bar{X} * \bar{Y} \qquad (6)$$

Cov= 0.075 means that there is no possible direct linear association between X and Y.

Substituting the values calculated in table 3 in equations (3, 4 and 5) gives the following system of equations:

$$5c+5.89b+9.68a= 7.5 \qquad (7)$$

$$5.89c+9.68b+18.2a= 9.15 \qquad (8)$$

$$9.68c+18.2\,b+36.7a=15.02 \quad (9)$$

To calculate the coefficient matrix Δ, the Sarrus method is used.

$$\Delta=\begin{vmatrix} 5 & 5.89 & 9.68 \\ 5.89 & 9.68 & 18.2 \\ 9.68 & 18.2 & 36.7 \end{vmatrix}= 15.19 \quad (10)$$

$$\Delta_c=\begin{vmatrix} 7.5 & 5.89 & 9.68 \\ 9.15 & 9.68 & 18.2 \\ 15.02 & 18.2 & 36.7 \end{vmatrix}= 16.94 \quad (11)$$

$$\Delta_b=\begin{vmatrix} 5 & 7.5 & 9.68 \\ 5.89 & 9.15 & 18.2 \\ 9.68 & 15.02 & 36.7 \end{vmatrix}= 11.29 \quad (12)$$

$$\Delta_a=\begin{vmatrix} 5 & 5.89 & 7.5 \\ 5.89 & 9.68 & 9.15 \\ 9.68 & 18.2 & 15.02 \end{vmatrix}= -3.85 \quad (13)$$

$$\hat{a}=\frac{\Delta c}{\Delta}= 1.11 \quad (14)$$

$$\hat{b}=\frac{\Delta b}{\Delta}= 0.74 \quad (15)$$

$$\hat{c}=\frac{\Delta a}{\Delta}=- 0.25 \quad (16)$$

Nomenclature

Δ: coefficient matrix.

Δa: coefficient matrix in which the column of coefficients of *a* is replaced by the column of the second members of the equations

Δb: coefficient matrix in which the column of coefficients of *b* is replaced by the column of the second members of the equations.

Δc: coefficient matrix in which the column of coefficients of *c* is replaced by the column of the second members of the equations.

The prediction model by the traditional method is given by (17) replacing the values of (14), (15) and (16).

$$\hat{Y}=-0.25\,x^2+0.74x+1.11 \quad (17)$$

$$\hat{y}_i=-0.25(0.27)^2+0.74\,(0.27)+1.11= 1.29$$
$$\hat{y}_i=-0.25\,(0.55)^2+0.74\,(0.55)+1.11=1.44$$
$$\hat{y}_i=-0.25\,(1.03)^2+0.74\,(1.03)+1.11=1.60$$
$$\hat{y}_i=-0.25\,(1.8)^2+0.74\,(1.8)+1.11= 1.63$$
$$\hat{y}_i=-0.25\,(2.24)^2+0.74(2.24)+1.11=1.51$$

Table 4: Prediction of values

$\hat{y}_i$	y_i	$\bar{y}$	$(\hat{y}_i-\bar{y})^2$	$(y_i-\bar{y})^2$
1.29	1.1	1.5	0.04	0.16
1.44	1.4	1.5	0.003	0.01
1.60	1.9	1.5	0.01	0.16
1.63	1.7	1.5	0.016	0.04
1.51	1.4	1.5	0.0001	0.01

Source: Author

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} 100\ \% = \frac{0.073}{0.38} 100\% = 19\ \% \tag{18}$$

Table 5: Volumetric efficiency from the mathematical model

	η_v	(V_c/V_b)	$(V_c/V_b)^2$	$(V_c/V_b)^3$	$(V_c/V_b)^4$	η_v * (V_c/V_b)	η_v $*(V_c/V_b)^2$	η vest	e_{est}	$(\eta_{vest}-\bar{\eta}_v)^2$	e_{es}^2
	1.1	0.27	0.07	0.019	0.005	0.29	0.08	1.29	-0.19	0.04	0.03
	1.4	0.55	0.30	0.16	0.09	0.77	0.42	1.44	-0.04	0.003	0.0016
	1.9	1.03	1.06	1.09	1.12	1.9	2.01	1.60	0.3	0.01	0.09
	1.7	1.8	3.24	5.8	10.4	3.06	5.5	1.63	0.07	0.016	0.0049
	1.4	2.24	5.01	11.23	25.17	3.13	7.01	1.51	-0.11	0.0001	0.012
addition	7.5	5.89	9.68	18.2	36.7	9.15	15.02	7.47	0.03	0.069	0.137
Average	$\bar{\eta} = 1.5$					Average	η_{est}	1.49	R^2=	0.19	
addition $(\eta-\bar{\eta})^2$	36				addition	$(\eta_{est}-\bar{\eta})^2$		0.069			
Parame-ters	a=-0.25	b=0.74	c=1.11	n=5							

Source: Author

Nomenclature
- ηest – Estimated volumetric efficiency
- eest - Estimated error
- $\bar{\eta}$ – Average volumetric efficiency
- ηv – Volumetric efficiency

By replacing the values from Table 5 in the coefficient of determination (18), it is shown that 19% of the volumetric efficiency behavior is explained by the volume ratio of the air chamber and pumping pipe, using the parabolic method. The above differs from the solution using Excel software, which is 96%.
General equation of a polynomial of the second degree (19)

$$\eta_V = -a\left(\frac{V_c}{V_b}\right)^2 + b\left(\frac{V_c}{V_b}\right) + c\ \ \% \tag{19}$$

Equation solution from calculations using the least square method

$$\eta_V = -0.25\left(\frac{V_c}{V_b}\right)^2 + 0.74\left(\frac{V_c}{V_b}\right) + 1.11\ \ \% \tag{20}$$

Finding the roots of equation (19) and linking it with the energy efficiency equation (21) and the volumetric performance [10] (22), equation (23) is determined.

$$\eta_E = \frac{q\ h}{QH}\ 100\% \tag{21}$$

$$\eta_v = \frac{q}{Q}\ 100\% \tag{22}$$

$$\frac{V_c}{V_b} = \left(\frac{-b \mp \sqrt{b^2 - 4a\left(\frac{hc - \eta_E H}{h}\right)}}{2a} \right) \tag{23}$$

The volumetric efficiency has its maximum value defined in equation (24)

$$\frac{V_c}{V_b} = \frac{-b}{2a} = 1.5 \tag{24}$$

Substituting the values in (23) and clearing, the values of the chamber volumes for which the volumetric efficiency is minimum are obtained.

$$V_{c1} = 0.3\ V_b \tag{25}$$

$$V_{c2} = 2.8 V_b \tag{26}$$

4. Results

By replacing the values of the magnitudes in (20) the following percentage efficiency results are obtained:

$$\eta_V = -0.25\left(\frac{V_c}{V_b}\right)^2 + 0.74\left(\frac{V_c}{V_b}\right) + 1.11\ \%$$

$V_c = V_b = 0.058\ m^3$	$\eta_V = 1.59\%$
$V_c = 0.3V_b = 0.017 m^3$	$\eta_V = 1.3\ \%$
$V_c = 2.8V_b = 0.162\ m^3$	$\eta_V = 1.2\%$
$V_c = 1.5V_b = 0.087\ m^3$	$\eta_V = 1.65\%$
$V_c = 2\ V_b = 0.116\ m^3$	$\eta_V = 1.59\%$
$V_c = 0.5V_b = 0.029\ m^3$	$\eta_V = 1.41\%$
$V_c = 1.2\ V_b = 0.069\ m^3$	$\eta_V = 1.63\%$

This shows that the highest volumetric efficiency is achieved for a volume 1.5 times the volume of the pumping or discharge pipe. Using Microsoft Excel grids, this is shown in Figure 6.

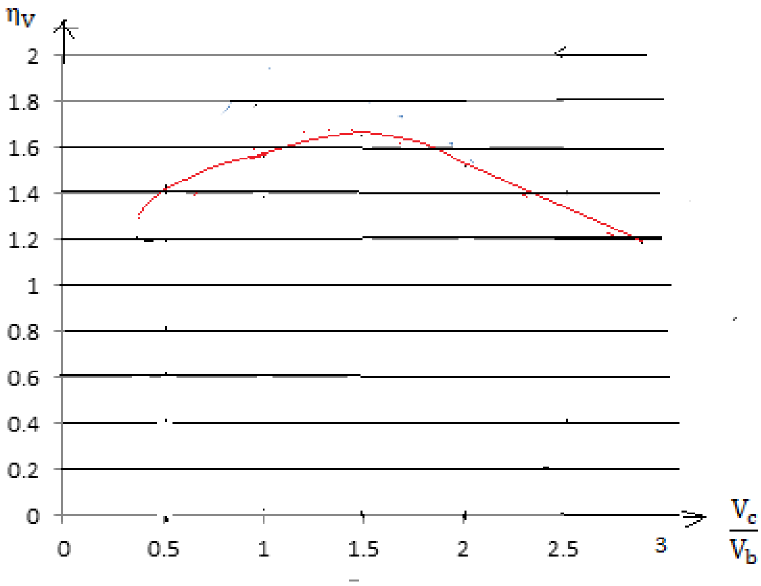

Fig. 6. Volumetric efficiency and volume ratio

With the methodology of the Microsoft Word Excel tool, the equation (27) is achieved.

The prediction solution equation by Excel software is

$$\hat{Y} = -0.68x^2 + 1.85x + 0.63 \quad \text{Excel} \tag{27}$$

The coefficient of determination for this last equation is R2 = 0.96.

$V_c = V_b = 0.058\ m^3$ $\eta_V = 1.8\%$

$V_c = 0.3V_b = 0.017 m^3$ $\eta_V = 0.98\ \%$

$V_c = 2.8V_b = 0.162\ m^3$ $\eta_V = 1.10\%$

$V_c = 1.36V_b = 0.075\ m^3$ $\eta_V = 1.89\%$

$V_c = 1.5V_b = 0.087\ m^3$ $\eta_V = 1.87\%$

$V_c = 2\ V_b = 0.116\ m^3$ $\eta_V = 1.61\%$

$V_c = 0.5V_b = 0.029\ m^3$ $\eta_V = 1.38\%$

$V_c = 1.2\ V_b = 0.069\ m^3$ $\eta_V = 1.87\%$

Its characterization is determined in Figure 7.

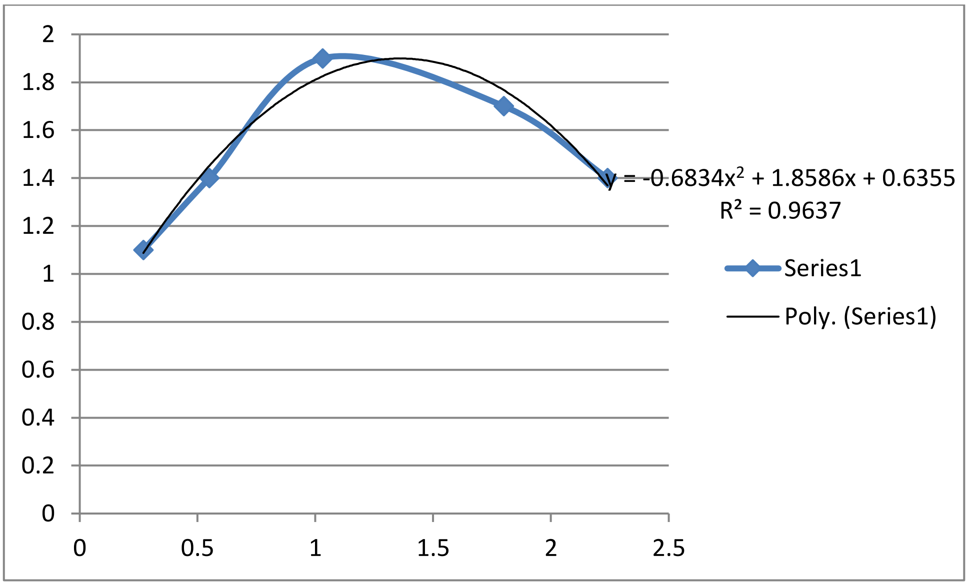

Fig. 7. Volumetric efficiency and volume ratio

e) Comparison of the model with the real situation.

The theoretical polynomial equation describing the experimental results obtained using the traditional procedure (20). The maximum efficiency with equation (24) when solving for the volume of the air chamber shows that it is fifty percent greater than the volume of the pumping pipe.

$$V_c = V_b + 50\%\ V_b$$

$$V_c = 1.5V_b$$

Table 6 defines that the law of conservation of energy is fulfilled, where it is shown that the energy efficiency of 6.7% is never exceeded by the volumetric efficiency.

Table 6: Volumetric efficiency of the hydraulic ram

Volume (m^3) Air chamber	Volumetric Efficiency (%)
0.016	1.1
0.032	1.4
0.058	1.9
0.105	1.7
0.130	1.4

Source: Author

The theoretical polynomial equation that describes the experimental results is obtained using Excel software (27). The maximum performance with equation (24) when clearing the volume of the air chamber shows that it is greater by thirty-eight percent more than the volume of the pumping pipe.

5. Discussion

f) Analysis of the restrictions to the model.
Restricted for pumping systems for renewable energy sources, with pumping pipe volumes exceeding hundreds of meters, as otherwise, it requires a disproportionate volume of the air chamber.
g) Application scenario of the model, as well as its interpretation
In multi-drive hydraulic ram pumps that have alternative air chambers, where it is interpreted that their volume is directly proportional to the volume of the pumping pipe.
The results show that the volume is equivalent to the volume of the pumping line plus fifty percent of that same volume, an analysis performed using the traditional method. Using Excel software, it shows that the volume is equivalent to the volume of the pumping line plus thirty-eight percent of that same volume. The entire study corroborates compliance with the law of conservation of energy, and the formula found is a specific example of the general formula for energy efficiency.

6. Conclusions

The volume of the air chamber is directly proportional to the volume of the pumping pipe, and to the square root of fourfold times the energy efficiency.
The mathematical modeling of volumetric efficiency is characterized with the help of Microsoft Excel software, and by the traditional method.
The highest volumetric efficiency is achieved when the volume of the air chamber is equal to the volume of the pumping pipe plus thirty-eight percent of the latter, a solution found by the Microsoft Excel software.
The traditional method shows that the highest volumetric efficiency is achieved when the volume of the air chamber is equal to the volume of the pumping pipe plus fifty percent of the latter.

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