Analysis of the Limitations of Bivariate Methods for Estimating Design Floods and Proposal of the IIUNAM Method

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Abstract: *Starting from the evident idea that the design of the spillway, that is, the capacity of the spillway and the volume allocated for flood regulation, in dams where this volume is considerable, depends on the maximum discharge and the volume of the inflows entering the reservoir; a large number of studies have been developed based on the fitting of bivariate probability distribution functions for these two variables. This paper presents an analysis that provides arguments to support the authors' opinion that the use of bivariate functions for the estimation of design floods in dams with considerable regulation capacity is not convenient. Instead, the method developed at the Institute of Engineering of UNAM is proposed.*

Keywords: *Probability distribution function, discharge-duration curve for return period, joint return period, base time, design floods shape*

1. Introduction

Bivariate distribution functions have been widely used since the mid-1990s for the estimation of design floods in reservoirs with significant regulation capacity, considering both peak discharge and flood volume (see, for example, the works of [1-6]; among others.

Three fundamental aspects can be identified that make it difficult—if not impossible—to achieve successful results in the application of bivariate methods: (1) the criteria used to define the volume of annual maximum flood events and to form peak discharge–volume pairs; (2) the definition of the design or failure domain based on the bivariate probability distribution function of the peak discharge–volume pairs (i.e., the definition of the joint return period); and (3) the criteria used to shape the design hydrographs. This study will primarily focus on the first aspect, with brief consideration of the second and third. In contrast, the method developed by the Institute of Engineering overcomes the limitations associated with the use of bivariate distribution functions for the estimation of design hydrographs in dams with significant regulation capacity.

1.1 Definition of Annual Maximum Flood Volumes

The selection of the base time for flood events allows for the definition of annual maximum flood volumes, which are then incorporated into the series of peak discharges and volumes used to estimate the joint distribution function of these two variables. However, only a few studies explicitly present the criteria proposed for this calculation. Some of these include: Pegram and Deacon,1992 [7] suggest using 10% of the peak discharge value to determine the base time. Yin et al., 2018 [8] propose a 7-day period for estimating flood volume, based on the observation that a longer duration better captures the total flood volume. Escalante, 1998 [9] recommends determining the peak time and recession time using correlation and multiple regression analysis. Other studies, such as those by [2,4,10], suggest—though not always explicitly—that the entire flood event should be captured and incorporated into the series of annual maximum volumes.

These investigations share a common limitation: they do not consider the specific characteristics of the site under design. In the case of a reservoir, it is essential to account for the available storage volume for flood regulation, the spillway capacity, and its operational policy, among other factors, in order to obtain at least a preliminary idea of the flood durations that are critical for the particular case. In practice, the duration of flood events critical to the problem under study is not known with certainty, therefore it is not possible to adequately define the start and end of the annual maximum flood events.

1.2 Peak Discharge–Volume–Return Period (Qp–V–Tr) Curves

Qp–V–Tr curves are essential for interpreting the results of bivariate analysis. This section presents several approaches used by different authors to define the Joint Return Period (JRP).
In various studies related to bivariate probability distributions of peak discharge and volume, F(Qp, V), different methods have been proposed to define the "exceedance region". Early works introduced regions referred to as OR and AND. The OR region corresponds to the domain in which either peak discharge or volume is exceeded for a given (Qp, V) pair, while the AND region corresponds to the domain in which both variables are simultaneously exceeded [9,11].
Although it is now recognized that approaches defining the failure domain based on the probability of exceeding both peak discharge and volume (AND approach), as well as those considering exceedance of either variable (OR approach), do not adequately represent the actual failure domain for a given structure [2,3], efforts to adapt the definition of the failure domain to a specific structure have only achieved partial success.

1.3 Construction of Design Hydrographs

The construction of design hydrographs is essential for representing the natural variability of hydrological events. Several approaches proposed by different authors are presented below. Escalante,1998 [9] suggests obtaining the time to peak and recession time using regional methods based on physiographic characteristics and multiple linear regressions to shape the flood events [12]. Ramírez y Aldama, 2000 [13] use triparametric Hermite hydrographs to define design floods. Requena, 2015 [4] discusses the construction of flood shapes, providing a detailed approach for the design of hydrographs.

In syntesis, several authors highlight the advantages offered by bivariate models in the estimation of design floods, including: the ability of bivariate models to better represent the relationship between peak discharge and flood volume [14]; the improved accuracy in risk estimation associated with extreme flood events when considering two variables jointly [15]; and the flexibility in modeling, which allows for the use of different marginal distributions, thereby adapting to the specific characteristics of the basin and the available data [14].
In contrast, the main objective of this study is to emphasize the limitations identified in the use of bivariate models for the estimation of design floods in reservoirs with significant regulation capacity, particularly regarding the determination of annual maximum flood volumes. Instead, we proposed the IIUNAM method [16-19], which overcomes the inherent disadvantages of bivariate models and provides a more effective tool for estimating design floods in reservoirs with substantial regulation capacity.
This work is organized as follows: the introduction presented here; the methodology section, which describes the main concepts related to univariate frequency analysis, followed by a discussion on bivariate frequency analysis, emphasizing its limitations when applied to the design or review of spillway structures in regulated reservoirs. The IIUNAM method is then described, including the underlying hypothesis and the stages involved in its development. The following chapter presents the application of the IIUNAM method to the estimation of design floods for the spillway of El Infiernillo reservoir, including calculations to verify the results. The discussion section highlights the benefits of applying the IIUNAM method compared to approaches based on bivariate distribution functions. Finally, the conclusions and recommendations derived from this study are presented.

2. Methodology

2.1 Univariate Frequency Analysis

Univariate frequency analysis is essential in hydraulic and hydrologic engineering for evaluating the probability of extreme hydrological events, such as intense rainfall or peak discharges, by fitting probability distributions to observed data [20]. The most commonly used distributions include the Gumbel, Generalized Extreme Value and Log-Pearson III [21]. In Mexico, the Gumbel, Double Gumbel, and Generalized Extreme Value (GEV) distributions are commonly employed to model extreme events. In particular, the Double Gumbel function (Equation 1) offers greater flexibility by considering that annual maximum rainfall or discharge values may originate from two different populations [22].

$$F(x) = pe^{-e^{-\alpha_1(x-\beta_1)}} + (1-p)e^{-e^{-\alpha_2(x-\beta_2)}} \tag{1}$$

Where: p is the probability that the random variable x belongs to population 1; α_1, β_1, α_2, β_2 are the scale and location parameters of populations 1 and 2, respectively.
The process involves collecting historical data, selecting the appropriate distribution [23], and estimating its parameters [24]. These parameters allow for the construction of frequency curves (Figure 1), which are essential for the design of hydraulic infrastructure [25].

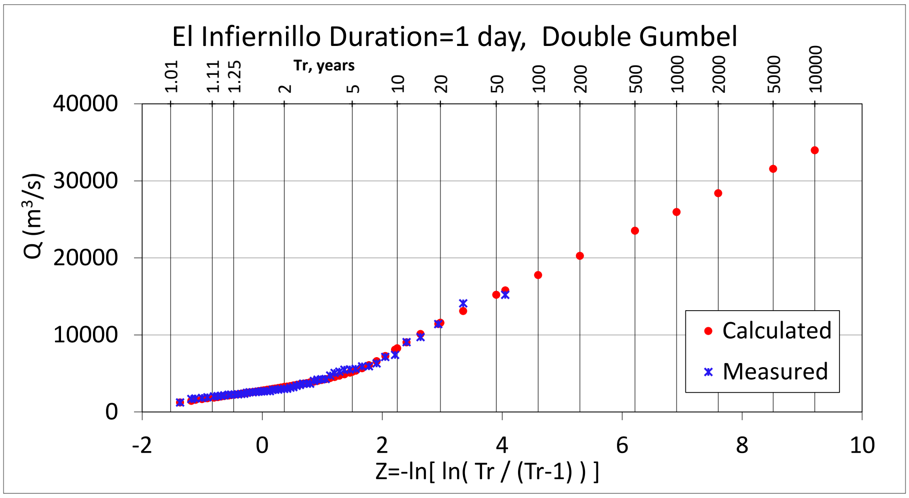

Fig. 1. Example of a graph used in univariate frequency analysis

2.2 Bivariate Frequency Analysis

The bivariate frequency analysis method in hydrology evaluates the relationship between two hydrological variables, such as peak discharge and the volume of annual maximum flood events, using bivariate logistic models or copula functions to model the dependence between variables, along with appropriate marginal distributions [13].
As previously mentioned, this study focuses on the shortcomings or limitations associated with the use of bivariate distribution functions. Considering the relative importance of these limitations, we will first present those related to the procedure used to define the series of annual maximum flood volumes. Next, we will address the limitations concerning the definition of the flood shape for design purposes, given a peak discharge–volume pair. Finally, we will discuss the limitations in estimating the Joint Return Period (JRP) based on a bivariate probability distribution of peak discharge and volume.

2.2.1 Estimation of Annual Maximum Flood Volumes

In 2022, the hydrograph shown in Figure 2 entered El Infiernillo reservoir, located in the state of Guerrero, Mexico. If specific data related to the problem—such as turbine capacity, the relationship between stored volume and spillway discharge, etc.—are not considered, it remains unclear whether the flood volume should be defined from day 1 (corresponding to August 29) to day 36, or from day 6 to day 29, or from day 12 to day 29.

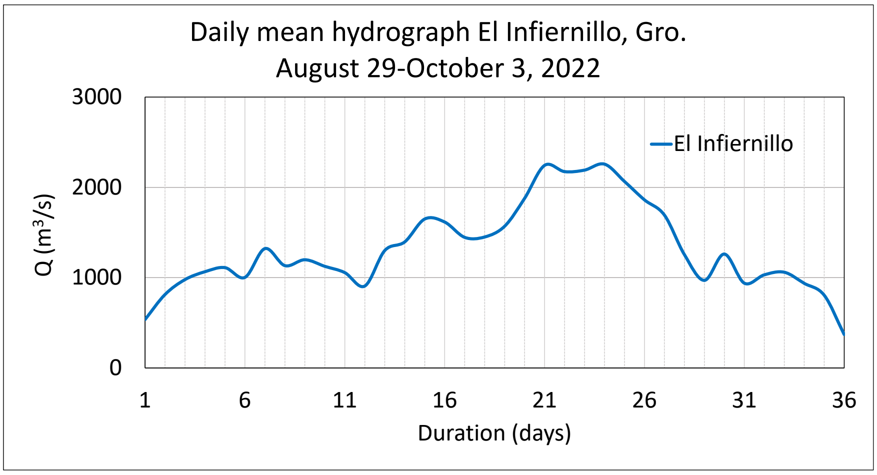

Fig. 2. Inflow hydrograph for El Infiernillo Dam

A similar situation occurs with the following hydrographs recorded at different sites in Mexico, as shown in Figure 3.

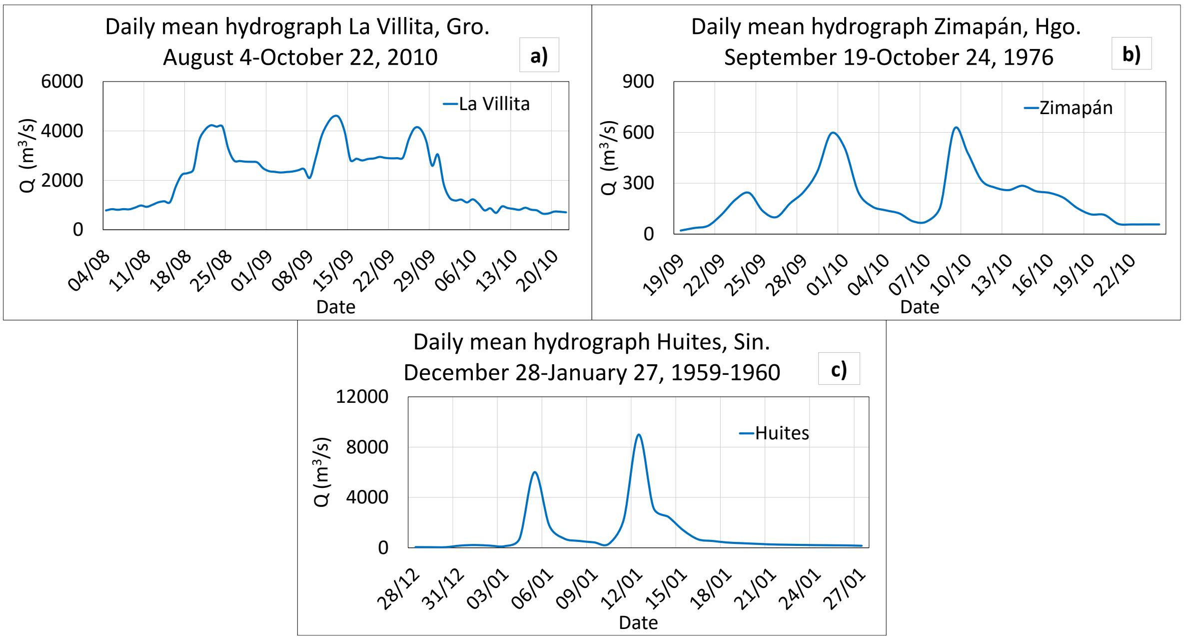

Fig. 3. Recorded inflows at the entrance of La Villita (a), Zimapán (b), and Huites (c) reservoirs

As shown in Figures 2 and 3, estimating the duration of annual maximum flood events—and therefore their volume—is not a trivial task. On the other hand, doing so properly is fundamental, since the entire process of constructing bivariate distribution functions is based on the definition of peak discharge–volume pairs.The example in Figure 4, demonstrates that when simulating the historical flood event (from late December 1959 to early January 1960) for Huites reservoir in Sinaloa,México, the critical conditions of maximum stored volume and maximum discharge are reached through a sequence of two flood waves.

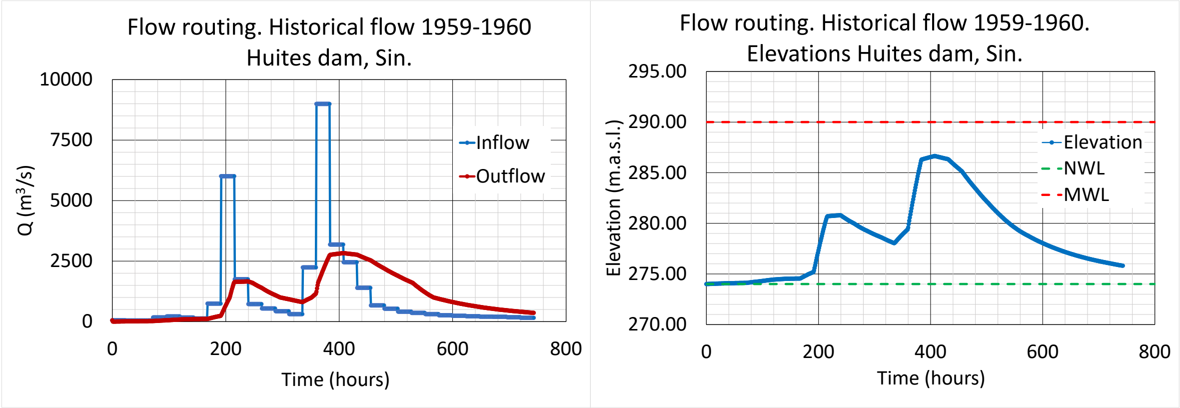

Fig. 4. Flood routing at Huites reservoir (Historical flood Dec 1959-Jan 1960). Source: Own design

Despite the importance of properly defining annual maximum flood volumes, few studies address this topic in depth. Peagram and Deacon, 1992 [7] suggest using 10% of the peak discharge to determine the base time. Shiau, 2003 [26] proposes establishing a threshold to define the duration of the flood event. Yin et al., 2018 [8] recommend a 7-day period for volume estimation, arguing that a longer duration allows for better capture of the total flood volume. Other studies [2,4,6,10] suggest—though not always explicitly—that the entire flood event should be captured and incorporated into the series of annual maximum volumes.
It is evident that applying the various criteria described can lead to considerable differences in the estimation of annual maximum volumes. The main limitation shared by these approaches is that they do not consider that, for the volume estimation, the characteristics of the reservoir under study should be taken into account—primarily the regulation volume and spillway capacity. That is, what matters is not so much the total flood volume, but rather the volume that enters the reservoir during a critical duration, which depends on the specific characteristics of the reservoir being analyzed.

2.2.2 Definition of the Shape of Design Floods

Once the design (Qp, V) pairs are obtained, the corresponding design hydrographs are constructed. The literature on bivariate functions for estimating design floods in reservoirs with regulation capacity presents various approaches for constructing these hydrographs; some methods give hypothetical typical hydrograph shapes and other methods attempt to give shapes more similar to historical ones; for example, Ayuso et al., 2007 [27] used triparametric Hermite hydrographs. De Michele and Salvadori, 2003 [10] developed a storm intensity–duration model based on copulas. Mediero et al., 2010 [28] proposed generating multiple (Qp, V) pairs and adjusting the design hydrographs to match observed ones. Rivera y Escalante, 1999 [12] recommended an approach based on historical flood events, using a convolution method to construct hydrographs. On the other hand, Jiménez and Mediero, 2014 [3] presented a procedure that applies bivariate frequency analysis and scales typical hydrographs. Requena, 2015 [4] proposed a copula-based bivariate model, including flood routing to ensure the accuracy of the hydrograph.

2.2.3 Joint Return Period (JRP)

Peak discharge–volume–return period curves are essential for interpreting the results of bivariate analysis.
Rivera y Escalante, 1999 [12] proposed a bivariate joint estimation technique (ECB) that models peak discharge (Qp) and total flood volume (V), defining the failure region using OR criteria. In this approach, the joint return period associated with a (Qp, V) pair is obtained using the following relationship (Equation 2):

$$JRP = \frac{1}{1-F(Q_p,V)} \quad (2)$$

Ramírez y Aldama, 2000 [13] use Equation 3, which corresponds to the AND-type approach.

$$JRP = \frac{1}{1-F(Q_p)-F(V)+F(Q_p,V)} \quad (3)$$

Where F(Qp, V) is the joint distribution function of peak discharge (Qp) and volume (V), respectively, and F(Qp) and F(V) are the marginal distribution functions of Qp and V, respectively.
Volvi y Fiori, 2012 [1] emphasized the importance of considering both AND-type and OR-type joint return periods (Figure 5 and Equations 2 and 3).

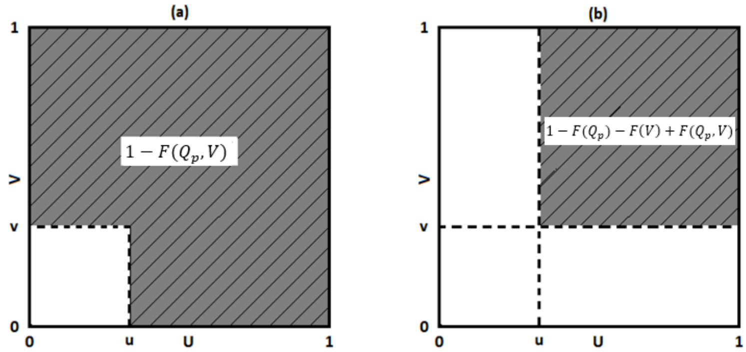

Fig. 5. Definition of exceedance regions using OR and AND criteria, respectively. Source: Modified from [11]

Brunner et al., 2016 [29] discuss the importance of bivariate return periods for estimating flood peaks and volumes, explaining that AND-type and OR-type joint return periods can have a significant impact on the design and risk assessment of hydraulic structures.
The Kendall return period, introduced by [30] and revisited by [31], is an alternative measure to the traditional definition. Unlike classical approaches based on joint probabilities (such as the AND and OR types), the Kendall return period is based on Kendall's dependence index and considers the expected frequency of occurrence of extreme events according to their joint ranking. It is defined as the inverse of the probability that a pair of random variables (e.g., discharge and volume) simultaneously exceed certain quantiles, weighted by their dependence structure. In terms of the bivariate joint distribution function F(Qp, V) and its marginals F(Qp) and F(V), the Kendall return period Tk is expressed according to Equation 4.

$$Tk = \frac{1}{1-K_{Ct}(t)} \quad (4)$$

Where: t is the probability associated with the joint event, which can be approximated using the copula function $t = C\left(F\left(Q_p\right), F(V)\right)$, K_{Ct} represents the joint non-exceedance probability (Kendall's function).
This formulation reflects the probability that a future event will simultaneously exceed both thresholds, taking into account their dependence.
Some recent efforts typically involve simulating the routing of a large number of synthetic flood events to define the actual failure domain in terms of the maximum volume stored in the reservoir under study and selecting the approach that best represents it. See, for example [4].

2.3 Method of the Institute of Engineering

The method developed by the Institute of Engineering [16-19,32] is based on the hypothesis that, for the operation of the spillway structure of a given reservoir with significant regulation capacity (evaluated in terms of the maximum water surface elevation in the reservoir and the peak discharge), there exists a critical duration. This critical duration corresponds to the time interval during which the inflow exceeds the outflow (as illustrated in Figure 6), and it depends on the specific characteristics of the reservoir under study (essentially the available volume for flood regulation and the spillway capacity) as well as the operational policy of the spillway structure.

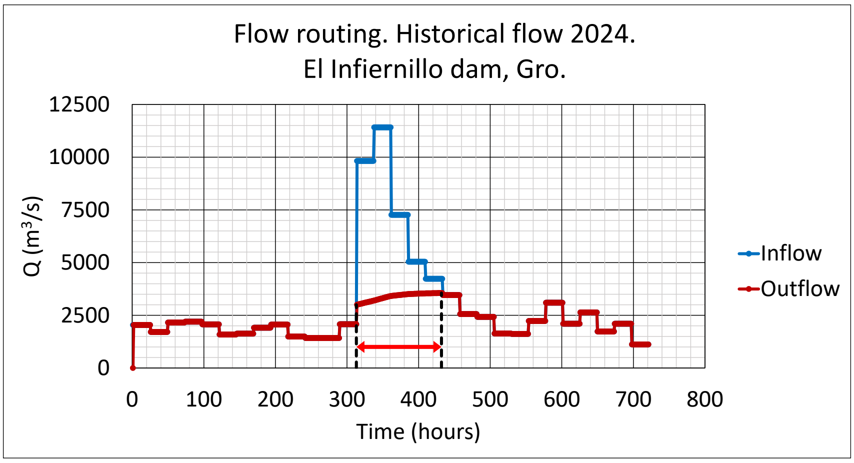

Fig. 6. Example of flow routing through a reservoir

For a given reservoir, the critical duration can only be estimated approximately (for example, by routing the maximum recorded flood), since it varies depending both on the magnitude of the flood and on the operational policy of the spillway structure. According to the basic hypothesis, it is necessary to determine the distribution functions of the mean inflows associated with different durations -ranging from a short duration, much less than the critical duration estimated by routing the maximum recorded flood, to a much longer duration-.
The method allows for the estimation of design floods considering their peak discharge, volume, and shape.
The method involves the following steps (as illustrated in Figure 7).

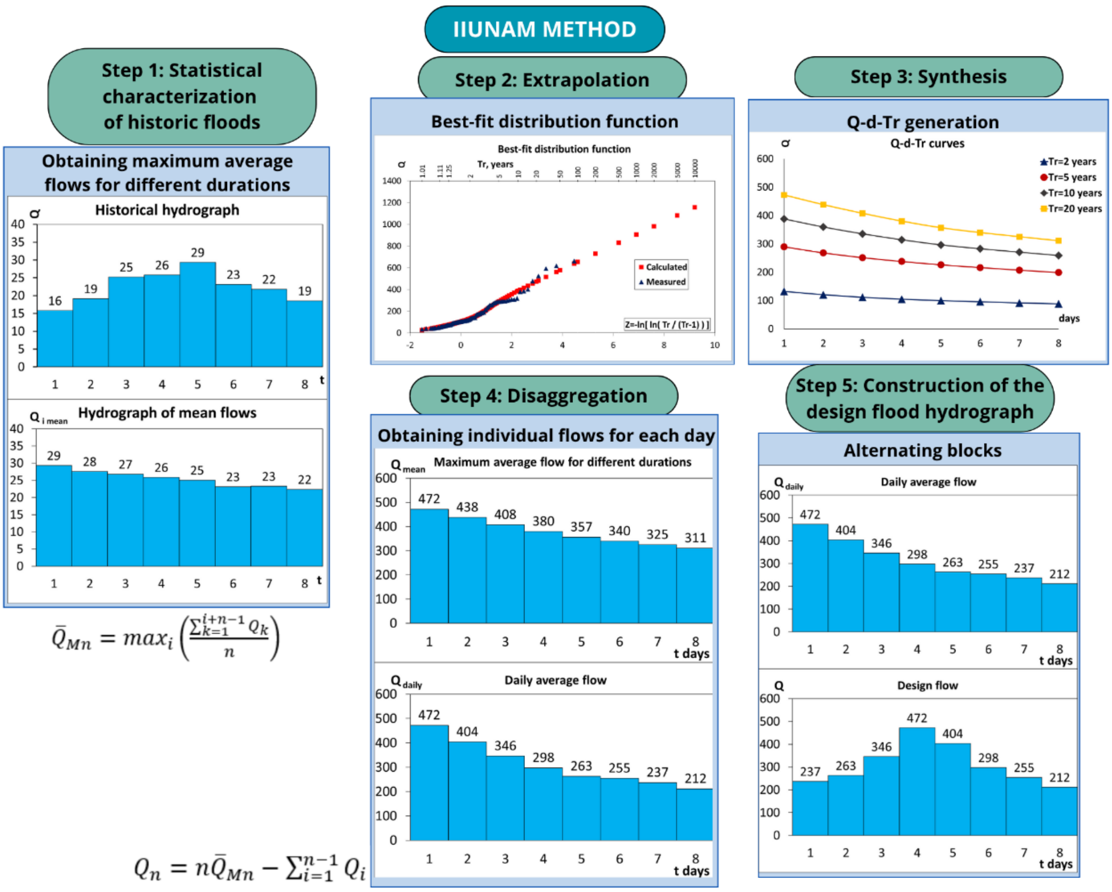

Fig. 7. Flowchart of the algorithm for the Institute of Engineering method, UNAM. Source: Original design and images adapted from [32]

2.3.1 Statistical characterization of historic floods

For the statistical characterization of the historic floods a determination is made of the maximum annual mean flow rates for different durations, ranging from one day to a large enough number of days *n*. For a given year, the maximum mean flow rate for one-day duration corresponds directly to the maximum mean daily flow rate. To obtain the maximum annual mean flows for other duration *n* the following equation is applied:

$$\bar{Q}_{Mn} = max_i \left(\frac{\sum_{k=1}^{i+n-1} Q_k}{n} \right) \; (n = 1,2,3, \dots, N; \quad i = 1,2, \dots, 365) \tag{5}$$

Where $\bar{Q}_{Mn}$ is the maximum mean flow for *n* days duration, Q_k is the mean daily flow on the kth day, and *i* is the counter of the day when duration *n* starts.

2.3.2 Extrapolation

The extrapolation to estimate values associated with different return periods is carried out separately for each duration by fitting a probability distribution function (as shown the Step 2 in Figure 7) to each of the samples of maximum annual values obtained from equation (5).

2.3.3 Synthesis

The results obtained in step 2.3.2 are summarized in a Mean discharge-duration-return period graph (Step 3, Figure 7).

2.3.4 Disaggregation

For each return period considered, determination is made of the daily flows through the recursive equation (Step 4, Figure 7):

$$Q_n = n\bar{Q}_{Mn} - \sum_{i=1}^{n-1} Q_i \;\; (n = 2,3, \dots, N) \tag{6}$$

where *N* is the total number of days considered; $\bar{Q}_{Mn}$ is the maximum mean flow for *n* days duration; and Q_i and Q_n are the individual flows for *i* = 1 to *N*.

2.3.4 Construction of the design flood hydrograph

The hydrograph of the design flood is constructed using the alternating block method [18,32] with the daily flows obtained from equation (6) (Step 5, Figure 7).
In the IINGEN method, it is not necessary to establish the total flood volume; it is only required that a large enough number of days *N* be defined to guarantee the inclusion of the complete shape of all historical floods.

3. Results

This section presents the case of El Infiernillo reservoir (Figure 8), officially named Adolfo López Mateos, located on the Balsas River between the states of Michoacán and Guerrero. It is one of Mexico's main hydroelectric infrastructures.A total of 57 years of hydrological records were analyzed, covering the periods from 1965 to 1994 and from 1998 to 2024. The watershed spans an area of 108 square kilometers, with a maximum annual runoff volume reaching 15,000 hm^3. El Infiernillo Dam features a flood control storage capacity of 2,660 hm^3. Its spillway discharge is regulated by nine radial gates, capable of handling a maximum flow rate of 13,800 m^3/s. The reservoir's normal water level is set at an elevation of 165 masl (meters above sea level), while the maximum water level reaches 183.20 masl.

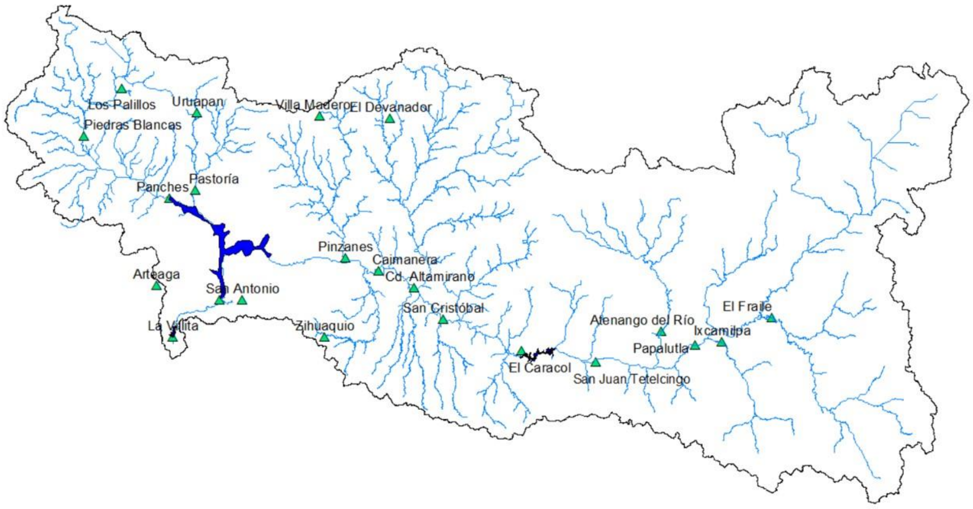

Fig. 8. The Balsas River Hydroelectric System. Source: Own design

Since its inauguration in 1964, the reservoir has faced extreme flood events associated with hurricanes, such as Beulah (1967), Manuel (2013), and John (2024), which caused sudden increases in reservoir levels, challenging its regulation capacity. These events highlight the need to incorporate extreme scenarios to characterize annual maximum floods and enable subsequent statistical analysis.

3.1 Statistical Characterization of Annual Maximum Floods

To characterize annual maximum floods and enable subsequent statistical analysis, the maximum averages associated with different durations were obtained for each year of record, as shown in Figure 9.

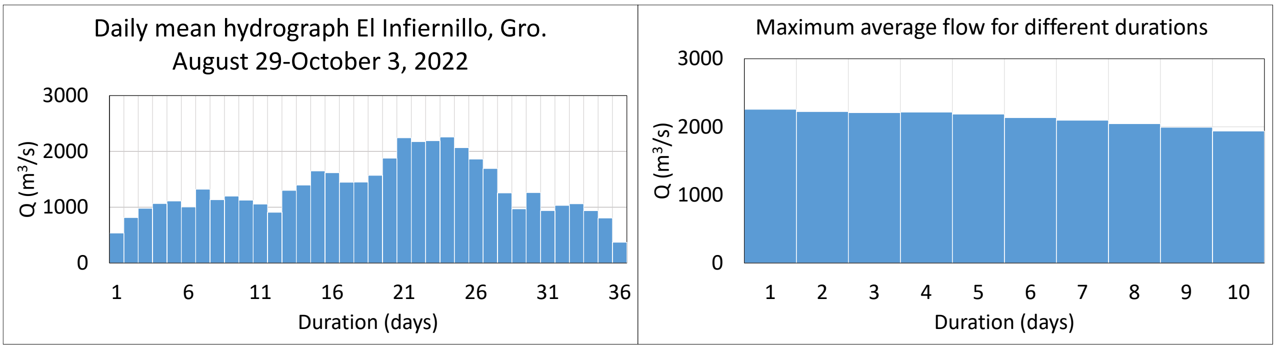

Fig. 9. Daily mean discharges for durations from 1 to 30 days for the 2022 maximum flood event

3.2 Extrapolation

For each duration, the best-fit probability distribution function was estimated, along with the maximum mean discharges for different return periods (Figure 10 corresponds to the 1-day duration). It is noteworthy that the best fit is obtained using the Double Gumbel function, as the maximum discharges originate from two distinct populations, with the second corresponding to the direct impact of hurricanes on the contributing watershed.

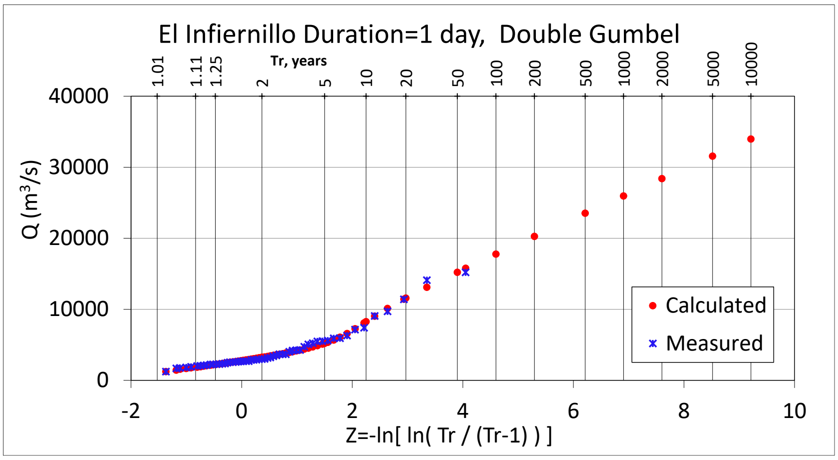

Fig. 10. Double Gumbel distribution fit for 1-day maximum mean discharges

3.3 Synthesis

Mean discharge–duration–return period curves were constructed (Figure 10, top section). To avoid inconsistencies observed for durations of 1 to 3 days and return periods greater than 100 years, the evolution of the parameter $\alpha 2$ from the Double Gumbel function was smoothed, as shown on the right side of Figure 11, bottom section.

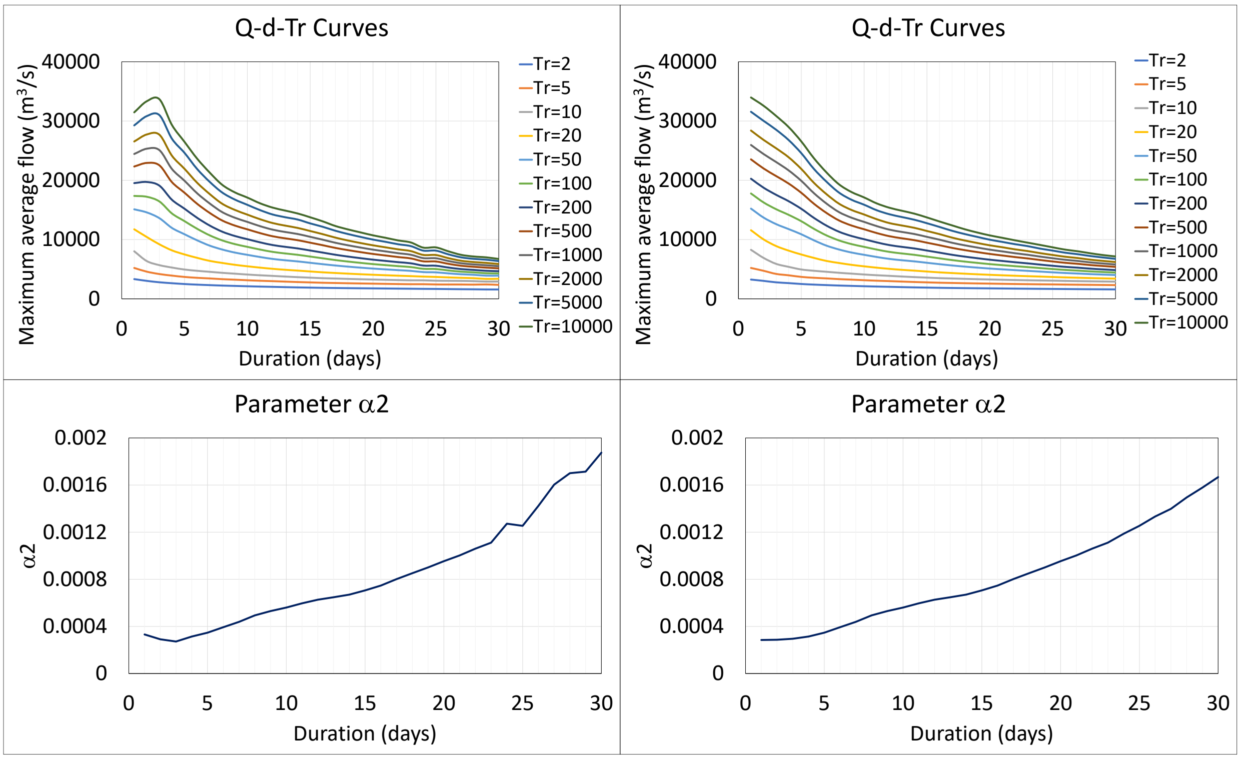

Fig. 11. Q–d–Tr curves before and after parameter adjustments

3.4 Disaggregation

For each return period, the daily mean discharges (individual discharges) were obtained for each day using Equation 6. Figure 12 ilustrate de disagregation procces usin the results obtained for days 1 to 10 for a return period of 20 years.

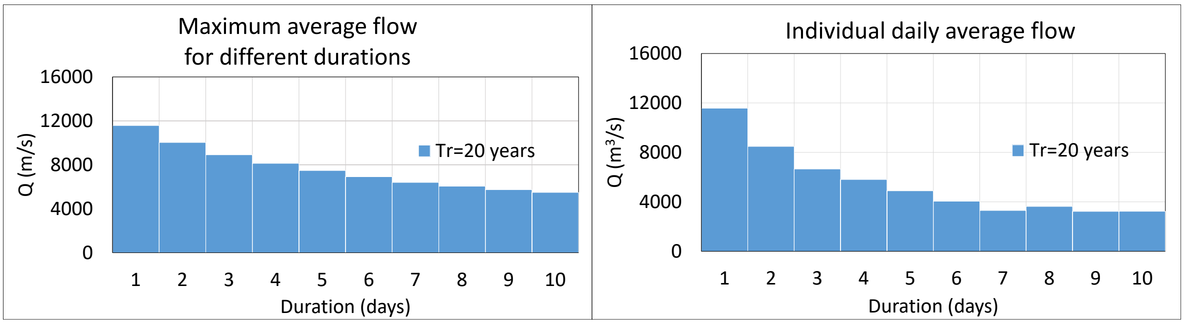

Fig. 12. Estimation of individual daily mean discharges

3.5 Sequencing for Design Flood Construction

The design flood was shaped using the alternating block method, placing the individual discharge for day 1 at the center, the discharge for day 2 to the right, the discharge for day 3 to the left, and so on (Figure 13).

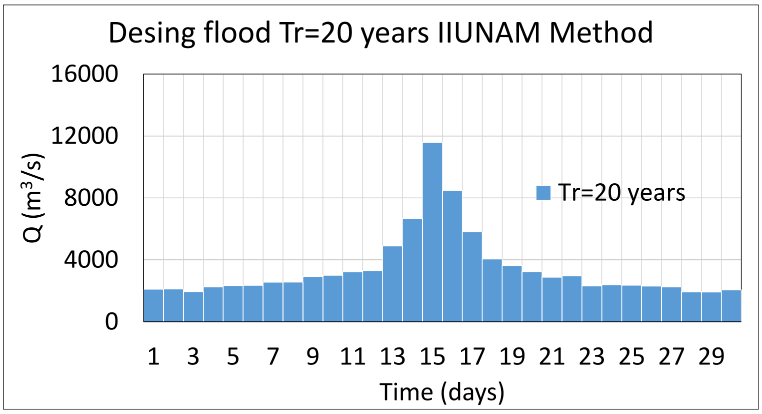

Fig. 13. Design flood for Tr = 20 years

3.6 Results Verification

Figure 14 presents the maximum average discharges for durations ranging from 1 to 10 days, recorded during the five most significant flood events. It is evident that the largest event occurred in 2013, followed closely by the one in 1967, then by the 2024 event, and finally by those in 1976 or 1984, depending on which duration proves to be critical. Considering that the record spans 57 years, these curves were compared with those corresponding to return periods of 58, 29, 19.33, 14.5, and 11.6 years.

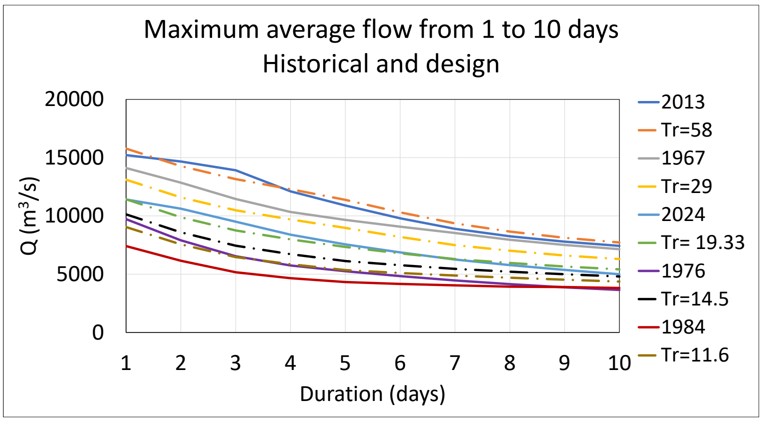

Fig. 14. Historical maximum average discharges and comparison with estimates for different return periods

In Figure 15 we presented the comparison between each historical flood and the estimated design flood regarding both the hydrograph shape and the relationship between average discharge and duration.

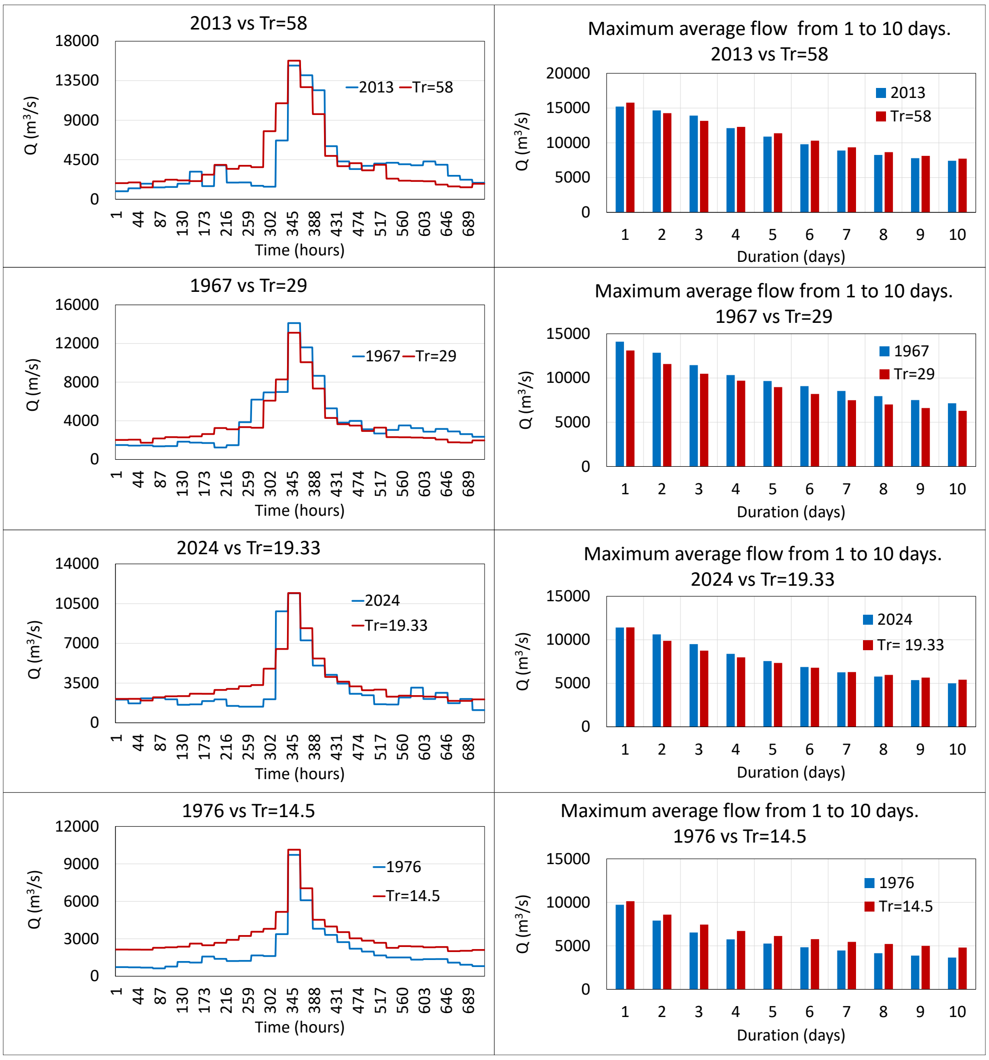

Fig. 15. Comparison of the largest recorded floods with those estimated for different return periods

Additionally, the routing of the five largest recorded floods through the reservoir was simulated and compared with the results obtained from routing the corresponding design floods. Figure 16 shows the comparison between the results of routing the largest recorded flood and the flood corresponding to a return period of 58 years. The maximum water surface elevations obtained for the five largest recorded floods and for the design floods corresponding to return periods of 58, 29, 19.33, 14.5, and 11.6 years are presented in Figure 17.

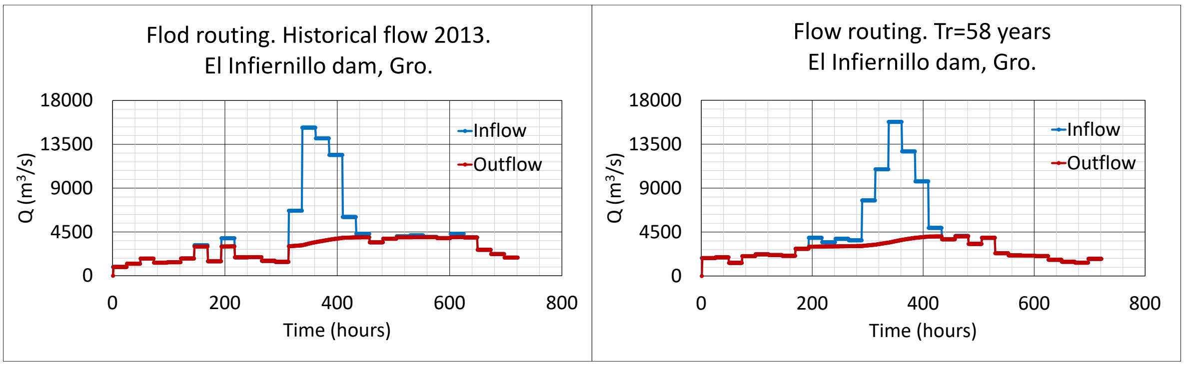

Fig. 16. Results of the simulation of the 2013 recorded flood routing and the design flood with a return period of 58 years

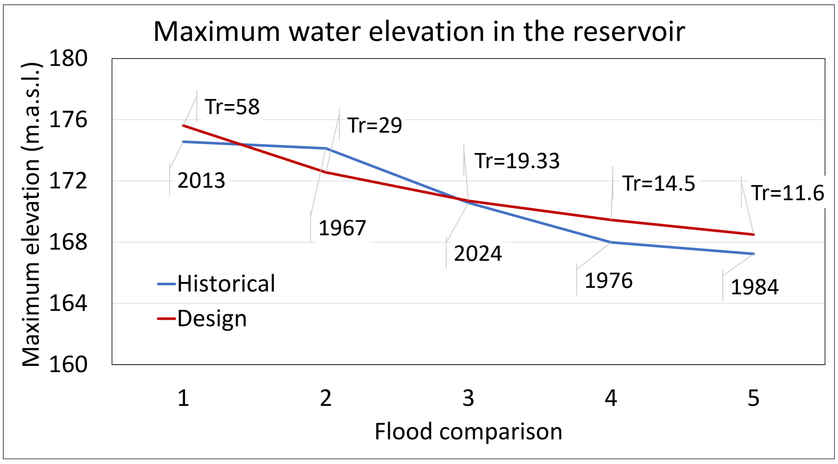

Fig. 17. Comparison of the maximum water surface elevations obtained from the simulation of the five largest recorded floods and the corresponding design floods

4. Discussion

When bivariate functions are used to estimate the design flood for dams with significant regulation capacity, various issues arise that, in our view, have not been adequately addressed.

The methodology chapter discusses the problems encountered in three stages of a study aimed at estimating design floods for dams with considerable regulation capacity using bivariate functions: the estimation of the volumes of the annual maximum recorded floods, the assignment of the joint return period for a given pair (Qp,V), and the definition of the shape of the design floods. Although recent studies propose procedures to address the latter two issues, we believe the first one—the estimation of the volumes of the annual maximum recorded floods—remains unresolved. This volume should be selected considering a critical duration that depends not only on the characteristics of the floods but also on the features of the dam under study and the way its spillway is operated.

Since the critical duration cannot be established a priori and is not unique (it varies with the magnitude and shape of the floods, the length of the spillway, the operation policy, etc.), we propose using the method developed by the Instituto de Ingeniería, which estimates the distribution function of maximum annual average discharges associated with multiple durations, ranging from a short one (typically 1 day when the dam has significant regulation capacity) to a much longer duration than the critical one obtained by routing the maximum recorded flood.

Regarding the criticisms by [3] of the Instituto de Ingeniería's method, we consider that the lack of sub-daily data is not essential for the method. In fact, in several studies [32], the instantaneous peak discharge has been incorporated, demonstrating the method's flexibility. Moreover, although the method does not guarantee the preservation of the correlation between peak discharge and volume—since the statistical analysis of average discharges for each duration is performed independently—the annual maxima generally originate from the same event. Thus, the shape of the historical hydrographs is approximately preserved in the design hydrographs, as demonstrated in Figures 13 to 16 for El Infiernillo Dam and in other case studies, including Malpaso Dam [17].

5. Conclusion

In the hydrological design of dams with significant flood regulation capacity, it is essential to define the return period in terms of the probability that a certain reservoir level will be exceeded. This probability directly depends on the specific characteristics of the dam, highlighting the importance of a case-specific approach.

This study has outlined the inherent limitations of using bivariate distribution functions to estimate design floods for dams with considerable regulation capacity. In particular, the authors argue that the estimation of the annual maximum flood volume, in addition to having a subjective component (which varies among authors), does not account for the critical duration specific to the case under

study. Moreover, even for a given case, this critical duration is not unique; it varies depending on both the dam’s characteristics and the spillway operation policy.
Given the limitations identified in the use of bivariate functions, we propose a method for estimating design floods based on the hypothesis that it is necessary to know the probability distribution function of the average discharge associated with a critical duration that is not precisely known. Therefore, the method considers multiple durations, ranging from a short one (typically 1 day) to a duration significantly longer than the critical one estimated by routing the largest recorded flood.
Considering that both the proposed method and those based on bivariate functions rely on certain assumptions, we recommend that, in all cases, verification tests be conducted to ensure that the resulting design floods are appropriate for the specific case under study. These tests should include a comparison between the recorded maximum floods and the design floods, as shown in Figures 13 to 16 of this study. Most importantly, given that the key design parameter is the maximum elevation reached in the reservoir, the routing of both the design floods and the recorded maximum floods should be simulated to enable a comparison such as that shown in Figure 16 or in the study of the Malpaso Dam previously mentioned, where similar comparisons were made using synthetic hydrographs to evaluate even larger return periods.

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