

## Comparing Water Quality Evaluations from a Fuzzy Model and a Multi-Parameter Model

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**Abstract:** Water quality in Lake Zirahuén (Mexico) was evaluated using two conceptually different approaches: a classical multi-parameter Water Quality Index (NSF model) and a fuzzy logic-based model. Data from four monitoring sites and three sampling campaigns in 2018 were used, considering biochemical oxygen demand ( $BOD_5$ ), chemical oxygen demand (COD), and total suspended solids (TSS). The NSF model produced WQI values close to 80/100, classifying the lake as acceptable except for sensitive organisms. In contrast, the fuzzy model yielded values near 50/100, frequently corresponding to the category only suitable for very resistant organisms. The fuzzy model also showed greater variability between sampling campaigns, indicating higher sensitivity to parameter fluctuations. These differences arise from the structural nature of the models: the NSF index applies compensatory averaging, whereas the fuzzy model follows rule-based inference that emphasizes deteriorated conditions. Although neither model indicates extreme pollution, both suggest that current water quality may impose ecological stress, particularly on sensitive aquatic species. The results highlight the importance of incorporating non-compensatory approaches such as fuzzy logic into water quality assessment, particularly in systems where conservation of aquatic life is a management priority.

**Keywords:** Water quality index, fuzzy model, multi-parameter model, water body

### 1. Introduction

The water quality assessment uses methodologies based on measurements of physical, chemical, biological, and bacteriological parameters [1, 5]. These components are the inputs for any of the well-known models that give a water quality index (WQI) [6], a value to describe the state of water, and follow up on the changes over time. The reliability of these models will always depend on the accuracy of the measurements, their continuity, and the reduction in uncertainty of external factors, such as having similar conditions in sampling campaigns (start and end of sampling times, control labels, and adequate transportation of samples), adequate calibration of equipment, and so on. However, since there is no globally applicable index to assess water quality, most countries use indices adapted to the specific conditions of local water bodies (rivers, lakes, and aquifers) [6, 16]. In México, as in other countries, a quick way to assess water quality is through the biochemical oxygen demand at five days ( $BOD_5$ ), the chemical oxygen demand (COD), and the total suspended solids (TSS) measurements [17]. There are limits determined by different official regulations for each of the three variables, which are related to the respective water quality classifications, expressed in qualitative terms (Table 1) rather than as a number between 0 and 1 or between 0 and 100, which is the way water quality is generally described.

**Table 1:** BOD<sub>5</sub>, COD, and TSS water quality classification scale [17]

CRITERION (mg/l)			CLASSIFICATION
BOD <sub>5</sub> ≤ 3	COD ≤ 10	TSS ≤ 25	EXCELLENT
3 < BOD <sub>5</sub> ≤ 6	10 < COD ≤ 20	25 < TSS ≤ 75	GOOD
6 < BOD <sub>5</sub> ≤ 30	20 < COD ≤ 40	75 < TSS ≤ 150	ACCEPTABLE
30 < BOD <sub>5</sub> ≤ 120	40 < COD ≤ 200	150 < TSS ≤ 400	POLLUTED
BOD <sub>5</sub> > 120	COD > 200	TSS > 400	HEAVILY POLLUTED

As the classification labels of these variables are verbal expressions, we thought of using the tools of Fuzzy Logic (FL) to develop a model that, based on logical rules, allows the joint evaluation of the variables to obtain a value that will coincide with the classification ranges established by the National Water Commission and the Ministry of the Environment and Natural Resources (CONAGUA and SEMARNAT, are the respective acronyms in Spanish), for the different water uses (Table 2).

With information from a public database for a water body in Michoacán, Mexico, for 2018, for BOD<sub>5</sub>, COD, and TSS, we calculated the water quality index with the fuzzy model. We then compared these values with those obtained in a previous study [18], which used the index developed in the USA by the National Sanitation Foundation (NSF), widely used to assess water quality.

**Table 2:** WQI classification scale according to the water use [19]

Value (%)	General criterion	Public supply	General recreation	Fishing and aquatic life	Industrial and Irrigation	Navigation
100	Not Polluted	No purification required	Acceptable for all aquatic sports	Acceptable for all organisms	No purification required	Acceptable
90						
85						
80	Acceptable	Slight purification required	Acceptable but not recommended	Acceptable, except for susceptible organisms	Slight purification is required for some processes	
70						
60	Slightly Polluted	Increased need for treatment		Doubtful for sensitive organisms	No treatment is required for industrial use	
50						
40	Polluted	Doubtful	Doubtful	Only very resistant organisms	Treatment is required for most industrial uses	
30		Not acceptable	Avoid water contact			
20	Heavily Polluted		Slightly Polluted	Not acceptable	Restricted use	Polluted
10			Not acceptable		Not acceptable	
0			Not acceptable		Not acceptable	

## 2. Methodology

### 2.1 Study Area and Sampling Sites

Lake Zirahuén lies in the basin of the same name, in the state of Michoacán, central México (Fig. 1), between 19°21'10" and 19°29'24" North, and 101°29'37" and 101°49'37" West, 2075 m above sea level. It is an endorheic-type basin, covering 260 km<sup>2</sup> of highlands of the Transverse Volcanic system, with elevations ranging from 2080 m, near the lake, to 3280 m, to the northwest and northeast of the lake. The lake is almost rectangular, covering approximately 10 km<sup>2</sup>. There is only one permanent inflow, El Silencio River, which is found in the eastern part of the basin [20] (Fig. 1). There are many settlements in the basin, the most important in terms of population and economic activities are Zirahuén (3263 inhabitants), Santa Clara del Cobre (16748 inhabitants), and Opopeo (11304 inhabitants) [21]. The main economic activities here are tourism, handmade copper crafts, and wooden furniture, respectively. The National Water Commission (NWC) is responsible for the management and protection of all water bodies in México, and has been sampling the water at four sites in Lake Zirahuén from 2012 to the present time. Following the NWC terminology, these sites are Entrada 1, Muelle principal, Entrada 2, and Centro, as seen in Figure 1.

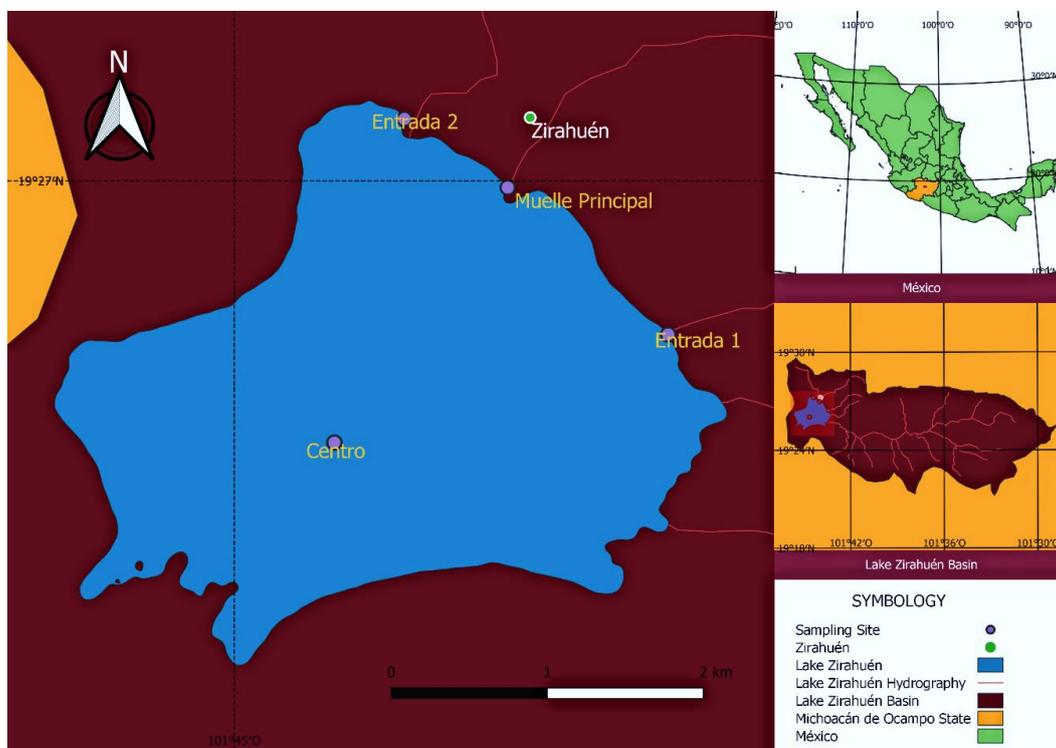


Fig. 1. Location of Lake Zirahuén and sampling sites

### 2.2 The National Sanitation Foundation (NSF) model

The NSF method was developed in 1970 [2], and is a multi-parameter model that uses nine variables to calculate the WQI: fecal coliforms (FC), dissolved oxygen (DO), biochemical oxygen demand (BOD), turbidity (TU), nitrates (NO<sub>3</sub>), potential of hydrogen (pH), total phosphates (TP), change of temperature between air and sample ( $\Delta T$ ), and total dissolved solids (TDS). Each parameter has a weighting, and the value for each measured variable is transformed into a quality value (Q), using a function curve. Q is then multiplied by the weighting of the parameter to give a subtotal, one for each variable (nine in total). These subtotals are summed to obtain the WQI value, which is classified according to the established ranges in Table 2.

## 2.3 The Fuzzy model (FM)

### a) Basic Theory

In 1965 Lofti A. Zadeh developed a form of processing information, in which data could have a degree of partial membership of sets associated with them, calling it Fuzzy Logic (FL) [22]. This means of processing uses validity functions, operations between fuzzy collections, and logic rules. The FM has three stages (Fig. 2): fuzzification, inference process, and defuzzification.

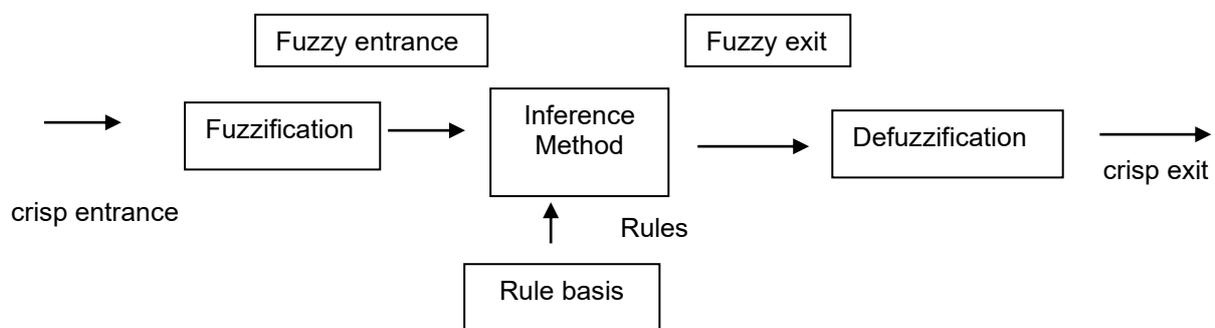


Fig. 2. Fuzzy system [23]

The first stage converts real values (in this case these are the limits of the variables  $BOD_5$ , COD, and TSS established in Table 1) into fuzzy values, assigning membership grades with membership functions (triangular, trapezoidal, S -,  $\pi$  -, beta, and Gaussian shape, are the most typical). We used the triangular function (Fig. 3) as a first approach because it is the easiest function to define; it only needs two base points and a third point, which is the peak of the triangle. The second stage uses a mapping process to make decisions or distinguish patterns. The most important are those developed by [24, 26]. We use the Mamdani fuzzy inference system. The third stage in the process takes the fuzzy outputs and converts them into a single value using mathematical methods such as the centroid, bisector, mean value of maximum (mom), the smallest value of maximum (som), and the largest value of maximum (lom). In this case, the centroid method was used.

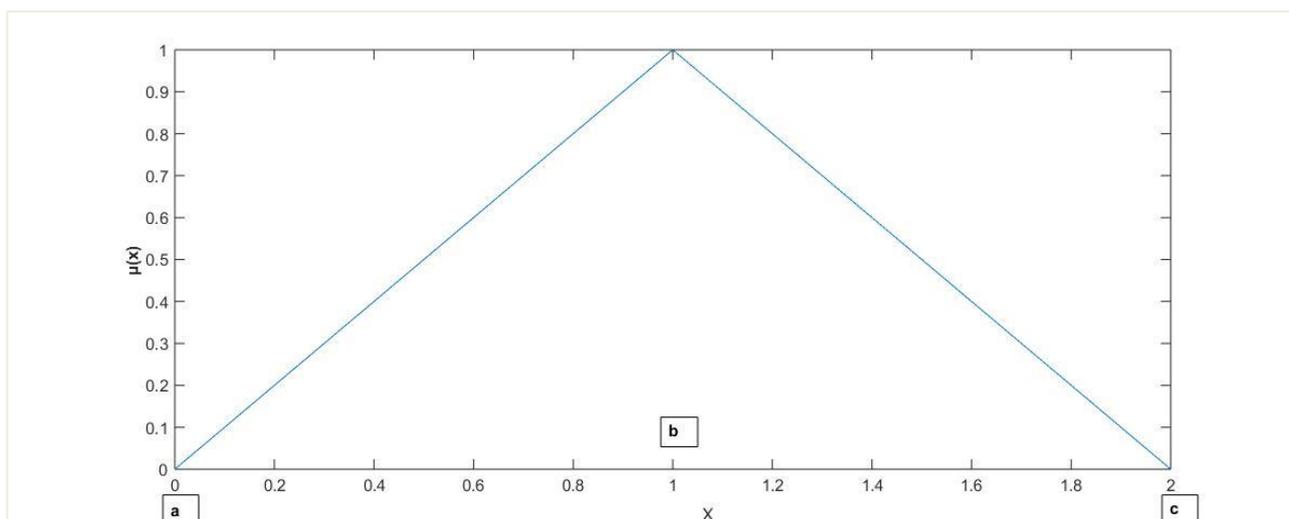


Fig. 3. Triangular membership function

### b) Linguistic concepts

The input variables  $BOD_5$ , COD, and TSS (ranges shown in Table 1), are fuzzy sets representing linguistic concepts: “E” (excellent), “G” (good), “A” (acceptable), “P” (polluted), and “HP” (heavily polluted). The WQI is the output variable, values shown in Table 2. For Lake Zirahuén, the column corresponding to Fishing and Aquatic Life was taken, and the fuzzy sets were defined with the

linguistic concepts: “AAO” (acceptable for all organisms), “AESO” (acceptable except susceptible organisms), “DSO” (doubtful for sensitive organisms), “OVRO” (only for very resistant organisms), and “NA” (not acceptable).

### c) Logical Rules

Logical rules are defined using simple if-then statements, or more complex if-then statements using the and-or connectors. For the output variable, the following cases were defined:

- 1) “AAO”: if the three input variables are “E”
- 2) “AESO”: if the three input variables are “G”, or if the input variables are “G”, “A” or “E”, or if two input variables are “E”, or “A”, and the third is “E”, “G”, or “A”
- 3) “DSO”: if the three input variables are “A”, or if any of the input variables is “P”
- 4) “OVRO”: if the three input variables are “P”, or if two input variables are “P”
- 5) “NA”: if the three input variables are “HP”, or if two input variables are “HP”

### d) Input and output variables membership functions

With the established limits (see Table 1) and the linguistic concepts, we defined the membership functions for each input variable and the output variable (Figs. 4 and 5).

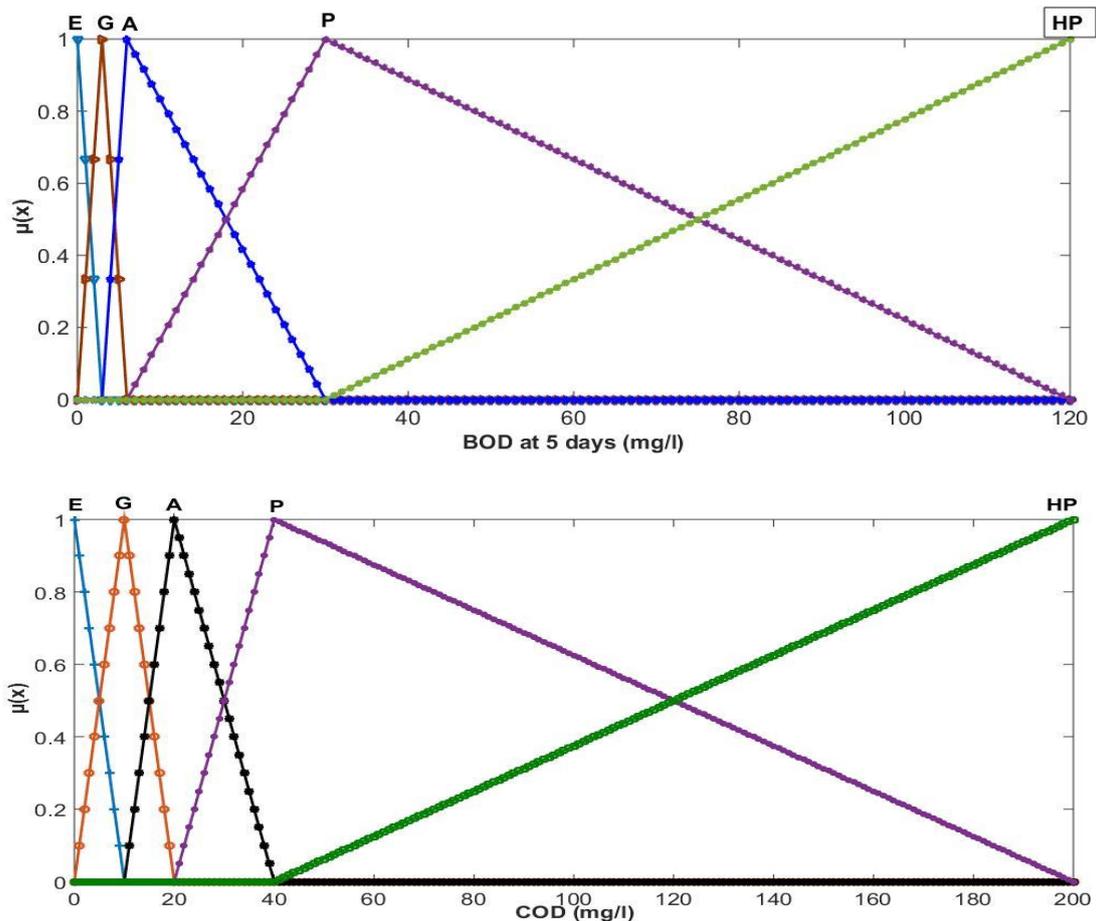


Fig. 4. Membership functions

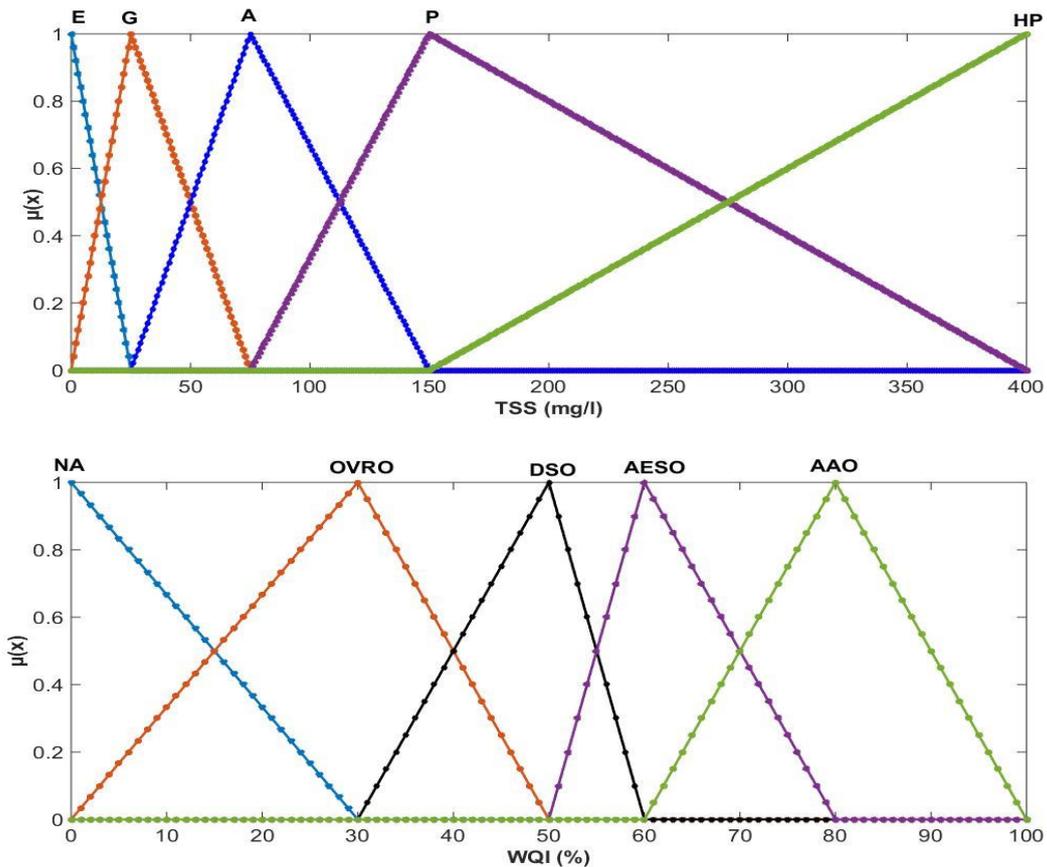


Fig. 5. Membership functions

### 3. Results

During 2018, three monitoring campaigns were conducted at the four sampling sites of Lake Zirahuén (Fig. 1). The measured concentrations of BOD<sub>5</sub>, COD, and TSS are presented in Table 3.

Table 3: 2018 campaigns: Values for each variable

Sampling date	Sampling site	Variables (mg/l)		
		BOD <sub>5</sub>	COD	TSS
2018/04/19	Entrada 1	7	44	9
	Muelle principal	7	60	9
	Entrada 2	7	64	9
	Centro	6	64	9
2018/06/21	Entrada 1	3	62	9
	Muelle principal	3	62	9
	Entrada 2	3	21	9
	Centro	3	77	9
2018/11/08	Entrada 1	6	43	9
	Muelle principal	4	45	9
	Entrada 2	6	42	9
	Centro	6	36	9

The numerical value of the WQI allows us to see what classification range this falls into, and which verbal description is given to define the water quality (Table 2), taking into account that the uses of this lake water are fishing and the conservation of aquatic life. Table 4 shows the results obtained with the two models, from which it can be seen that:

Table 4: WQI results with the two models

Sampling site: Entrada 1			
Sampling date	2018/04/19	2018/06/21	2018/11/08
WQI NSF	77	78	77
WQI FM	46	46	46
Sampling site: Muelle principal			
Sampling date	2018/04/19	2018/06/21	2018/11/08
WQI NSF	74	78	79
WQI FM	46	46	46
Sampling site: Entrada 2			
Sampling date	2018/04/19	2018/06/21	2018/11/08
WQI NSF	74	78	75
WQI FM	46	63	46
Sampling site: Centro			
Sampling date	2018/04/19	2018/06/21	2018/11/08
WQI NSF	76	77	76
WQI FM	46	46	52

**a) The NSF model:**

- a.1) In the April campaign, the best WQI value was obtained at the Entrada 1 site, followed by Centro, with Entrada 2 and Muelle principal being the same.
- a.2) Entrada 1, Muelle principal, and Entrada 2 tie with 78% (of the possible 100 percent) in the June campaign. The last place is Centro, but with a value of only one percent less.
- a.3) The four sites have different values in the November campaign, the best is for Muelle principal (79%), and the worst Entrada 2 (75%).
- a.4) Regarding numerical values, the best site is Entrada 1, then Muelle principal, next Centro, and last Entrada 2.
- a.5) The water quality for all four sampling sites, in all the sampling campaigns, falls into the category “Acceptable except for sensitive organisms” ( $60\% \leq WQI < 80\%$ ), according to the classification of Table 2.

**b) The Fuzzy Model:**

- b.1) For the four sites, the most common value is 46, the only exceptions being Entrada 2 (63%) in the June campaign, and Centro (52%) in the November campaign.
- b.2) The best average value, in terms of percentages, is obtained by Entrada 2 (52), then Centro (48), and then Entrada 1 and Muelle principal, which tie (46).
- b.3) According to Table 2: The water quality values are in the category “Only very resistant organisms” for all the campaigns at Entrada 1 and Muelle principal. For Entrada 2 and Centro, the results for one sampling campaign are higher than the other two campaigns. Only Entrada 2, in the June campaign, achieved a rating of “Acceptable, except for susceptible organisms”, thus matching the results of the NSF model. The WQI for the Centro sampling site in the November campaign was somewhat higher than the values of the other two campaigns: “Doubtful for sensitive organisms”.

**c) Similarities**

- c.1) The only overlap between the two models is that the Centro site in the June campaign had a rating of “Acceptable except for sensitive organisms”, which is the rating that encompasses all the values of the four sampling sites with the NSF model.

**d) Differences**

- d.1) Between the maximum and minimum numerical values of the two models, the difference is notable: 5 points for the NSF model and 17 points for the FM.
- d.2) With the NSF model, there is no homogeneity in the numerical values for any of the sites. With the FM model, Entrada 1 and Muelle principal have homogeneous values.

d.3) As shown in Table 2, the values obtained with the FM are classified into three categories: values between 30 – 49 % are for “Only very resistant organisms”, values between 50 – 59% are “Doubtful for sensitive organisms”, and finally values between 60-69% are “Acceptable, except for susceptible organisms”. While with the NSF model, the water quality for all four sampling sites, in all the sampling campaigns, falls into the category “Acceptable except for sensitive organisms” ( $60\% \leq WQI < 80\%$ ).

d.4) The NSF model produced consistently higher WQI ratings for the lake compared to the FM.

#### 4. Conclusions

This research compared a classical multi-parameter water quality index (NSF) with a three-input fuzzy logic model for Lake Zirahuén. Although both approaches indicated moderate water quality conditions, the fuzzy model consistently produced lower values and identified more restrictive ecological classifications. The differences arise from the structural nature of the models: the NSF index uses compensatory averaging, while the fuzzy model applies rule-based inference that emphasizes deteriorated parameters. Consequently, the fuzzy model acts as a precautionary indicator, more aligned with ecological threshold responses.

The results suggest that, while general physicochemical conditions may appear acceptable, biological communities could already be under stress. Therefore, relying solely on averaging indices may underestimate ecological risk. This paper highlights the value of fuzzy logic as a complementary tool in water quality assessment, particularly for water bodies where conservation of aquatic life is a priority. The approach can be extended to other lakes and regulatory systems, providing a framework that integrates environmental standards with ecological sensitivity.

Future work should incorporate biological indicators and additional physicochemical parameters to further validate the fuzzy model and refine its rule base.

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